Efficiency-Enhancing Taxation and Nonlinear Pricing*

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Abstract

This paper considers commodity taxation in presence of a monopolist that prices access to and usage of a good/service (e.g., with a two-part tariff). We examine how underprovision can be corrected by (ad-valorem) taxation, allowing for differentiated tax rates on usage and access. In a simple model with identical consumers, we show that if the marginal cost is small, the usage fee decreases with the respective tax rate, and consumption increases. Hence, despite underprovision, the optimal ad-valorem tax rate on usage is positive. With heterogeneous consumers, and the monopolist engaging in second-degree price discrimination, we show that a tax targeting usage is optimal when the marginal cost or the information rent left to the high types are small. By contrast, when the marginal cost or the information rent are large, a tax targeting access is optimal. Applications of our model include mobile and fixed telephony, Internet access, energy distribution and transportation.

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1 Introduction

This paper studies commodity taxation when producers have market power and charge multi-part tariffs (e.g., a hook-up and a usage fee). The markets for several goods and services fit this description, including telecommunications, Internet access, energy distribution, transportation services and digital intermediation.[1] Our focus is on the effects of taxation on economic efficiency. We therefore ignore equity and redistribution considerations. We consider a monopolist that charges (a menu of) two-part tariffs and restricting supply of the good in comparison to the competitive equilibrium.[2] We examine unit and ad-valorem commodity taxes, allowing, in the latter case, for differentiated tax rates on usage of and access to the good/service. We characterize conditions such that usage fees *decrease* with either the tax rate applied to usage or that applied to access, and, thus, equilibrium consumption of the good/service *increases* with the tax rates. Hence, we show that despite underprovision, positive (ad-valorem) taxes targeting either usage or access can be optimal.

Our interest in this subject stems from two main reasons. First, the markets mentioned above play a key role in modern economies. Therefore, it is of capital importance to understand how to design the fiscal instruments that concern them. This need has become increasingly evident in recent years. Consider, for example, the case of digital services. Faced with the rapidly growing share of economic activity that takes place online, governments have

[1] Consider broadband Internet access. In the US, the main cable operators have strong market power in high-speed landline services in several local markets. A recent report by the FCC shows that about 20% of homes in the US have access to a single broadband provider for a service of up to 4Mbits/s. The figure rises to 30% and 55% for speeds up to 10Mbit/s and 25Mbits/s, respectively (http://www.washingtonpost.com/blogs/the-switch/wp/2014/09/04/fcc-chairman-a-duopoly-dominates-basic-internet-service-in-america/). As pointed out by, e.g., Economides and Hermalin (2015), landline and mobile ISPs often adopt capped pricing plans whereby the consumer is charged extra fees when exceeding a certain amount of usage (i.e., a three-part tariff). Another example is energy distribution. In more than half of US states (including California, Texas, Massachussets, Virginia and Florida) there is either no or highly limited competition in either the retail gas or electricity distribution market (see https://en.wikipedia.org/wiki/Electricity_provider_switching). Furthermore, as of 2013, former state monopoly EDF retained a market share in the French residential electricity market of more than 80% (http://www.datamonitorenergy.com/2013/03/12/edf-and-gdf-still-dominate-the-french-retail-power-market/). Additional examples are online platforms that provide intermediation services (e.g., eBay, Amazon and Tmall), airlines, health insurance providers and local monopolies such as private road operators and amusement parks.

[2] As we argue below, in our setting there is no loss of generality in restricting attention to this simple tariff scheme.

been looking for ways to tax these services, without stifling their far-reaching contribution to economic growth, and without compromising equity. Yet, so far, little consesus has been reached (see, e.g., European Commission (2014)). Internet access services are a case in point. The US Congress recently passed the Permanent Internet Tax Freedom Act, which strongly restricts governments' ability to tax, essentially on the premise that taxation must result in reduced consumption.[3] By contrast, other governments (e.g., France) have reportedly considered taxing this service.[4] Second, previous literature on commodity taxation in imperfectly competitive markets (briefly surveyed below) has to some extent overlooked the case of nonlinear pricing suppliers. Specifically, it has ignored the question of how to correct the underprovision due to market power. Furthermore, it has ignored the implications of allowing for differentiated tax rates on usage and access, despite this differentiation being feasible (and not rare) in reality.[5] As we show, allowing for this possibility leads to novel and counterintuitive findings.

We begin from a simple setup in which identical consumers acquire a service (e.g., Internet connection) from a monopolistic provider (e.g., an Internet Service Provider) that charges an access fee and a (linear) usage fee. We assume that the service is instrumental for consumption of a final good, provided by downstream firms (e.g., providers of digital content, such as movies, music, news, etc.). In this setup, if the supply of the final good is perfectly competitive, the usage fee is equal to marginal cost, and total surplus is maximized (Oi, 1971). However, if the final good is supplied by imperfectly competitive firms, the monopolist charges a usage fee

[3] The law prohibits federal, state and local governments from taxing Internet access and from imposing discriminatory Internet-only taxes such as bit taxes, bandwidth taxes, and email taxes. See https://www.congress.gov/bill/114th-congress/house-bill/235

[4] Former president Sarkozy proposed in 2008 "an infinitesimal sales tax on new communication methods, like internet access and mobile telephony." (see http://content.time.com/time/world/article/0,8599,1702223,00.html). More recently, minister Pellerin proposed "a new tax on the use of bandwidth by large operators", such as NetFlix and Facebook (http://www.theregister.co.uk/2015/02/11/french_minister_hit_google_facebook_apple_netflix_et_al_with_bandwidth

[5] We here provide some examples. In some US states (e.g., Illinois), subscribers to wireless telecommunication services pay a separate per-line fee on top of VAT and other state-level taxes. Until 2012, the Federal government levied a 3% ad-valorem federal telephone excise tax on long distance calls. Furthermore, several governments (including the US, Brazil, Peru, Tanzania, Pakistan, etc.) apply excise taxes or custom duties on handsets, which are comparable to taxing access to telphony services. Another example are excise taxes on calls, SMS and mobile broadband, charged by governments such as Argentina. See ITU (2013) for further examples of differentiated taxes in the telecom sector.

above marginal cost (Economides and Hermalin, 2015). Therefore, the equilibrium quantity falls short of the optimal one. We obtain that when the marginal cost of the good/service is small enough, the government can correct the inefficiency with commodity *tax*es. Specifically, we find that the optimal ad-valorem tax rate on usage is positive and larger than that on access.

The intution is simple. Because the monopolist recovers consumer surplus via the access fee, it is in its best interest to charge a relatively small usage fee. Assume that, in the absence of taxes, this fee is small enough that the equilibrium quantity is on the inelastic part of the consumer's demand curve.[6] Then an increase in the tax rate applied to usage fee triggers a reduction in the respective fee, and an increase in the access fee. As a result, consumption increases. Nevertheless, for the equilibrium quantity to be on the inelastic part of the demand curve, the marginal cost of production must not be exceedingly large.

We then extend the model to incorporate consumer heterogeneity. Specifically, we assume consumers differ in terms of preferences for the final good (private information). The monopolist engages in second-degree price discrimination, offering a menu of two-part tariffs. In the laissez-faire equilibrium, all consumers except the highest types are charged a usage fee that exceeds marginal cost, i.e., the good is underprovided. In line with our baseline results, we find that a an ad-valorem tax targeting usage produces an increase in consumption for all types, and is thus optimal, provided the marginal cost or the information rent left to the high types are relatively small. This condition is harder to satisfy the greater the share of high types in the population and the more high types differ from the low types. Furthermore, when either the marginal cost or the information rent are large, an ad-valorem tax targeting access is optimal.

Albeit preliminary, we believe our findings have important policy implications. They suggest that taxation can be used as a tool to correct the undeprovision due to market power

[6]This cannot occur in the standard (i.e., linear) monopoly pricing case, because the monopolist always operates on the *elastic* part of the demand curve (Mas-Colell et al., 1995). Indeed, if the monopolist is constrained to adopt linear pricing, the optimal response to a higher tax rate is necessarily to increase the price (Keen, 1998).

when goods or services are priced nonlinearly. However, this calls for differentiated tax rates on usage and access. Indeed, when one takes into account the adjustment in nonlinear tariffs applied by the monopolist, one sees that it is not necessarily the case that the consumer price of the service increases with the tax rates. Therefore, albeit derived from a very stylized model, our results suggest that exempting services like Internet access from taxation may not be justified, at least from the perspective of economic efficiency.

Related Literature. There is a vast literature analyzing commodity taxation in imperfectly competitive markets (see Auerbach and Hines (2002) for a survey). This literature has studied extensively the properties of specific (i.e., unit) and ad-valorem taxes, focusing on both incidence and efficiency considerations. The primary focus of the literature has been on firms engaging in linear pricing strategies. Anderson et al. (2001) show that, while ad-valorem taxes are unambiguosly welfare superior in a monopoly setup, specific taxes may be preferable in oligopoly, depending on the strategic interactions between firms. However, taxation is generally not efficiency-enhancing: if producers underprovide the good in the laissez-faire equilibrium, the optimal corrective instrument is a subsidy. Carbonnier (2014) extends these findings by allowing for non-linear tax schedules. He finds that, if appropriately designed, nonlinear (price-dependent) taxes can induce price reductions and, thus, increase efficiency. Peitz and Reisinger (2014) study commodity taxation in markets with downstream and upstream oligopoly. They find that ad valorem taxes dominate specific taxes and are better levied downstream than upstream. Cremer and Thisse (1994) show that if product quality is endogenous, an ad-valorem tax may induce a reduction in equilibrium prices. Furthermore, a small tax dominates a small subsidy.

A far smaller number of papers analyzes commodity taxation in markets where firms set nonlinear prices. In a model of second-degree price discrimination, Laffont (1987) analyzes the properties of unit taxes. McCalman (2010) analyzes trade policy instruments in a similar setup. Jensen and Schjelderup (2011) compare unit and ad-valorem taxation. They find that an increase in either tax rate raises usage fees for all consumers, but the access fee may fall.

Nonetheless, consumption decreases with either tax rate. In line with general findings in previous literature, ad-valorem taxes are preferable to unit taxes. The key difference with respect to our framework is that we allow for differentiated ad-valorem tax rates on usage and access fees. Furthermore, we explicitly focus on efficiency-enhancing fiscal policy.

Finally, Kind et al. (2008) study efficiency enhancing taxation in two-sided markets. They show that, under certain conditions, consumption of one good may increase with the ad valorem tax rate. Hence, if the monopolist underprovides the good, ad valorem taxation is efficiency-enhancing. This result is due to the consumption externalities between the two markets: by reducing the price of one good (and, thus, increasing consumption) the monopolist may increase consumers' willingness to pay for the other good. By contrast, unit taxes can increase efficiency only with overprovision.

The remainder of the paper is organized as follows. We present the baseline setup in Section 2, we then solve the model and present the optimal tax rates in Section 3. Section 4 considers the extended setup studying second-degree price discrimination. Section 5 concludes. Unless otherwise stated, proofs are relegated to the Appendix.

2 Representative consumer setup

We consider two goods: good 1 is an intermediate good that consumers combine (in fixed proportions) with good 2. Consumers do not obtain utility from 1 directly, but it is essential to consume 2. We assume the following market structure: there is a monopolist providing good 1 and N firms provide good 2. For simplicity, we assume there exist N differentiated varieties of good 2, that each firm selling good 2 provides a unique one (i.e., each is a monopolist for its own variety), and that these firms are identical in all other respects.

As a driving example, we refer to good 1 as data transfered on last-mile Internet connection by an Internet Service Provider (e.g. ComCast, Orange, Deutsche Telekom...) and to good 2 as the content (e.g. movies) embedded in such data, which is provided by N online streaming websites (e.g., NetFlix, YouTube, Amazon Prime, HBO Go, etc.). However, the setup is consistent with several other examples. For instance, one can think of good 1 as an energy good (e.g., natural gas or electricity) that consumers combine with another good (e.g., household appliances) for consumption. Furthermore, 1 could be a transportation good (e.g. vehicle-miles on tolled roads), that consumers combine with some good available at their destination (e.g., hotels or restaurants). Other relevant examples include credit cards, online intermediation platforms (e.g., ebay) and shopping clubs.

Our baseline setup is similar to Economides and Hermalin (forthcoming), who study nonlinear pricing by a monopolist providing an essential service (specifically, internet connection) in the case where complementary goods are provided by firms that are not price takers. The key difference is that we consider commodity taxation.

Consumers. There is a unit mass of identical consumers, whose utility function is specified as follows

$$U = \sum_{i=1}^{N} \int_0^{x_j} (\alpha_j - r_j)\, dr_j + y.$$

The first term refers to consumption of good 2 (e.g., online content: movies, music, etc.). α_j is a positive parameter and x_j is the consumed quantity of the j-th variety of 2, where $j = 1, .., N$. The second term y is consumption of a numeraire good. In order to consume $x \equiv \sum_{j=1}^{N} x_j$ units, we assume the consumer needs to purchase x units of good 1 (e.g., data transfered on the internet connection). Therefore, the consumer's budget constraint is

$$I - (T_1(x) + T_2(x)) = y,$$

where I is exogenous income and $T_i(x),\ i = 1, 2$ is the total payment to firms 1 and 2.

Suppliers of good 2. We assume each firm of type 2 sets linear tariffs and denote by q_j the price charged by the j-th firm selling good 2. For simplicity, we ignore costs for firms of type 2. Hence the profit of a type-2 firm is $\pi_{2j} = T_{2j}(x) = q_j x_j$. Without loss of generality, we normalize the number of type-2 firms to one, i.e. set $N = 1$. Consequently, we drop subscripts when referring to prices and quantities of good 2.

Supplier of good 1. We assume the provider of good 1 charges a two-part tariff to consumers. Specifically, we have

$$T_1(x) = \begin{cases} A + px & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

where A is the " access" or " hook-up" fee that consumers pay irrespectively of the actual quantity x consumed (except if $x = 0$), and p is the price paid for every unit. As we show in Appendix B, there is no loss of generality in restricting attention to two-part tariffs in this simple setup. We assume firm 1 produces at a constant marginal cost, denoted c. Fixed costs are ignored for simplicity. Firm 1's profit is $\pi_1 = T_1(x) - b_1(x) - cx$, where $b_1(x)$ denotes firm 1's tax burden (to be specified below).

Note that because firms that provide good 2 are not price takers, and because the consumer's willingness to pay for it depends on the prices set by 1, q_j depends on the tariff $T_1(x)$ chosen by 1. This implies that firm 1 has an incentive to manipulate q_j through its own tariff structure.

Government and Social Welfare. Several kinds of commodity taxes could be considered. In order to focus on the most salient results, we restrict attention to commodity taxes levied on good 1.[7] Specifically, we consider ad-valorem and specific taxes (to be included in the next revision of this paper). In the former case, we assume the government sets *differentiated* ad-valorem taxes: there is a tax rate t_A applying to the hookup fee A, and a tax rate t_p applying to the unit price p. To simplify the analytics below, we assume tax rates are nonnegative (i.e., we rule out subsidies) and have an upper bound equal to one, i.e. $t_k \in [0;1]$, $k = p, A$. These restrictions are of no consequence for the main results.

[7] It is easy to show that taxation of good 2 does generate additional results.

Denoting by R the government's revenue, we have

$$R = b_1(x) = t_A A + t_p px.$$

We can therefore rewrite firm 1's profit as $\pi_1 = (1 - t_A) A + (1 - t_p) px$.

Social welfare is given by the sum of consumer surplus, firm profits and government revenues. That is,

$$W = U + \sum_{i=1,2} \pi_i + R.$$

Using the fact that taxes and tariffs are simply transfers, welfare immediately boils down to

$$W = \int_0^x (\alpha - r)\, dr - cx + I.$$

To avoid situations in which the socially optimal x is zero, we assume throughout that $\alpha > c$. Under this assumption, the socially optimal consumption level $x^*$is such that $\frac{\partial W}{\partial x} = 0$, yielding

$$\alpha - x = c \Rightarrow x^* = \alpha - c.$$

In words, the socially optimal consumption level is such that marginal utility of an additional unit of goods 1 and 2 is equal to marginal cost.

Observe that our setup is such that consumers combine goods from two monopolistic suppliers. However, because the goods are complementary and consumed in fixed proportions, social welfare depends only on x. Thus, to correct the market failure, the government only needs to control one quantity. We will return to this point below.

Timing. At the start of the game, the government sets tax rates t_A and t_p. Next, firm 1 sets A and p. Then, firms 2 sets q and, finally, consumers choose x. The solution concept is Subgame-Perfect Nash Equilibrium. We solve the model backwards.

2.1 Solving the model

2.1.1 Equilibrium

Anticipating that the consumer's budget constraint is binding and replacing for y, her problem can be written as

$$\max_{x} \quad U = \begin{cases} \int_0^x (\alpha - r)\, dr + I - A - px - qx & \text{if } x > 0 \\ I & \text{otherwise} \end{cases}.$$

Under the condition that $A \leq \int_0^x (\alpha - r)\, dr - px - qx$, the solution to this problem is positive. We anticipate that this condition holds in equilibrium (if it did not, we would have $x^e = 0$ and firms would not make positive profit). The first order condition reads

$$\frac{\partial U}{\partial x} = \alpha - x - p - q = 0,$$

and the unique solution is $x(q,p) = \alpha - q - p$.

Consider now the problem faced by firm 2. Anticipating consumer behavior and taking as given the tariffs set by firm 1, the problem is

$$\max_{p} \quad qx = q\left(\alpha - q - p\right),$$

which yields $q(p) = \frac{\alpha - p}{2}$. Observe that the equilibrium price set by firm 2 decreases with p. The demand for firm 1 is therefore

$$x(p) = \frac{\alpha - p}{2}.$$

Consider now the problem faced by Firm 1, which writes

$$\max_{A,p} \quad \begin{cases} A\left(1 - t_A\right) + p\left(1 - t_p\right) x(p) - cx(p) & \text{if } x(p) > 0 \\ 0 & \text{otherwise} \end{cases}.$$

As mentioned above, $x(p) > 0$ if and only if $A \leq \int_0^{x(p)} (\alpha - r)\, dr - px(p) - q(p) \cdot x(p)$ holds. Profit maximization by 1 thus calls for setting

$$A = \int_0^{x(p)} (\alpha - r)\, dr - px(p) - q(p) \cdot x(p).$$

Replacing $x(p)$ and $q(p)$, one obtains

$$A = \int_0^{\frac{\alpha - p}{2}} (\alpha - r)\, dr - p\frac{\alpha - p}{2} - \left(\frac{\alpha - p}{2}\right)^2 = \frac{(\alpha - p)^2}{8}.$$

We can therefore rewrite firm 1's problem as

$$\max_p \quad \frac{(\alpha - p)^2}{8} (1 - t_A) + (p(1 - t_p) - c) \left(\frac{\alpha - p}{2}\right).$$

The solution to this problem is

$$p^e = \frac{\alpha (1 - 2t_p + t_A) + 2c}{3 + t_A - 4t_p}, \tag{1}$$

which implies that

$$x^e = \frac{\alpha (1 - t_p) - c}{3 + t_A - 4t_p}. \tag{2}$$

To streamline the presentation, we restrict attention to equilibria in which $p^e \geq 0$ and $x^e \geq 0$, which requires that $t_p \leq min \left[1 - \frac{c}{\alpha}; \frac{1+t_A}{2} + \frac{c}{\alpha}; \frac{3+t_A}{4}\right]$.[8]

Two remarks are in order before proceeding. First, note that, without taxes, firm 1 charges a per unit fee p^e which is strictly larger than marginal cost c. Indeed, when $t_A = t_p = 0$, we have $p^e = \frac{\alpha+2c}{3} > c$. As shown by Economides and Hermalin (forthcoming), this is a consequence of imperfect competition in the market for the complementary good 2. A marginal increase in p triggers a reduction in the price of good 2. Because the price increase is partly absorbed by producers, charging $p^e > c$ is optimal for firm 1, despite the fact that consumer surplus

[8]Equilibria with positive prices and quantities could also be obtained for $t_p > max \left[1 - \frac{c}{\alpha}; \frac{1+t_A}{2} + \frac{c}{\alpha}; \frac{3+t_A}{4}\right]$. However, for small values of c, this would require $t_p > 1$. We therefore focus on the first set of conditions.

can be recovered via the access fee. Second, the distortion produced by the fact that $p^e > c$ creates the necessary conditions for government internvention. Indeed, if sector 2 is perfectly competitive (i.e., $q = 0$), firm 1 sets $p^e = c$ without taxes and $x^e = x^*$ as a result. This implies that the laissez-faire allocation is optimal, and optimal tax rates are zero.

From (1) and (2), we can derive the following comparative statics

$$\frac{\partial p^e}{\partial t_p} = -\frac{2\left(\alpha\left(1-t_A\right)-4c\right)}{\left(3+t_A-4t_p\right)^2}, \qquad \frac{\partial p^e}{\partial t_A} = \frac{2\left(\alpha\left(1-t_p\right)-c\right)}{\left(3+t_A-4t_p\right)^2}, \tag{3}$$

$$\frac{\partial A^e}{\partial t_p} = 2\left(\alpha - p^e\right)\frac{\alpha\left(1-t_A\right)-4c}{\left(3+t_A-4t_p\right)^2}, \qquad \frac{\partial A^e}{\partial t_A} = -2\left(\alpha - p^e\right)\frac{\alpha\left(1-t_p\right)-c}{\left(3+t_A-4t_p\right)^2}, \tag{4}$$

$$\frac{\partial x^e}{\partial t_p} = \frac{\alpha\left(1-t_A\right)-4c}{\left(3+t_A-4t_p\right)^2}, \qquad \frac{\partial x^e}{\partial t_A} = -\frac{\alpha\left(1-t_p\right)-c}{\left(3+t_A-4t_p\right)^2}. \tag{5}$$

These expressions show that the monopolist's response to changes in the commodity tax rates may diverge from canonical results. In particular, (3) suggest that the response to an increase in t_p is to raise the unit price p only if either the marginal cost c or the tax rate t_A are large enough. Otherwise, the monopolist responds by reducing p, which results in an increase in equliibrium consumption x^e (see (5)). Specifically, we have

$$\frac{\partial p^e}{\partial t_p} < 0, \frac{\partial x^e}{\partial t_p} > 0 \Leftrightarrow \frac{\alpha}{4}\left(1-t_A\right) > c. \tag{6}$$

To grasp the intuition, it is useful to start by considering the case of linear pricing, i.e., $A = 0$. The monopolist would then set p equating marginal cost to marginal revenue, and total revenues $p \cdot x(p)$ would not increase if p were to be raised further. As a result, the response to a marginal increase in the ad valorem tax rate t_p would be to raise p. However, with a two-part tariff, because the monopolist can recover consumer surplus via A, it is in its best interest to charge a relatively small p. Indeed, we find that, when evaluated at p^e, $p \cdot x(p)$ is increasing in p if and only if the right hand side of (6) holds. Under this condition, the monopolist's optimal response to a marginal increase in t_p is to reduce p, because this minimizes the extra tax burden. Nevertheless, the second inequality in (6) fails when either c or t_A are relatively large. The intuition is that p^e is increasing in both c and t_A: a higher marginal cost increases incentives to restrict supply, whereas a higher t_A implies that a larger share of consumer surplus has to be relinquished in the form of taxes, diminshing the incentive to keep p low. When either of these variables is large enough, p^e is big enough that increasing it further reduces the revenues $p \cdot x(p)$. Therefore, p^e decreases with t_p.

Turn now to the effect of the tax rate on access, t_A. Because $t_p \leq 1 - \frac{c}{\alpha}$, we have

$$\frac{\partial p^e}{\partial t_A} > 0, \frac{\partial x^e}{\partial t_A} < 0. \tag{7}$$

Therefore, a marginal increase in the tax rate on access always increases the per unit fee, thereby reducing the equilibrium quantity. The intuition is that a higher tax on access reduces the monopolist's incentive to maintain a low usage fee. This is because a larger share of the surplus extracted from consumers via the access fee A is taxed away.

2.1.2 Optimal tax rates

We now turn to the characterization of the optimal fiscal policy in this setup. To facilitate comparison with standard results, we will start by imposing the restriction that ad-valorem tax rates are uniform. That is, there is no differentiation between the rate applied to access and applied to usage. We then turn to the case of differentiated tax rates.

2.1.3 Uniform ad-valorem taxes

Let us momentarily impose the restriction that $t_A = t_p = t$. We can rewrite (1) and (2) as

$$p^e = \frac{\alpha}{3} + \frac{2c}{3(1-t)}, \qquad q^e = \frac{\alpha}{3} - \frac{c}{3\,(1-t)}, \qquad x^e = \frac{\alpha}{3} - \frac{c}{3\,(1-t)}.$$

From which one can immediately derive the effect of the tax rate on the equilibrium prices and quantity, which are in line with standard results: a higher tax rate increases the per unit fee and reduces the equilibrium quantity. This immediately leads to the following conclusion

Lemma 1. *The optimal uniform ad-valorem tax rate t cannot be positive.*

Proof. The social welfare function is $\int_0^{x^e} (\alpha - r)\, dr - cx^e + I$. Taking the derivative of this function with respect to t we have $\frac{\partial W}{\partial t} = (p^e + q^e - c)\, \frac{\partial x^e}{\partial t}$. Because $p^e + q^e > c$ for any positive t and $\frac{\partial x^e}{\partial t} = -\frac{c}{3(1-t)^2} < 0$, we have $\frac{\partial W}{\partial t} < 0$ for any $t \geq 0$. □

2.1.4 Differentiated ad-valorem taxes

Consider now the case of differentiated taxes. The government's problem writes

$$\max_{t_A, t_p} W = \int_0^{x^e} (\alpha - r)\, dr - cx^e + I. \tag{8}$$

The first order derivatives of the objective function are

$$\frac{\partial W}{\partial t_k} = (p^e + q^e - c)\, \frac{\partial x^e}{\partial t_k}, \quad k = A, p.$$

These derivatives simply tell us that social welfare increases with consumption of goods 1 and 2 if and only if the combined (unit) consumer prices are larger than the marginal cost of good 1 (recall that the marginal cost of 2 is set to zero). Thus, when $p^e + q^e > c$, any change in tax rates t_p and t_A such that consumption increases is welfare-enhancing.

As a first step towards characterization of the optimal tax rates, in Lemma 2 we take t_A as given, and present the optimal t_p (conditionally on all prices and quantities being nonnegative).

Lemma 2. *Consider differentiated ad valorem tax rates t_A and t_p. For any $t_A \in [0; 1]$,*

- *if $\frac{\alpha(1-t_A)}{4} > c$, the optimal ad-valorem tax rate applied to usage is $t_p = \frac{1+t_A}{2} + \frac{c}{\alpha} > t_A$. As a result, we have $p^e = 0$ and $q^e = x^e = \frac{\alpha}{2}$.*
- *if $\frac{\alpha(1-t_A)}{4} \leq c$, the optimal ad-valorem tax rate applied to usage is $t_p = 0$. As a result, we have $p^e = \frac{\alpha(1+t_A)+2c}{3+t_A}$ and $q^e = x^e = \frac{\alpha - c}{3+t_A}$.*

Lemma 2 establishes that when the marginal cost of supplying good 1 is not too large, the optimal (ad-valorem) tax rate on usage is strictly positive, and larger than the tax rate applied to the hookup fee. Hence, we find that the optimal corrective instrument is a *tax* on usage (i.e., consumption) of good 1. This stands in contrast with canonical results, which suggest that governments should subsidize goods for which supply is restricted due to market power. The intuition follows from above: when marginal cost is relatively small, a nonlinear pricing monopolist responds to an increase in the ad-valorem tax on usage by reducing the per unit price. By contrast, when the marginal cost is relatively large, we find that taxing usage cannot be optimal. This is in accordance with canonical results.

We can now turn to the characterization of the optimal set of policy instruments. We obtain the following

Proposition 1. *The optimal ad-valorem tax rates are as follows:*

- *if* $\frac{\alpha}{4} > c$, $t_p = \frac{1+t_A}{2} + \frac{c}{\alpha}$ *and* $t_A \in [0; 1 - \frac{4c}{\alpha})$.
- *if* $\frac{\alpha}{4} \leq c$, $t_p = t_A = 0$.

We therefore conclude that welfare-maximization calls for *taxing* the monopolist providing good 1, except if the marginal cost of production is quite large. Specifically, when $\frac{\alpha}{4} > c$, the optimal tax rate applied to the unit price p is strictly positive, whereas the tax rate applied to the access fee A is undetermined (and may be set it to zero without welfare loss). By contrast, when $\frac{\alpha}{4} \leq c$, positive taxes cannot be optimal. In fact, if we allowed the government to subsidize good 1, we would get that the optimal tax rates are negative.

3 Heterogeneous consumers and second-degree price discrimination

We have so far made the simplifying assumption that consumers are identical. We now extend the model to incorporate consumer heterogeneity. Specifically, we assume consumers differ with respect to their preferences for the final good, assumed private information. The monopolist firm 1 offers a menu of nonlinear tariffs. To simplify, we assume that supply of the final good 2 is perfectly competitive, and normalize its price to zero. Therefore, our problem is a standard problem of second-degree price discrimination (see, e.g., Maskin and Riley, 1984), except for the fact that we incorporate commodity taxes.

3.1 Setup

There are two types of consumers, high and low, denoted h and l. The utility function is specified as

$$U_i = \int_0^x (\alpha_i - r)\, dr + y,\ i = h, l$$

with $\alpha_h > \alpha_l > 0$. We assume α_i is private information. The share of consumers of type h (resp. l) is denoted by v (resp. $1 - v$). We normalize the total quantity of consumers to one and assume that consumers have identical income I.

As anticipated, firm 1 proposes a menu of two-part tariffs $\{T_1(x, i)\}$, $i = h, l$. Specifically, we have

$$T_1(x, i) = A_i + p_i x \quad i = h, l.$$

As usual, consumers self-select on the tariff which is most convenient given their type. Similarly to the baseline model, it can be shown (see Appendix B) that there is no loss of generality in restricting attention to menus of two-part tariffs. Because we assume perfect competition in the market for good 2, its price is equal to marginal cost, i.e., $q = 0$.

As in the baseline setup, we allow for differentiated ad-valorem tax rates, denoted t_A and t_p. Anticipating that the tariff menu $\{T_1(x, i)\}$ satisfies the incentive and participation constraints (we assume it is optimal to serve both types), tax revenue (equal to firm 1's total tax burden) is

$$R = t_p (p_h x_h v + p_l x_l (1 - v)) + t_A (A_h v + A_l (1 - v)), \tag{9}$$

where x_i denotes equilibrium consumption of type $i = h, l$.

Social welfare is given by the sum of consumer surplus, firm profits and government revenues. That is,

$$W = \sum_{i=h,l} U_i + \sum_{i=1,2} \pi_i + R,$$

Tariffs paid by consumers are simply transfers to firm 1. Similarly, commodity taxes are transfers to the government. Therefore, social welfare boils down to

$$W = v \left(\int_0^{x_h} (\alpha_h - r)\, dr - c x_h \right) + (1 - v) \left(\int_0^{x_l} (\alpha_l - r)\, dr - c x_l \right) + I \tag{10}$$

To avoid situations in which the socially optimal quantities $x_i, i = h, l$ are zero, we assume throughout that $\alpha_h > \alpha_l > c$. The socially optimal consumption level $x_i^*, i = h, l$ is therefore such that

$$\alpha_i - x_i = c \Rightarrow x_i^* = \alpha_i - c \quad i = h, l.$$

3.2 Solving the model

We solve the model backwards. Consider the problem of a type-i consumer that chooses the tariff intended for type $j = h, l$:

$$\max_x \quad \begin{cases} \int_0^x (\alpha_i - r)\, dr + I - A_j - p_j x & \text{if } x > 0 \\ I & \text{otherwise} \end{cases}, \quad i, j = h, l$$

Under the condition that $A_j \leq \int_0^x (\alpha_i - r)\, dr - p_j x$, the solution to this problem is positive. Assuming that firm 1 serves all types, this condition holds in equilibrium. The first order condition reads therefore

$$\alpha_i - x - p_j = 0, \quad \forall i, j,$$

and it is easily established that second order conditions hold. Hence, the solution is

$$x_{ij} = \alpha_i - p_j.$$

To simplify notation, when $i = j$, i.e. when the consumer chooses the tariff intended for his/her own type, we drop the double index and denote consumption by x_i $i = h, l$. Recall that $q = 0$.

Consider now firm 1's problem:

$$\max_{A_h, p_h, A_l, p_l} \quad v\left(\left(1 - t_A\right) A_h + \left(1 - t_p\right) p_h x_h\right) + (1 - v)\left(\left(1 - t_A\right) A_l + \left(1 - t_p\right) p_l x_l\right) \qquad \text{s.t.}$$

$$V_i \geq I \quad i = h, l \tag{11}$$

$$V_i \geq V_{ij} \quad i, j = h, l \tag{12}$$

where

$$V_i = \int_0^{x_i} \left(\alpha_i - r\right) dr + I - A_i - p_i x_i - q x_i \quad i = h, l,$$

is the indirect utility of a type-i consumer adopting the tariff intended for him, and

$$V_{ij} = \int_0^{x_{ij}} \left(\alpha_i - r\right) dr + I - A_j - p_j x_{ij} - q x_{ij} \quad i, j = h, l,\ i \neq j,$$

is the indirect utility of a mimicker. Recall that any consumer obtains utility I if not buying the good. Following standard steps (see, e.g., Laffont and Martimort, 2001), it can be shown that there is no loss of generality in (i) treating constraints (11) for $i = h$ and (12) for $i = l$ and $j = h$ as slack, and (ii) treating the remaining two constraints as binding. Hence, we rewrite firm 1's problem as

$$\max_{A_h, p_h, A_l, p_l} \quad v\left(\left(1 - t_A\right) A_h + \left(1 - t_p\right) p_h x_h\right) + (1 - v)\left(\left(1 - t_A\right) A_l + \left(1 - t_p\right) p_l\right) \qquad \text{s.t.}$$

$$V_l = I \tag{13}$$

$$V_h = V_{hl}. \tag{14}$$

Using (13) we write

$$A_l = \int_0^{x_l} \left(\alpha_l - r\right) dr - p_l x_l, \tag{15}$$

and from (14)

$$A_h = A_l + \int_0^{x_h} (\alpha_h - r)\, dr - p_h x_h - \left(\int_0^{x_{hl}} (\alpha_h - r)\, dr - p_l x_{hl} \right). \tag{16}$$

In words, the access fee charged to low types extracts their entire surplus. As for the high types, some information rent has to be granted (this is represented by the term in square brackets on the right hand side). We can therefore rewrite the firm 1's problem as

$$\begin{aligned} \max_{p_h, p_l} \quad & v\,(1 - t_A) \left(\int_0^{x_l} (\alpha_l - r)\, dr - p_l x_l + \int_0^{x_h} (\alpha_h - r)\, dr - p_h x_h \right) + \\ & -v\,(1 - t_A) \left(\int_0^{x_{hl}} (\alpha_h - r)\, dr - p_l x_{hl} \right) + v\,(1 - t_p)\, p_h x_h + \\ & + (1 - v)\,(1 - t_A) \left(\int_0^{x_l} (\alpha_l - r)\, dr - p_l x_l \right) + (1 - v)\,(1 - t_p)\, p_l x_l. \end{aligned}$$

Let us look at the first order conditions of this problem. After some simplification and using $\frac{dx_h}{dp_h} = \frac{dx_l}{dp_l} = \frac{dx_{hl}}{dp_l} = -1$, we get

$$\frac{d\pi}{dp_h} = (1 - t_A)\,(-\alpha_h + p_h) + (1 - t_p)\,(x_h - p_h) + c = 0$$

and

$$\frac{d\pi}{dp_l} = v\,(1 - t_A)\,(\alpha_h - \alpha_l) + (1 - v)\,[x_l\,(t_A - t_p) - (1 - t_p)\, p_l] + c\,(1 - v) = 0.$$

Rearranging the above expressions, we obtain

$$p_h = \frac{\alpha_h\,(t_A - t_p) + c}{1 - 2t_p + t_A}, \tag{17}$$

$$p_l = \frac{\frac{v}{1-v}\,(\alpha_h - \alpha_l)\,(1 - t_A) + \alpha_l\,(t_A - t_p) + c}{1 - 2t_p + t_A}. \tag{18}$$

As a result, equilibrium consumption levels are

$$x_h = \frac{\alpha_h (1 - t_p) - c}{1 - 2t_p + t_A}, \tag{19}$$

$$x_l = \frac{\alpha_l (1 - t_p) - \frac{v}{1-v} (\alpha_h - \alpha_l)(1 - t_A) - c}{1 - 2t_p + t_A}. \tag{20}$$

We restrict attention to equilibria such that all quantities are positive. Using (19) and (20), this implies the following restrictions on tax rates: $t_A \in [0, 1]$ and $0 \leq t_p < min \left[1 - \frac{c}{\alpha_l} - \frac{v}{1-v} \left(\frac{\alpha_h - \alpha_l}{\alpha_l}\right); \frac{1+t_A}{2}\right]$.

Derivation of (17) and (18) gives us the following comparative statics

$$\frac{\partial p_h}{\partial t_p} = -\frac{\alpha_h (1 - t_A) - 2c}{(1 - 2t_p + t_A)^2}, \qquad \frac{\partial p_h}{\partial t_A} = \frac{\alpha_h (1 - t_p) - c}{(1 - 2t_p + t_A)^2},$$

$$\frac{\partial p_l}{\partial t_p} = \frac{\left(\frac{2v}{1-v} (\alpha_h - \alpha_l) - \alpha_l\right)(1 - t_A) + 2c}{(1 - 2t_p + t_A)^2}, \qquad \frac{\partial p_l}{\partial t_A} = \frac{-\left(\frac{2v}{1-v} (\alpha_h - \alpha_l) - \alpha_l\right)(1 - t_p) - c}{(1 - 2t_p + t_A)^2}$$

Except for $\frac{\partial p_h}{\partial t_A}$, which is positive because $x_h > 0$ by assumption, the sign of these derivatives is ambiguous in general. It is possible that unit prices intended for both types decrease (and consumed quantities increase) with the tax rate applied to the per unit price. Indeed, we get

$$\frac{\partial p_h}{\partial t_p} < 0, \frac{\partial x_h}{\partial t_p} > 0 \Leftrightarrow c < (1 - t_A) \frac{\alpha_h}{2}, \tag{21}$$

$$\frac{\partial p_l}{\partial t_p} < 0, \frac{\partial x_l}{\partial t_p} > 0 \Leftrightarrow c < (1 - t_A) \left(\frac{\alpha_l}{2} - \frac{v}{1 - v} (\alpha_h - \alpha_l)\right). \tag{22}$$

The intutition is similar to that for (6). The monopolist's optimal response to a marginal increase in t_p is to raise p_h (resp. p_l) if and only if doing so reduces the burden directly associated to the tax increase. Namely, the first term in brackets in (9).[9] Now, $p_h x_h v$ (resp.

[9]Indeed, $p_h x_h v + p_l x_l (1 - v)$ increases with p_h (resp. p_l), when evaluated in (17) and (18), if and only if

$p_l x_l (1 - v)$) increases with p_h (resp. p_l) if and only if the initial value of p_h (p_l) is small enough. Therefore, to characterize the conditions such that p_h and p_l decrease with t_p, we need to look at under which conditions these unit prices are small. As indicated by (17) and (18), p_h and p_l increase with the marginal cost c. Furthermore, p_l increases with the distortion ascribing to the information rent (see the last term on the right hand side of (22)). This distortion is relatively small when either α_l is sufficiently close to α_h (i.e. the difference between consumers is relatively mild) or v is small (i.e. there are relatively few high types and many low types).

Consider now the effect of the tax rate applied to the access fee A. We have

$$\frac{\partial p_h}{\partial t_A} > 0, \frac{\partial x_h}{\partial t_A} < 0, \tag{23}$$

$$\frac{\partial p_l}{\partial t_A} < 0, \frac{\partial x_l}{\partial t_A} > 0 \Leftrightarrow c > 2(1 - t_p)\left(\frac{\alpha_l}{2} - \frac{v}{1-v}(\alpha_h - \alpha_l)\right). \tag{24}$$

Therefore, a marginal increase in the tax rate applied to access always increases the usage fee intended for the high type, but may either increase or decrease in the usage fee intended for the low type. The intuition is as follows. An increase in t_A reduces the revenue the monopolist can extract through A. As a result, it reduces the incentive to restrain the usage fees. This effect is stronger the larger the marginal cost and the smaller the low-type consumer's marginal surplus. However, there is also a second, and more subtle, effect of the tax increase: it affects the monopolist's incentives to reduce the high type's information rent. Intuitively, the higher t_A, the weaker the incentive to reduce this rent by raising the usage fee intended for the low type (and distort the quantity they consume downward). The net effect of these two forces explains the effect of t_A on p_h and p_l. Because nobody wants to mimick type h in equilibrium, only the first effect is relevant for p_h, and so the sign of (23) is unambiguous. By contrast, the sign of (24) depends on which effect dominates. Specifically, when either the marginal cost or the information rent to the high type are large (last term on the right hand side of (24)), the second effect dominates, so p_l decreases with t_A.

the condition on the right hand side of (21) (resp. (22)) holds.

3.3 Optimal tax rates

Let us now turn to the characterization of the optimal ad-valorem tax rates t_A and t_p. Observe, to begin, that when no taxes are applied, i.e. $t_A = t_p = 0$, we get the standard outcome

$$p_h = c, \qquad p_l = \frac{v}{1-v}(\alpha_h - \alpha_l) + c. \tag{25}$$

Thus, in the laissez-faire equilibrium, the monopolist charges a per unit price equal to marginal cost to the high type, whereas it distorts the price charged to the low type in order to reduce the high-type's information rent (Maskin and Riley, 1984). This distortion calls for government intervention: in principle, there is room for increasing social welfare by an appropriate design of fiscal instruments.

3.3.1 Uniform ad-valorem taxes

To facilitate comparison with standard results, we begin from the case of a uniform ad valorem tax rate. Therefore $t_p = t_A = t$. With this restriction in place, we find the following straightforward result.

Lemma 3. *Consider the setting with heterogeneous consumer types and assume there is a uniform ad valorem tax rate t. The optimal t cannot be positive.*

Proof. Consider the first-order derivative of W in (10) with respect to t: $\frac{\partial W}{\partial t_k} = v(p_h - c)\frac{\partial x_h}{\partial t} + (1-v)(p_l - c)\frac{\partial x_l}{\partial t}$. Note that, when $t_p = t_A = t$, then

$$x_l = \alpha_l - \frac{v}{1-v}(\alpha_h - \alpha_l) - \frac{c}{1-t}, \qquad x_h = \alpha_h - \frac{c}{1-t},$$

$$p_l - c = \frac{v}{1-v}(\alpha_h - \alpha_l) + \frac{tc}{1-t}, \qquad p_h - c = \frac{tc}{1-t}.$$

Because when $t > 0$ both $p_l - c$ and $p_h - c$ are strictly positive, and because $\frac{\partial x_h}{\partial t} = \frac{\partial x_l}{\partial t} = -\frac{c}{(1-t)^2} < 0$, we get that $\frac{\partial W}{\partial t_k} < 0$ for any nonnegative tax rate. The claim follows. □

This finding is in line with standard results in the literature. The monopolist restricts supply for the low type, and a positive tax rate would only tighten the restriction. Hence, the optimal uniform tax rate cannot be positive.

3.3.2 Differentiated tax rates

We now relax the uniformity restriction on t_p and t_A. Using (10), the first-order derivatives of social welfare W are

$$\frac{\partial W}{\partial t_k} = v\left(p_h - c\right)\frac{\partial x_h}{\partial t_k} + (1-v)\left(p_l - c\right)\frac{\partial x_l}{\partial t_k}, \quad k = A, p, \tag{26}$$

where

$$p_h - c = \frac{\alpha_h\left(t_A - t_p\right) + c\left(2t_p - t_A\right)}{1 - 2t_p + t_A}, \quad p_l - c = \frac{\frac{v}{1-v}\left(\alpha_h - \alpha_l\right)\left(1 - t_A\right) + \alpha_l\left(t_A - t_p\right) + c\left(2t_p - t_A\right)}{1 - 2t_p + t_A}.$$

We begin by investigating the following question: given that it is never optimal to set a uniform tax (see Lemma 3), is it possible to increase social welfare by setting a small tax targeting either usage or access? Proposition 2 provides the answer.

Proposition 2. *Let $\phi \equiv \frac{\alpha_l}{2} - \frac{v}{1-v}\left(\alpha_h - \alpha_l\right)$ and assume that $\phi > 0$. If $c < \phi$, the optimal tax rates are such that $t_p > 0$ and $t_A = 0$. If $\phi \leq c \leq 2\phi$ the optimal tax rates are $t_p = t_A = 0$.. If $c \geq 2\phi$, then the optimal tax rates are such that $t_p = 0$ and $t_A > 0$. When $\phi \leq 0$, then the optimal tax rates are such that $t_p = 0$ and $t_A > 0$.*

Proof. (Sketch) Consider the initial situation where $t_p = t_A = 0$. Because $p_h = c$ and $p_l - c = \frac{v}{1-v}\left(\alpha_h - \alpha_l\right)$, then (26) becomes $\frac{\partial W}{\partial t_k} = v\left(\alpha_h - \alpha_l\right)\frac{\partial x_l}{\partial t_k}$, $\quad k = A, p$. Therefore, the sign of $\frac{\partial W}{\partial t_k}$ is identical to the sign of $\frac{\partial x_l}{\partial t_k}$. From (22), we know that $\frac{\partial x_l}{\partial t_p} > 0$ if and only if $c < \phi$. Furthermore, from (24), we know that $\frac{\partial x_l}{\partial t_A} > 0$ if and only if $c > 2\phi$. The claim follows. □

The result indicates that, if the government can apply differentiated tax rates, taxing either usage or access may increase efficiency. Therefore, despite the fact that the monopolist restricts supply, the proposition suggests that exempting the good/service it provides from taxation (or even a settign small subsidy) is not the optimal policy, at least on pure efficiency grounds. Specifically, we find that when the marginal cost is small enough, a tax on usage is optimal. By contrast, when the marginal cost is large enough, or when the information rent to the high type is so large that $\phi < 0$, a tax on access is optimal. Finally, for intermediate values of marginal cost, neither tax is desirable. The intuition follows from the comparative statics provided above.

4 Concluding Remarks

We have studied commodity taxation in a market where a monopolist prices access to and usage of an essential service. We have shown that, if marginal costs of production are small, usage fees *decrease* with the ad valorem tax rate applied on them and, thus, equilibrium consumption of the good/service *increases*. Hence, if usage is priced above marginal cost (i.e., the monopolist underprovides the service), positive (ad-valorem) tax rates are optimal. We have also shown that the results carry over to the case of second-degree price discrimination.

Our analysis is still preliminary and will be extended in several directions, inclduing the following. First, we will consider an environment in which the essential service is provided by an oligopoly rather than a monopoly. Second, we will allow for a more general setup with more than two types, a more general distribution of types and utility function. Third, we will consider the case where market coverage is less than complete (i.e., some consumers do not buy the good provided by the monopolist). Finally, one could consider the optimal tax rates if the government has a revenue requirement.

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A Proofs of Propositions and Lemmas

A.1 Proof of Lemma 2

Using the expressions for p^e and q^e obtained above, we get

$$p^e + q^e - c = \frac{(\alpha - c)(2 + t_A - 3t_p) + ct_p}{3 + t_A - 4t_p}. \tag{27}$$

Assume that $\frac{\alpha(1-t_A)}{4} > c$, implying that $min\left[1 - \frac{c}{\alpha}; \frac{1+t_A}{2} + \frac{c}{\alpha}; \frac{3+t_A}{4}\right] = \frac{1+t_A}{2} + \frac{c}{\alpha}$. Hence, to ensure quantities and prices are nonnegative, we restrict attention to $t_p \in \left[0; \frac{1+t_A}{2} + \frac{c}{\alpha}\right]$. Because $t_p \leq \frac{3+t_A}{4}$, the right hand side of (27) is positive if and only if $(2 + t_A) \frac{\alpha-c}{3\alpha-4c} > t_p$. Replacing the upper bound of t_p in the latter condition, this inequality can be rewritten as $\alpha^2 (1 - t_A) + 2\alpha c t_A > 2c (3\alpha - 4c)$. We note that the left hand side is strictly decreasing in t_A, because $t_A < 1 - \frac{4c}{\alpha}$ by assumption. Hence, the lower bound of the left hand side is $2c (3\alpha - 4c) + \varepsilon (\alpha^2 - 2\alpha c)$, where ε and the last term in brackets are strictly positive. As a result, we have $p^e + q^e > c$ and, hence, $\frac{\partial W}{\partial t_p} > 0$ for any $t_A \in [0; 1]$ and $t_p \leq \frac{1+t_A}{2} + \frac{c}{\alpha}$. It follows that $t_p = \frac{1+t_A}{2} + \frac{c}{\alpha}$. Observe that $\frac{1+t_A}{2} + \frac{c}{\alpha} \geq t_A$ because $t_A \in [0; 1]$.

Assume now that $\frac{\alpha(1-t_A)}{4} \leq c$. This condition implies that $min\left[1 - \frac{c}{\alpha}; \frac{1+t_A}{2} + \frac{c}{\alpha}; \frac{3+t_A}{4}\right] = 1 - \frac{c}{\alpha}$. Hence, to ensure quantities and prices are nonnegative, we restrict attention to $t_p \in \left[0; 1 - \frac{c}{\alpha}\right]$. Because $t_p \leq \frac{3+t_A}{4}$, the right hand side of (27) is positive if and only if $(2 + t_A) \frac{\alpha-c}{3\alpha-4c} \geq t_p$. Replacing t_p by its upper bound, and after simplification, this condition becomes $\frac{\alpha(1-t_A)}{4} \leq c$, which holds by assumption. Recall that $\frac{\partial p^e}{\partial t_p} \geq 0$ and $\frac{\partial x^e}{\partial t_p} \leq 0$ when $\frac{\alpha(1-t_A)}{4} \leq c$. It follows that, $\frac{\partial W}{\partial t_p} \leq 0$ for any $t_A \in [0, 1]$ and $t_p \leq 1 - \frac{c}{\alpha}$. Therefore, $t_p = 0$.

A.2 Proof of Proposition 1

Assume that condition $\frac{\alpha(1-t_A)}{4} \leq c \Rightarrow t_A \geq 1-\frac{4c}{\alpha}$ holds. By Lemma 2, we have $p^e = \frac{\alpha(1+t_A)+2c}{3+t_A}$ and $q^e = x^e = \frac{\alpha-c}{3+t_A}$ for any $t_A \in \left[1-\frac{4c}{\alpha};1\right]$. As a result $\frac{\partial W}{\partial t_A} = (p^e+q^e-c)\frac{\partial x^e}{\partial t_A} = -\frac{(\alpha-c)^2(2+t_A)}{(3+t_A)^3} < 0$. Therefore, the optimal t_A within the interval $\left[1-\frac{4c}{\alpha};1\right]$ is $t_A = 1-\frac{4c}{\alpha}$. This results in $x^e = \frac{\alpha}{4}$. Recalling the expression for social welfare in (8), we obtain $W = \frac{\alpha}{4}\left(\frac{7\alpha}{8}-c\right)$. Assume now that $\frac{\alpha(1-t_A)}{4} > c \Rightarrow t_A < 1-\frac{4c}{\alpha}$. Observe that, because $t_A \in [0;1]$, this condition can be satisfied only if $\alpha > 4c$. By Lemma 2, we have $t_p = \frac{1+t_A}{2}+\frac{c}{\alpha}$, $p^e = 0$ and $q^e = x^e = \frac{\alpha}{2}$ for any $t_A \in [0;1-\frac{4c}{\alpha})$. As a result $\frac{\partial W}{\partial t_A} = (p^e+q^e-c)\frac{\partial x^e}{\partial t_A} = \left(\frac{\alpha}{2}-c\right)\frac{\partial x^e}{\partial t_A} = 0$. Therefore, any $t_A \in [0;1-\frac{4c}{\alpha})$ yields the same level of social welfare, equal to $W = \frac{\alpha}{2}\left(\frac{3\alpha}{4}-c\right)$. Comparing the two welfare levels obtained, we get that the first one dominates if and only if $\alpha \leq \frac{8c}{5}$. However, we also know that the second one is attainable only if $\alpha > 4c$. Therefore, we have that when $\frac{\alpha}{4} > c$, $t_p = \frac{1+t_A}{2}+\frac{c}{\alpha}$ and $t_A \in [0;1-\frac{4c}{\alpha})$ are optimal. Otherwise, the optimal set of tax rates is $t_p = t_A = 0$.

A.3 Proof of Proposition 3

Consider the first order condition (26) and assume second order conditions are satisfied. Using the expressions for $p_h - c$, $p_l - c$, $\frac{\partial x_h}{\partial t_p}$ and $\frac{\partial x_l}{\partial t_p}$ provided in the text, the first order condition of the problem can be written as

$$v\left(\alpha_h\left(t_p-t_A\right)+c\left(2t_p-t_A\right)\right)\cdot\left(\alpha_h\left(1-t_A\right)-2c\right)+$$

$$+\left(1-v\right)\left(\frac{v}{1-v}\left(\alpha_h-\alpha_l\right)\left(1-t_A\right)+\alpha_l\left(t_A-t_p\right)+c\left(2t_p-t_A\right)\right)\cdot\left(2\left(\phi-c\right)\right)=0.$$

Solving for t_p we obtain

$$t_p = \frac{t_A\left[v\left(\alpha_h^2\left(1-t_A\right)-3c\alpha_h+ct_A\alpha_h+2c^2\right)\right]+\left(\phi-c\right)\left[t_A\left(1-v\right)\left(\alpha_l-c\right)+v\left(1-t_A\right)\left(\alpha_h-\alpha_l\right)\right]}{v\left(\alpha_h^2\left(1-t_A\right)-2c\left(\alpha_h\left(2-t_A\right)-2c\right)\right)+\left(1-v\right)\left(\phi-c\right)\left(\alpha_l-2c\right)},$$

Rearranging this expression, we obtain (**??**). Observe that, using the definition of ϕ provided in the text, and under the assumption that $c < \phi$, all the terms in square brackets in (**??**) are strictly positive. Therefore, the second term on the right hand side of the equality is positive.

Assume now that $c \geq \phi$. Because the derivative $\frac{\partial x_l}{\partial t_p}$ is negative under this condition, the optimal tax rate t_p will certainly be zero (hence, smaller or equal to t_A) for certain parameter values, but may be larger than t_A for others. To see this, it suffices to consider some special cases. To begin, consider the case in which the share of high types v in the population tends to one. Then, $(1-v)(p_l - c) = \frac{v(\alpha_h - \alpha_l)(1-t_A)+(1-v)(\alpha_l(t_A - t_p)+c(2t_p - t_A))}{1-2t_p+t_A}$ is strictly positive, but $\frac{\partial x_l}{\partial t_p} = \frac{-\left(\frac{2v}{1-v}(\alpha_h - \alpha_l) - \alpha_l\right)(1-t_A)-2c}{(1-2t_p+t_A)^2}$ is infinitely large in absolute value. Therefore, the first order derivative $\frac{\partial SW}{\partial t_p}$ is everywhere negative, and the optimal t_p is zero. By contrast, consider the case in which v tends to zero and $t_A = 0$. Then $\frac{\partial SW}{\partial t_p} = -t_p \frac{(\alpha_l - 2c)^2}{(1-2t_p)^3}$. Hence, because $t_p < \frac{1+t_A}{2}$, the optimal t_p is zero.

B Proof: no loss of generality in two-part tariffs

B.1 One-type model (Section 3)

Let $T(x)$ be a generic tariff by firm 1 and let $B(T(x), x)$ be the associated tax burden. Let t_p be the ad-valorem tax rate applied to the marginal part of $T(x)$ and t_A be the tax rate applied to the remaining revenues of firm 1. Specifically, for any quantity x supplied in equilibrium, we have

$$B(T(x), x) = t_p T'(x) \cdot x + t_A \left(T(x) - T'(x) \cdot x\right).$$

Define the two part tariff as $T_{TP}(x) = A + px$, therefore $T'_{TP}(x) \cdot x = px$ and $T_{TP}(x) - T'_{TP}(x) \cdot x = A$. Finally, let the firm's net-of-tax profit π be

$$\pi\left(T(x), x\right) = T(x) - cx - \left(t_p T'(x) \cdot x + t_A \left(T(x) - T'(x) \cdot x\right)\right).$$

Consider a tariff $T^*(x)$ which maximizes firm 1's profit and let $x^* > 0$ denote the quantity chosen by the consumer when $T^*(x)$ is implemented. By definition, $T^*(x)$ is such that the following conditions have to be satisfied

$$\pi\left(T^*(x^*), x^*\right) \geq \pi\left(T(x), x\right), \quad \forall x, T(x) \tag{28}$$

$$U(T^*(x^*), x^*) > U\left(T^*(0), 0\right) \tag{29}$$

$$T^{*\prime}(x^*) = \frac{\partial U}{\partial x}\left(x^*\right) \tag{30}$$

Condition (28) tells us that $T^*(x)$ maximizes profits of firm 1. Conditions (29) and (30) are necessary conditions for implementability. The first tells us that the quantity chosen by consumers is positive, the second that $x^*$is such that the marginal rate of substitution between x and generic consumption is equal to the marginal price at x^*. We will now show that any net-of-tax profit level π attainable with a generic tariff $T(x)$ can be attained with an appropriately designed $T_{TP}(x)$. Hence, for any couple of tax rates (t_p, t_A), there is no loss of generality in assuming that the firm implements a two part tariff.

Consider a two part tariff $T_{TP}(x)$ such that $p = \frac{\partial U}{\partial x}\left(x^*\right)$ and $A = T^*(x^*) - \frac{\partial U}{\partial x}\left(x^*\right) \cdot x^*$. This satisfies condition (30) and implements quantity x^*. We assume that (29) is satisfied (we check this later). Therefore, we have

$$B(T_{TP}(x), x) = t_p p x + t_A A = t_p \frac{\partial U}{\partial x}\left(x^*\right) \cdot x + t_A \left(T^*(x^*) - \frac{\partial U}{\partial x}\left(x^*\right) \cdot x^*\right)$$

and

$$\pi\left(T_{TP}(x), x\right) = A + px - cx - \left(t_p p x + t_A A\right) =$$
$$T^*(x^*) - cx^* - \left(t_p \frac{\partial U}{\partial x}\left(x^*\right) \cdot x + t_A \left(T^*(x^*) - \frac{\partial U}{\partial x}\left(x^*\right) \cdot x^*\right)\right).$$

Therefore, implementing $T_{TP}(x)$ gives the firm exactly the maximum profit level $\pi\left(T^*(x^*), x^*\right)$. It remains to check that $T_{TP}(x)$ satisfies the participation constraint (29). Observe that, by definition, $T^*(x^*)$ is such that (29) holds. But since $A + px^* = T^*(x^*)$ by construction of $T_{TP}(x)$, the constraint must be satisfied by $T_{TP}(x)$ as well.

A similar proof applies for the case of a specific tax on x. We omit it for brevity.

B.2 Two-type model (Section 4)

Let $M = \{T_l(x), T_h(x)\}$ be a menu of generic tariffs by firm 1 and let $B(M, x_l, x_h)$ be the associated tax burden, where x_l and x_h are equilibrium quantities consumed, respectively, by type-l and type-h consumers. Let t_p be the ad-valorem tax rate applied to the marginal part of $T_i(x), i = l, h$, and t_A be the tax rate applied to the remaining revenues of firm 1. Specifically, for any quantity x supplied in equilibrium, we have

$$B(M, x_l, x_h) = t_p \left(T_l'(x_l) \cdot x_l (1 - v) + T_h'(x_h) \cdot x_h v\right) + t_A \left((1 - v) \left(T_l(x_l) - T_l'(x_l) \cdot x_l\right) + v \left(T_h(x_h) - T_h'(x_h) \cdot\right.\right.$$

Define an "augmented" two part tariff $T_{TP}(x) = A + px + X \cdot I\,(x > x_0)$, where x_0 is a positive threshold on x and $I\,(x > x_0)$ is an indicator function taking value one if and only if $x > x_0$. Therefore, to the extent that $x \leq x_0$, we have $T_{TP}'(x) \cdot x = px$ and $T_{TP}(x) - T_{TP}'(x) \cdot x = A$. Finally, let the firm's net-of-tax profit π be

$$\pi\,(M, x_l, x_h) = (1 - v)\, T_l(x_l) + vT_h(x_h) - c\,((1 - v)\, x_l + vx_h) +$$

$$-\left(t_p\left((1 - v)\, T_l'(x_l) \cdot x_l + vT_h'(x_h) \cdot x_h\right) + t_A\left((1 - v)\left(T_l(x_l) - T_l'(x_l) \cdot x_l\right) + v\left(T_h(x_h) - T_h'(x_h) \cdot x_h\right)\right)\right).$$

Consider a menu of tariffs M^* which maximizes firm 1's profit and let $x_l^*, x_h^* > 0$ denote the quantity chosen by consumers of type l and h, respectively, when M^* is implemented. By definition, M^* is such that the following conditions have to be satisfied

$$\pi\left(M^*, x_l^*, x_h^*\right) \geq \pi\left(M, x_l, x_h\right), \quad \forall x_l, x_h, M \tag{31}$$

$$U(T_h^*(x_h^*), x_h^*) \geq U\left(T_l^*(x_{hl}), x_{hl}\right) \tag{32}$$

$$U(T_l^*(x_l^*), x_l^*) \geq U\left(T_h^*(x_{lh}), x_{lh}\right) \tag{33}$$

$$U(T_i^*(x_i^*), x_i^*) \geq U\left(T^*(0), 0\right) \qquad i = h, l \tag{34}$$

$$T_i^{*,}(x_i^*) = \frac{\partial U_i}{\partial x}\left(x_i^*\right), \qquad i = h, l \tag{35}$$

Condition (31) tells us that M^* maximizes profits of firm 1. Conditions (32) and (33) tell us that the incentive compatitiblity constraints are satisfied. Conditions (34) and (35) are necessary conditions for implementability. The first tell us that the quantities chosen by consumers is positive (participation constraints), the second that $x_i^*, i = h, l$ is such that the marginal rate of substitution between x and generic consumption is equal to the marginal tariff at x_i^*. We will now show that the net-of-tax profit level π attainable with a generic tariff menu M^* can be attained with an appropriately designed menu of augmented two-part tariffs M_{TP}. Hence, for any couple of tax rates (t_p, t_A), there is no loss of generality in assuming that the firm implements a two part tariff.

Consider a menu of augmented two part tariffs $M_{TP} = \{T_{TP,l}(x), T_{TP,h}(x)\}$ such that $p_i = \frac{\partial U_i}{\partial x}\left(x_i^*\right)$ and $A_i = T_i^*(x_i^*) - \frac{\partial U_i}{\partial x}\left(x_i^*\right) \cdot x_i^*, i = h, l$. These satisfy conditions (35) and implement quantities $x_i^*, i = h, l$. We assume that M_{TP} is such that (32) - (34) are satisfied (we check later that this is indeed the case). Therefore, we have

$$B(M_{TP}, x_l^*, x_h^*) = t_p\left((1-v)\frac{\partial U_l}{\partial x}\left(x_l^*\right) \cdot x_l^* + v\frac{\partial U_h}{\partial x}\left(x_h^*\right) \cdot x_h^*\right) + t_A\left((1-v)\left(T_l^*(x_l^*) - \frac{\partial U_l}{\partial x}\left(x_l^*\right) \cdot x_l^*\right) + v\Bigg(T$$

and

$$\pi\left(M_{TP}, x_l, x_h\right) = (1-v)\, T_l^*(x_l^*) + vT_h^*(x_h^*) - c\left((1-v)\, x_l^* + vx_h^*\right) - B(M_{TP}, x_l^*, x_h^*).$$

Implementing M_{TP} gives exactly the maximum profit level $\pi\left(M^{*}, x_{l}^{*}, x_{h}^{*}\right)$. It remains to check that M_{TP} satisfies (32) - (34). Let us begin from (34). Observe that, by definition, $T_{i}^{*}(x_{i}^{*}), i = h, l$ is such that (29) holds. But since $A_{i} + p_{i}x_{i}^{*} = T_{i}^{*}(x_{i}^{*}), i = h, l$ by construction of M_{TP}, the constraints must be satisfied by M_{TP} as well.

Focus now on (32). Note that $p_{l} = \frac{\partial U_{l}}{\partial x}\left(x_{l}^{*}\right)$, which implies that $x_{hl} > x_{l}^{*}$, because $\frac{\partial U_{h}}{\partial x}\left(x\right) > \frac{\partial U_{l}}{\partial x}\left(x\right)$ for any x. Denote $CS_{gr}^{h}\left(x\right)$the gross consumer surplus of a type-h consumer buying quantity x. To satisfy (32), it is sufficient to set $x_{0,l} = x_{l}^{*} + \varepsilon$, where ε is positive and arbitrarily small, and $X_{l} = CS_{gr}^{h}\left(x_{hl}\right) - CS_{gr}^{h}\left(x_{l}^{*}\right)$, so the h-type mimicker cannot be better off than when choosing the tariff intended for her type. Note that since X_{l} is never paid in equilibrium, it does not affect the firm's tax burden. Hence, we ignore it in the main text.

Turn now to (33). We assume, without loss of generality, that any generic tariff menu M^{*} that maximizes profits is such that constraints (32) and (34) for $i = l$ are binding. Hence, these constraints must be binding also when M_{TP} is implemented. Therefore, to show that (33) is satisfied when M_{TP} is implemented, we proceed assuming conditions (15) and (16) hold. Using them, we can rewrite (33) as

$$\int_{0}^{x_{hl}}\left(\alpha_{l} - r\right)dr - \int_{0}^{x_{l}}\left(\alpha_{l} - r\right)dr - \int_{0}^{x_{h}}\left(\alpha_{h} - r\right)dr + p_{h}x_{h} + \int_{0}^{x_{hl}}\left(\alpha_{h} - r\right)dr - p_{l}x_{hl} - p_{h}x_{lh} \leq 0.$$

Using the fact that $x_{ij} = \alpha_{i} - p_{j}$ for $i, j = h, l$, this constraint simplifies to

$$-\left(2\alpha_{h} - \alpha_{l}\right)\left(p_{l} - p_{h}\right) - p_{l}\left(\alpha_{h} - p_{l}\right) \leq 0,$$

which is satisfied because $\alpha_{h} > \alpha_{l} > p_{l} > p_{h}$.

Finally, a similar proof applies for the case of a specific tax on x. We omit it for brevity.