Strategic Compromise in Contests with Two-Dimensional Policy Spaces*

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Abstract

Policy reforms are frequently multifaceted. However in the lobbying literature most policies are taken as one-dimensional. This paper models policy reform using a political contest with endogenous policy proposals and two-dimensional policy spaces. The two dimensions give players the opportunity to trade off one factor over another to make the opposition less aggressive. As a result, players make strategic compromises in an attempt to augment their winning probability. This finding is verified empirically using a novel dataset on California legislation between 2008-2013.

Keywords: contest, political reforms, lobbies

JEL classification codes: D72, D86, H4

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1 Introduction

Policy reforms are frequently multifaceted. They are composed of a collection of provisions involving more than one policy dimension. In international trade for instance, decisions must be made about the commodities to be regulated as well as the level of tariffs. In health care policy, insurance coverage and insurance policy standards may fall under a single health reform bill. In education, a reform could contain provisions on the salary of teachers as well as on the curriculum. Often, these different elements of reform are used by opposing interests as tools for bargaining in the political arena. A group who prioritizes one policy dimension over another may use the other dimension as an instrument for negotiation.

In this paper we explore such strategic behavior in a political contest model with two policy elements that may be traded off to augment the probability of winning. We consider a policy-oriented government interested in reforming a bundle of two policies, one of which is opposed and the other supported by an interest group. Our two-stage setting builds on the framework of Epstein and Nitzan (2004) where the first stage determines the proposed policy bundle, and the second stage involves a contest over the enactment of this policy bundle. Contrary to Epstein and Nitzan (2004), however, we do not model two opposing interest groups who make proposals and engage in the contest. Instead, the government makes the proposal and the contest over its enactment involves the government and an interest group, both exerting effort to augment the probability of their desired outcome. We therefore abstract from the recent contest lobbying literature that takes lobbying effort to be favors or bribes that enter the government's utility function[1]. We follow instead Skaperdas and Vaidya (2009) and assume that the lobbying effort exerted by both sides are costs associated with measures taken to nudge the public debate in their favor. Examples of this are the time costs of organizing protests, and the costs of producing publicity materials. With lobbying effort defined as the cost of "persuasion", it is sensible to imagine the government also expending resources for this purpose. For instance, during the 2009 legislative process involving United States' Patient Protection and Affordable Care Act, colloquially, Obamacare, time and effort were spent in negotiations to break the threat of a Republican filibuster. Barack Obama himself delivered a speech to a joint session of Congress to emphasize his commitment to the reform. Another example of government lobbying is the video released by the Italian government in May 2015 about the highly-controversial and much opposed *La Buona Scuola* (The Good School) education reform. The video showed Italian prime minister Matteo Renzi discussing in detail the benefits of his administration's proposed reform.

Our attempt to model policy determination in the face of opposition is related to the literature on policy formation under lobbying. There are two camps in this literature, the first being the "compromise" camp that says that the lobbying induces a compromise between the policy preferences of the stakeholders. Studies of this kind include Grossman and Helpman (1996), Epstein and Nitzan (2004), Münster (2006) and Felli and Merlo (2006). Grossman and Helpman's (1996) Downsian model considered lobbying as a "menu-auction" and found that the equilibrium policy is a compromise between the policy preferences of the lobbies and the policy preferences of the voters. Felli and Merlo (2006) also used "menu-auction" lobbying to develop a citizen-candidate model of

[1] See for example Epstein and Nitzan (2004); Münster (2006); Epstein and Nitzan (2006a,b)

political competition where the politician selects the lobbies endogenously. In this setting, the policy outcome is a compromise between the policy preferred by the candidate and the policy preferred by the chosen lobbies. Epstein and Nitzan (2004) used contests to model lobbying and relaxes the assumption commonly made in contest games that the policy proposals are exogenous. They develop a two-stage political contest between interest groups where first stage determines the choice of policy proposal and the second stage is where the contest ensues. They show that under these circumstances groups have an incentive to strategically moderate their proposals in order to reduce the effort of the opposition, thereby increasing their chances of winning. The resulting proposals are therefore less polarized than they would be without opposition. In their model, the proposals will never coincide. Münster (2006) explored the same setup for perfectly discriminating contests and found that the proposals of the two groups will not only be less polarized, but will coincide.

The opposite camp in this strand of literature provides circumstances under which lobbying could result in extreme policies. Glazer et al. (1998) developed a simple framework showing that an incumbent may choose to implement an extreme policy if he is strongly office motivated and the costs of a challenger reversing the policy is substantial. The intuition behind this is that the median voter with moderate preferences will prefer to reelect the incumbent with an extreme policy than vote for the challenger and incur the high cost of changing the policy when the challenger comes into office. More recently, Epstein and Nitzan (2006a) studied a two-stage public policy contest in which a politician proposes a policy and interest groups compete for its approval or rejection. Building on the results of Epstein and Nitzan (2004) and Münster (2006), they find that a politician will propose an extreme policy if his marginal benefit from the lobbying expenditures exceeds his marginal losses from the disutility of the lobbies.

Inherent in these studies is the one-dimensionality of the policy space. The few studies involving theoretical models with two or more policy components make simplifying assumptions about how the components affect special interests. For instance, Glazer et al. (1998) assume that one of the two types of policy issues is fixed due to predetermined positions reflecting ideology. Other studies assume that interest groups have preferences over only one of the policy components (List and Sturm, 2006; Chaturvedi and Glazer, 2005). This gap in the theoretical literature has persisted despite a number of empirical work acknowledging that reforms are multi-faceted and each facet affects interest groups in various degrees (Kang, 2014; Lake, 2015). We attempt to fill this gap by proposing a model of policy reform in a two-dimensional policy space, the components of which both enter directly into the preferences of the interest groups. This provides insight into the decision to trade off one component over another to augment the probability of success. Indeed, we find that compared to the government's preferred reform bundle without opposition, the equilibrium proposal of the government will have more of the interest group's favored policy component and less of the opposed policy component. Hence, the government makes a strategic compromise in an attempt to make the interest group less aggressive.

In the next section, we present the basic setting. We then elaborate on the case of a single reform opposed by an interest group in Section 3. Section 4 extends the model to two policy dimensions and establishes the main results. In Section 5 the empirical case study of California is presented. Finally, Section 6 discusses the implications of the model and concludes.

2 Basic Setting

Consider a setting in which there are two actors, the government G and an interest group I. The government wishes to conduct a policy reform. Suppose this reform has two dimensions. These can be for example broadening access (A) versus improving quality (Q), in the sense that applies to health policy or education policy. For the rest of the discussion, this example shall be used.

Let A and Q be two non-negative, real-valued sets of public policies, and let $E \equiv A \times Q$ be the Cartesian product of A and Q. Without loss of generality, let $(0,0) \in E$ be the status quo. All other policies $(a,q) \in E$ such that $(a,q) \neq (0,0)$ shall be called reforms, r.

Let the preferences of the government G be described by $U_G(a,q)$, which is increasing and concave in both its arguments and normalized to zero at the status quo.[2] Moreover we assume that a and q are imperfect substitutes, so that $\partial^2 U_G/\partial a \partial q < 0$, and that there exists an optimal policy reform that the government wants to implement, denoted by $r_g^* = (a_g^*, q_g^*) \in E$ and $a_g^*, q_g^* \geq 0$. By the definition of an optimal point, r_g^* maximizes U_G.

Let the preferences of the interest group I be described by $U_I(a,q)$ and let I's optimal policy be denoted by $r_i^* = (a_i^*, q_i^*) \in E$. The behavior of the I towards the G's proposed reforms depends on the position of r_i^* relative to r_g^* in the Cartesian plane. Figure 1 illustrates the three possible cases. First, if $a_i^* < a_g^*$ and $q_i^* < q_g^*$, then r_i^* falls into the dark-gray region and I would be opposed to any reforms along both dimensions A and Q. On the other hand, if $a_i^* > a_g^*$ and $q_i^* > q_g^*$, then r_i^* falls into the white region where I would oppose neither A nor Q. The third case is represented by the two light-gray regions for which one component of r_i^* is larger than r_g^*, while the other component is smaller. In such a case I would support reform along one dimension but oppose reforms along the other. For the rest of this paper, we will assume that I opposes the reforms along Q and supports the reforms along A.

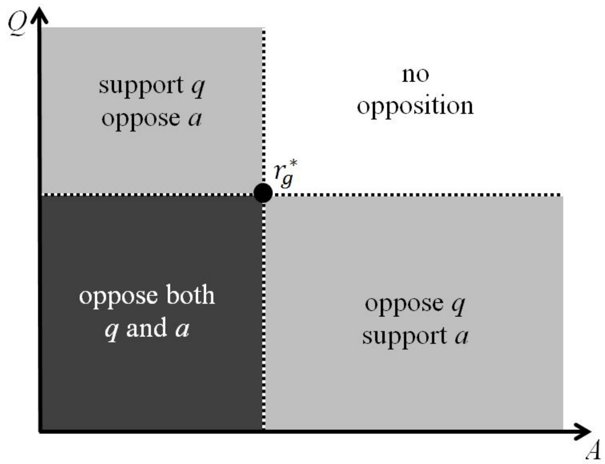

Figure 1: Position of r_i^* relative to r_g^* in the Cartesian plane.

In the absence of opposition, the government will enact its optimal reform r_g^*. Otherwise, the government will propose a reform $\widetilde{r} = (\widetilde{a}, \widetilde{q})$, which will enter as payoffs of a political contest between G and I in the next stage.

[2] $U_G(0,0) = 0$, $\partial U_G/\partial q > 0$, $\partial^2 U_G/\partial a^2 < 0$ and $\partial^2 U_G/\partial q^2 < 0$.

3 One-dimensional policy space

Let us first analyze the simplest case in which the interest group is opposing only one dimension of the reform. Without loss of generality, let q denote the opposed dimension of the reform and $U_I(q)$ and $U_G(q)$ be I's and G's preferences over q, respectively. Opposition to q means that $0 \leq q_i^* < q_g^*$. For simplicity, we assume here that $q_i^* = 0$. Within the context of our two-dimension policy setting, a one-dimensional opposition could occur if, for instance, I has no preference over policies in A (as in List and Sturm (2006), Chaturvedi and Glazer (2005), and Glazer et al. (1998)), or if both A and Q policies are opposed by I such that they may be collapsed into a single policy dimension akin to Epstein and Nitzan (2004).

The political game has two stages. The first stage determines the reform proposal, $\tilde{r}$, based on an optimization by G, and the second stage determines whether $\tilde{r}$, is enacted through a contest between G and I.

Denote the probability that the proposal is enacted by the contest success function $p(e_G, e_I) \in [0, 1]$, where e_G and e_I are the efforts exerted respectively by G and I. Following our assumption over government and interest group preferences, this success function is increasing and concave in the lobbying effort of the government, while is decreasing and convex in the one of the interest group.[3] These assumptions ensure a positive but diminishing marginal effect of each player's effort on his own probability of winning the contest, moreover they ensures that an increase in each player's effort harm the other, making strategically desirable for each to induce a lower effort from the other. In our case in particular, as we will see, makes it desirable for the first mover (the government) to make a different proposal. The contestants are assumed to be risk neutral.

$$p(e_G, e_I) = \frac{\alpha_G e_G}{\alpha_G e_G + \alpha_I e_I}, \tag{1}$$

and $\alpha_j, j = G, I$ denotes the "productivity" of each contestant's effort.

Denoting the stake of the I by $N(q) = U_I(0) - U_I(q)$ and recalling that G's utility of the status quo is normalized to zero, the expected payoffs of G and I are given by:

$$EU_G = pU_G(q) - e_G, \tag{2}$$

$$EU_I = U_I(0) - pN(q) - e_I. \tag{3}$$

By backward induction, contestants maximize their expected payoffs with respect to effort in the second stage contest.

Lemma 1. *The equilibrium lobbying effort levels are such that the ratio of their marginal productivity (in absolute value) is equal to the ratio of their respective gain in terms of utility from winning the contest.*

$$\left| \frac{\frac{\partial p}{\partial e_I}}{\frac{\partial p}{\partial e_G}} \right| = \frac{U_G}{N} \tag{4}$$

Proof. See appendix A.1 ∎

[3]Formally, $\partial p/\partial e_G > 0$, $\partial^2 p/\partial e_G^2 < 0$ and $\partial p/\partial e_I < 0$, $\partial^2 p/\partial e_I^2 > 0$.

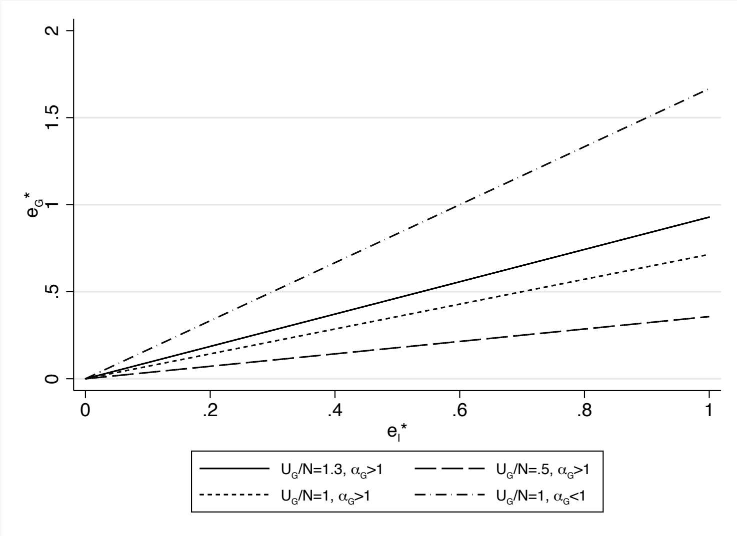

Figure 2: Best Responses

In figure 2 we can see the government's optimal effort, given the one exerted by the interest group for four different combinations of its effort productivity and the relative stakes.

In the first stage, G maximizes his expected utility with respect to policy q, taking into account the preferences of the interest group and the level of lobbying effort given in Lemma 1.

Lemma 2. *The government proposed policy $\tilde{q}$ is such that the marginal benefit from increased utility equals the marginal cost from a more aggressive opposition.*

Proof. The government maximization problem with respect to $\tilde{q}$ leads to the following first order condition:

$$\frac{\partial EU_G}{\partial q} = p\frac{\partial U_G}{\partial q} + U_G\left(\frac{\partial p}{\partial e_G}\frac{\partial e_G}{\partial q} + \frac{\partial p}{\partial e_I}\frac{\partial e_I}{\partial q}\right) - \frac{\partial e_G}{\partial q} = 0\ ,$$

which by (12) and (13) simplifies to

$$p\frac{\partial U_G}{\partial q} - \frac{U_G}{N}\frac{\partial e_I}{\partial q} = 0. \tag{5}$$

The first term in the above expression is the marginal utility gain from an increase in q, provided the policy is enacted. The second term represents the marginal cost of q brought about by the increase in the stake of the interest group, inducing him to exert more lobbying effort. ∎

Lemma 2 introduces the trade off that allows us to relate the proposed policy level $\tilde{q}$ with the ideal policy level q_g^*, for which $\partial U_G/\partial q = 0$.

Proposition 1. *In the presence of an opposition, the proposed reform $\tilde{q}$ will be such that $\tilde{q} < q_g^*$.*

Proof. See appendix A.2 ∎

In the face of an opposing interest group, it pays for the government to restrain his proposal of the reform to reduce the lobbying effort of the opposition, thereby increasing his winning probability. This result coincides with the strategic restraint result presented by Epstein and Nitzan (2004) . Essentially, the level of $\tilde{q}$ proposed by the government serves two functions: a policy reform that contributes to the utility of the government, and a "bargaining tool" that affects the incentive of the opposition to engage in rent-seeking efforts against the reform.

4 Two-dimensional policy space

Now let us assume that interest group I favors A-reforms and opposes Q-reforms. That is, we can describe the preferences of the interest group by $U_I(a,q)$, which is increasing in a and decreasing in q.[4]

In the first stage, G selects his proposed reforms $(\bar{a},\bar{q})$. In the second stage, G and I engage in a political contest over the enactment of $(\bar{a},\bar{q})$, exerting lobbying effort levels e_G and e_I, respectively, to influence their win probability, as in the previous model.

Denoting the stake of the interest group I by $N(a,q) = U_I(0,0) - U_I(a,q)$, the expected payoffs of G and I are given by:

$$EU_G = pU_G(a,q) - e_G, \tag{6}$$

$$EU_I = U_I(0,0) - pN(a,q) - e_I. \tag{7}$$

Proposition 2. *In the presence of an opposition, the proposed reforms $(\bar{a},\bar{q})$ are such that $\bar{a} > a_g^*$ and $\bar{q} < q_g^*$.*

Proof. See appendix A.3 ∎

That is, a government faced with an opposition restrains his proposal of the disfavored policy component q and compensates by over proposing on the favored policy component a. The government over proposes to appease the interest group. government moderates its proposal of the disfavored policy component q, proposing a quality reform $\bar{q}$ to the left of G's ideal level without opposition, q_g^*. Figure 3(a) and 3(b) illustrates these results graphically.

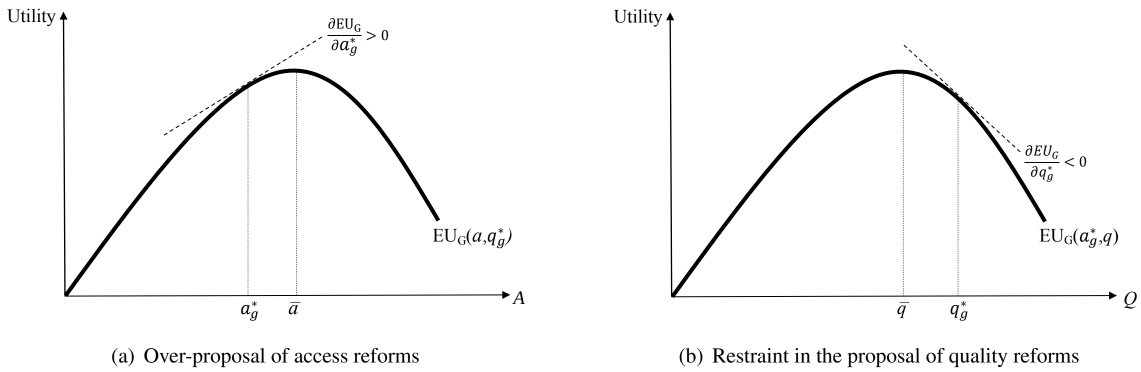

Figure 3: Ideal reforms $(a^* q^*)$ and proposed reforms $(\bar{a},\bar{q})$

In seeking to maximize expected payoffs, the government makes a strategic compromise by over-proposing in the policy component that the interest group favors and under-proposing in the component that the interest group opposes. The intuition behind the results is simple. In the face of opposition, the government recognizes that he is dealing with a group whose lobbying efforts are affected by the extent of his proposed reforms. As a result, although proposing an $\bar{a} > a_g^*$ and a $\bar{q} < q_g^*$ reduces his payoffs, doing so also reduces the stake of the interest group, thereby reducing their incentive to exert as much rent-seeking effort. In effect, he gets a higher probability

[4]In other words, the utility function $U_I(a,q)$ of I satisfies $\partial U_I/\partial a > 0$ and $\partial U_I/\partial q < 0$.

that the proposed reform will be enacted, which benefits the government. In the following section, we empirically analyze this theoretical prediction using data on California legislation.

5 Empirical case study: California

The primary elements of the political contest developed in this paper are the reform bundles in two dimensions, and the opposition. The main result in Proposition 2 reveals that, relative to the ideal policy levels of the one who proposes, opposition increases the level of the supported policy, and reduces the level of the opposed policy. We therefore need, for each reform bundle, a measure of the ideal level (a_g^*, q_g^*), the resulting reform level $(\bar{a}, \bar{q})$, and the amount of opposition faced by the reform. For this we use California legislative data on education legislation for the years 2008-2013. Education reforms are particularly interesting for our question because most of them may be grouped into two broad categories: broadening educational access, or improving educational quality. Often, these two reform categories are bundled together in a single piece of legislation (henceforth called a *bill*). It is well documented that a strong and organized interest group, the teachers' unions, have a stake in these bills, and are active in lobbying in congress in favor of access reforms and against quality reforms (Corrales, 1999; Grindle, 2004; Moe, 2012). This is because access reforms expand the extensive margin of education, providing more schools and textbooks, and improving school infrastructure, whereby providing jobs for teachers and enhancing their work environment. Meanwhile, quality reforms attempt to expand the intensive margin of education by changing the curriculum, imposing teacher performance evaluations and creating school accountability programs, which keep teachers on their toes.[5] The delineation between access and quality education reforms therefore matches the theoretical assumptions made in our model.

The reason our empirical investigation focuses on California are twofold. First, we need a state that has a substantial amount of education bills and a active interest group that will serve as the opposition. Second, we need specific information on each bill, in particular, an empirical counterpart for the ideal level of reforms (a_g^*, q_g^*) and the equilibrium level of reforms $(\bar{a}, \bar{q})$. California satisfies both these conditions. Between 2008-2013, California has enacted the second highest number of education bills (National Conference of State Legislatures, 2014). It also has the sixth strongest teachers' unions among all states in 2012 (Winkler et al., 2012).[6] Finally, the California State Legislature publicly provides legislative information for enacted bills from 2008-2013. Every bill contains information on (1) the final and all previous drafts of the legislative document as it went through the different committees, (2) the number of "yes" and "no" votes obtained at every juncture of the political process and (3) the names and party affiliations of the representatives who voted for and against the bill. This gives us the opportunity to obtain an measure for (a_g^*, q_g^*) from the initial draft of the bill and $(\bar{a}, \bar{q})$ from the final draft.

Our data consists of 272 enacted education bills in California from 2008 to 2013, of which 81.6% contain access and/or quality education reforms. The following subsections elaborate on the procedures used for the creation of our variables of interest: education reforms and bill opposition.

[5] For a more comprehensive discussion on the differences between access and quality education reforms, see Fabella (2015)

[6] By contrast, Arkansas, which has the most number of education bills during the same period, had the seventh *weakest* teachers' union in the US.

5.1 Access and quality education reforms

Measures of access and quality reforms were generated using the text contained in the initial and final drafts of each bill. Because these legal documents contain detailed explanations of the issues tackled in the bill and the policies or amendments proposed, they may contain access- and quality- related reform terms, or both. To measure the extent to which a bill contains access and quality reforms, we use the definitions:

$$\begin{aligned} Access_{ij} &= \left(\sum_{k \in A} \frac{f_{ijk}}{n_{ij}} \right) \times 100 \\ Quality_{ij} &= \left(\sum_{k \in Q} \frac{f_{ijk}}{n_{ij}} \right) \times 100 \end{aligned} \tag{8}$$

where draft j of bill i contains a total word count of n_{ij}, while f_{ijk} is the number of times the term k appears in the text of draft j. A and Q are the sets of access and quality education reform terms. The sets A and Q are based on Fabella (2015) who provides an extensive list of U.S. education reform terms that fall under the broad categories of access and quality. The expressions inside the parentheses are therefore the proportions of the total word count relating to access or quality reforms. Multiplying by a hundred converts the values into percentages. Table 1 below provides the summary statistics for these variables.

Table 1. Descriptive statistics

	Obs	Mean	SD	Min	Max
Access reform variables					
Access, initial draft	272	0.585	1.137	0	8.985
Access, final draft	272	0.857	1.164	0	8.789
Change in access	272	0.272	1.080	-5.139	5.473
Bill contains access reforms	272	0.794	0.405	0	1
Quality reform variables					
Quality, initial draft	272	0.155	0.372	0	2.655
Quality, final draft	272	0.189	0.456	0	3.922
Change in quality	272	0.034	0.317	-2.145	3.006
Bill contains quality reforms	272	0.441	0.497	0	1
Regressors					
Proportion of no votes in senate	272	0.089	0.135	0	0.436
Proportion of no votes in assembly	272	0.092	0.135	0	0.605
Proportion of democrats in senate	272	0.631	0.016	0.615	0.718
Proportion of democrats in assembly	272	0.638	0.021	0.600	0.684
Proportion of male senators	272	0.709	0.033	0.658	0.750
Proportion of male assembly members	272	0.743	0.013	0.725	0.762
Bill author is a democrat	272	0.835	0.372	0	1
Number of legislative actions	272	22.301	5.015	12	44
Senate bill	272	0.404	0.492	0	1
Election year	272	0.592	0.492	0	1

5.2 Bill opposition

The ideal measure for bill opposition would be the amount of lobbying effort the primary interest group (the teachers' unions) exert to keep each bill from being passed in congress. This information however is not readily available. Instead, we argue that the voting behavior of members of congress is a good proxy for bill opposition, given that one of the most common methods by which special interests lobby is through campaign contributions to politicians (Stratmann, 2005), and the primary motive of many of these contributions is to influence policy (Stratmann, 1991; Snyder, 1990, 1992, 1993). We therefore measure bill opposition as the proportion of "no" votes among all votes cast in both chambers of the California state legislature. Given that some bills went through both chambers more than once during the political process, we take only the last voting round for each chamber, which is presumably the vote that is associated with the final draft of the bill. Since the final draft is our measure for $(\bar{a}, \bar{q})$, this coincides with our theoretical setting in which the level of reforms that is used in the second-stage contest is $(\bar{a}, \bar{q})$.

5.3 Empirical strategy and results

Our model predicts that with stronger opposition, the equilibrium level of proposed access reforms will be higher, while the equilibrium level of proposed quality reforms will be lower, relative to the levels that are ideal for the proposer. For this reason we take as dependent variables the difference between the final and initial drafts of the bill, to capture the position of the resulting reform levels relative to the ideal level. A corollary of our main theoretical result is that the proposer bargains with a little bit more access reforms, when the bill contains some opposed quality reforms. Therefore, there are two ways in which we can test our theoretical predictions in the data. We can test (1) whether "no" votes have a positive (negative) influence on the change in access (quality) between the two drafts, and (2) whether bills containing quality reforms are compensated for by larger changes in access reforms. To test these two hypotheses, we employ a three-stage least squares analysis with the following two equations:

$$\begin{aligned} (A_{it}^{final} - A_{it}^{initial}) &= \alpha_0 + \alpha_1 dQ_{it} + \alpha_2 No_{it}^S + \alpha_3 No_{it}^A + \mathbf{X_{it}'}\boldsymbol{\mu}_\mathbf{A} + \delta_{1t} + u_{it} \\ (Q_{it}^{final} - Q_{it}^{initial}) &= \beta_0 + \beta_1 dAit + \beta_2 No^S it + \beta_3 No^A it + \mathbf{X_{it}'}\boldsymbol{\mu}_\mathbf{Q} + \delta_{2t} + u_{it} \end{aligned} \tag{9}$$

where the left hand side variables are the changes in the access or quality reform scores between initial and final drafts of bill i at year t, dQ_{it} and dA_{it} are dummy variables for whether the bill contained quality or access reforms respectively, No_{it}^S is the proportion of "no" votes in the senate, No_{it}^A is the proportion of "no" votes in the assembly and δ_{1t} and δ_{2t} are time-specific fixed effects. $\mathbf{X_{it}}$ is a vector of controls which include the proportion of democrats and males in both chambers, whether the is authored by a democrat, whether the bill was introduced in the senate, whether the bill was enacted in an election year, and the number of legislative actions from the introduction up to the enactment of the bill.

For the first hypothesis, our parameters of interest are the coefficients of No_{it}^S and No_{it}^A. In particular, we would like to test whether "no" votes correlate positively with access ($\alpha_2 > 0$ and $\alpha_3 > 0$) and negatively with quality ($\beta_2 < 0$ and $\beta_3 < 0$). For the second hypothesis, we are interested in the coefficient of dQ_{it}, to test whether there are more access reforms in bills containing quality reforms ($\alpha_1 > 0$).

Table 2 presents the baseline results for the equations in (9) using both an OLS and a 3SLS procedure. It is clear from the point estimates of the proportion of "no" votes in the senate that the first hypothesis is satisfied. The "no" votes in the senate have a positive influence on ΔAccess and a negative influence on ΔQuality, both being significant regardless of whether the OLS or the 3SLS model is used. The results also reveal an unexpected

Table 2. Baseline results

	OLS		3SLS	
VARIABLES	ΔAccess	ΔQuality	ΔAccess	ΔQuality
Bill contains quality reforms	0.401***		0.406***	
	(0.124)		(0.120)	
Bill contains access reforms		0.0618		0.0641
		(0.0496)		(0.0481)
Proportion of no votes in senate	1.577**	-0.714***	1.577**	-0.715***
	(0.721)	(0.229)	(0.700)	(0.222)
Proportion of no votes in assembly	0.472	0.725***	0.470	0.724***
	(0.753)	(0.239)	(0.730)	(0.232)
Proportion of democrats in senate	-3.544	-6.355	-3.515	-6.352
	(17.46)	(5.540)	(16.94)	(5.374)
Proportion of democrats in assembly	53.11***	-1.844	53.08***	-1.874
	(18.81)	(6.005)	(18.25)	(5.825)
Proportion of male senators	-52.21	3.853	-52.22	3.854
	(35.77)	(11.36)	(34.70)	(11.02)
Proportion of male assembly members	7.783	-0.403	7.841	-0.337
	(40.15)	(12.82)	(38.95)	(12.44)
Bill author is a democrat	-0.348**	0.0423	-0.348**	0.0425
	(0.168)	(0.0533)	(0.163)	(0.0517)
Number of legislative actions	-0.0315**	-0.00234	-0.0315**	-0.00235
	(0.0130)	(0.00413)	(0.0126)	(0.00401)
Senate bill	-0.386***	0.0157	-0.387***	0.0155
	(0.129)	(0.0411)	(0.125)	(0.0398)
Election year	5.337**	-0.806	5.062	-0.808
	(2.279)	(0.725)	(43.47)	(0.703)
Observations	272	272	272	272
R-squared	0.204	0.068	0.204	0.068
Year Fixed Effects	Yes	Yes	Yes	Yes

positive effect of "no" votes in the assembly on ΔQuality. One possible explanation for this could be that interest groups are more likely to target senators when lobbying since there are twice as many assembly members as there are senators, therefore a single vote weighs more in the senate than in the assembly.

As for the question of whether access reforms are used to compensate for bills that contain quality reforms, Table 2 reveals that indeed, keeping opposition constant, bills with quality reforms have a significantly larger ΔAccess than bills without quality reforms. Interestingly, the result does not hold in reverse. That is, the ΔQuality is not significantly different between bills with and without access reforms. This result, however, compares only those bills facing the same amount of opposition. A more complete picture of the compromise story would be to investigate, for bills containing quality reforms, how the compensation in access varies with the strength of the opposition. This could be answered by including interactions of the access and quality dummies with the proportion of "no" votes in both chambers. Table 3 presents the results of this exercise.

The base dummy of quality reforms continues to be positive and marginally significant for ΔAccess, however, the bulk of the positive effect we previously saw in the baseline table appear to be driven by the interaction between the quality dummy and the "no" votes in the senate. The positive and highly significant estimate for this interaction term coincides with the prediction that, for bills containing quality reforms, more compromises are made by adding more access reforms when the opposition to the bill is stronger. Indeed, the positive effect that we previously found for the proportion of "no" votes disappears when this interaction term in added, suggesting that opposition has no influence on access for bills without quality reforms.

Table 3. Results with interactions

	OLS		3SLS	
VARIABLES	ΔAccess	ΔQuality	ΔAccess	ΔQuality
Bill contains quality reforms	0.259*		0.260*	
	(0.152)		(0.147)	
Contains quality × no votes in senate	3.479**		3.469**	
	(1.441)		(1.392)	
Contains quality × no votes in assembly	-1.822		-1.812	
	(1.442)		(1.394)	
Bill contains access reforms		0.0287		0.0291
		(0.0565)		(0.0546)
Contains access × no votes in senate		-0.754		-0.755
		(0.717)		(0.693)
Contains access × no votes in assembly		1.288*		1.289*
		(0.720)		(0.695)
Proportion of no votes in senate	-0.333	-0.0398	-0.327	-0.0388
	(1.094)	(0.672)	(1.057)	(0.650)
Proportion of no votes in assembly	1.578	-0.412	1.572	-0.413
	(1.147)	(0.679)	(1.108)	(0.656)
Observations	272	272	272	272
R-squared	0.224	0.081	0.224	0.081
Controls	Yes	Yes	Yes	Yes
Year Fixed Effects	Yes	Yes	Yes	Yes

6 Conclusion

In this paper we show that in a game-theoretic setting with endogenously proposed reforms and two policy dimensions, the government who proposes the reform uses both policies as bargaining tools that results in a compromise between what the interest group supports and what it opposes. The resulting bundle of reforms contains more of the policy desired by the interest group, and less of the policy disfavored by the interest group.

These theoretical findings are validated empirically using legislative data on California education bills. We find that the proportion of "no" votes in the senate is associated positively with favored access reforms and negatively with disfavored quality reforms.

Our results are apparent in situations in which certain policies are traded off to gain the favor of opposed parties. During the heated 2009 debates regarding the United States' Obamacare health bill, certain concessions were made in order to secure the support of politicians to ensure the bill's passage, some of those policies on which compromises were made were the federal funding for abortion and the public health insurance option, which many agreed to be minor parts of the overall reform. Another example is Australia's recent and controversial Higher Education Bill of 2014, whose main purpose was to reform the funding system for Australian universities through deregulation of higher education fees. The bill has undergone a series of compromises since its introduction early in 2014, including the removal of the proposal to cut government funding to universities by 20%, in order to win over the labor party, the Greens, and resistant cross-bench politicians.

An extension of our analysis would be to see how our results hold in a generalized setting with more than one interest group and multiple policy dimensions.

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A Appendix

A.1 Proof of Lemma 1

The first order condition of the government and the interest group maximization problem are the following:

$$\Delta_G = \frac{\partial EU_G}{\partial e_G} = \frac{\partial p}{\partial e_G} U_G - 1 = 0 \tag{10}$$

$$\Delta_I = \frac{\partial EU_I}{\partial e_I} = -\frac{\partial p}{\partial e_I N} - 1 = 0 \tag{11}$$

Equations (10) and (11) imply

$$\frac{\partial p}{\partial e_G} = \frac{1}{U_G} \tag{12}$$

$$\frac{\partial p}{\partial e_I} = -\frac{1}{N} \tag{13}$$

The second order conditions are

$$\frac{\partial \Delta_G}{\partial e_G} = \frac{\partial^2 EU_G}{\partial e_G^2} = \frac{\partial^2 p}{\partial e_G^2} U_G \tag{14}$$

$$\frac{\partial \Delta_I}{\partial e_I} = \frac{\partial^2 EU_I}{\partial e_I^2} = -\frac{\partial^2 p}{\partial e_I^2} N. \tag{15}$$

which are as $\partial^2 p/\partial e_G^2 < 0$ and $\partial^2 p/\partial e_I^2 > 0$.

Given our contest success function defined as in 1, we have that:

$$\frac{\partial p}{\partial e_G} = \frac{\alpha_I e_I}{(\alpha_G e_G + \alpha_I e_I)^2} \tag{16}$$

$$\frac{\partial p}{\partial e_I} = -\frac{\alpha_I \,\alpha\text{GeG}}{(\alpha_G e_G + \alpha_I e_I)^2} \tag{17}$$

$$\frac{e_G}{e_I} = \frac{U_G}{\alpha_{\text{G}}\text{N}} \tag{18}$$

A.2 Proof of proposition 1

We can get to this result by showing that $\partial EU_G/\partial q\mid_{q=q_g^*}< 0$. Since q_g^* maximizes U_G by assumption, that is, $\partial U_G/\partial q\mid_{q=q_g^*}= 0$, then equation (5) collapses to

$$\left.\frac{\partial EU_G}{\partial q}\right|_{q=q_g^*} = -\frac{U_G}{N}\frac{\partial e_I}{\partial q_g^*}. \tag{19}$$

The above expression is negative if $\partial e_I/\partial q_g^* > 0$. To get the sign of $\partial e_I/\partial q_g^*$, totally differentiate (10) and (11) ,

$$\frac{\partial \Delta_G}{\partial e_G}de_G + \frac{\partial \Delta_G}{\partial e_I}de_I + \frac{\partial \Delta_G}{\partial q}dq = 0 \tag{20}$$

$$\frac{\partial \Delta_I}{\partial e_G}de_G + \frac{\partial \Delta_I}{\partial e_I}de_I + \frac{\partial \Delta_I}{\partial q}dq = 0. \tag{21}$$

Rearranging, we get

$$\frac{\partial \Delta_G}{\partial e_G}\frac{de_G}{dq} = -\frac{\partial \Delta_G}{\partial e_I}\frac{de_I}{dq} - \frac{\partial \Delta_G}{\partial q} \tag{22}$$

$$\frac{\partial \Delta_I}{\partial e_G}\frac{de_G}{dq} = -\frac{\partial \Delta_I}{\partial e_I}\frac{de_I}{dq} - \frac{\partial \Delta_I}{\partial q}. \tag{23}$$

Dividing (22) by (23) and cross multiplying yields

$$\frac{\partial \Delta_G}{\partial e_G}\left(\frac{\partial \Delta_I}{\partial e_I}\frac{de_I}{dq} + \frac{\partial \Delta_I}{\partial q}\right) = \frac{\partial \Delta_I}{\partial e_G}\left(\frac{\partial \Delta_G}{\partial e_I}\frac{de_I}{dq} + \frac{\partial \Delta_G}{\partial q}\right)$$

Finally, isolating de_I/dq,

$$\frac{de_I}{dq} = \frac{\dfrac{\partial \Delta_I}{\partial e_G}\dfrac{\partial \Delta_G}{\partial q} - \dfrac{\partial \Delta_G}{\partial e_G}\dfrac{\partial \Delta_I}{\partial q}}{\dfrac{\partial \Delta_G}{\partial e_G}\dfrac{\partial \Delta_I}{\partial e_I} - \dfrac{\partial \Delta_I}{\partial e_G}\dfrac{\partial \Delta_G}{\partial e_I}} \tag{24}$$

Obtaining the sign of (24) requires the signs of $\partial\Delta_i/\partial e_j$, $\partial\Delta_i/\partial e_i$, and $\partial\Delta_i/\partial q$, $\forall i, j = G, I$. Using equations (10) , (11) , (12) and (13) , we get

$$\frac{\partial \Delta_G}{\partial e_I} = \frac{\partial^2 p}{\partial e_G \partial e_I} U_G \tag{25}$$

$$\frac{\partial \Delta_I}{\partial e_G} = -\frac{\partial^2 p}{\partial e_I \partial e_G} N \tag{26}$$

$$\frac{\partial \Delta_G}{\partial e_G} = \frac{\partial^2 p}{\partial^2 e_G} U_G \tag{27}$$

$$\frac{\partial \Delta_I}{\partial e_I} = -\frac{\partial^2 p}{\partial^2 e_I} N \tag{28}$$

$$\frac{\partial \Delta_G}{\partial q} = \frac{\partial p}{\partial e_G} \frac{\partial U_G}{\partial q} = \frac{1}{U_G} \frac{\partial U_G}{\partial q} \tag{29}$$

$$\frac{\partial \Delta_I}{\partial q} = -\frac{\partial p}{\partial e_I} \frac{\partial N}{\partial q} = \frac{1}{N} \frac{\partial N}{\partial q} \tag{30}$$

Therefore, (24) becomes

$$\frac{de_I}{dq} = \frac{\frac{\partial^2 p}{\partial e_I \partial e_G} \frac{\partial U_G}{\partial q} \frac{N}{U_G} + \frac{\partial^2 p}{\partial^2 e_G} \frac{\partial N}{\partial q} \frac{U_G}{N}}{N U_G \frac{\partial^2 p}{\partial^2 e_I} \frac{\partial^2 p}{\partial^2 e_G} - \frac{\partial^2 p}{\partial e_I \partial e_G} \frac{\partial^2 p}{\partial e_G \partial e_I}} \tag{31}$$

At the ideal point q_g^*, we know that U_G is maximized such that $\partial U_G / \partial q|_{q=q_g^*} = 0$. Therefore the first term in the numerator drops out and we are left with

$$\left.\frac{de_I}{dq}\right|_{q=q_g^*} = \frac{\frac{\partial^2 p}{\partial^2 e_G} \frac{\partial N}{\partial q}}{N^2 \left[\frac{\partial^2 p}{\partial e_I^2} \frac{\partial^2 p}{\partial e_G^2} - \left(\frac{\partial^2 p}{\partial e_I \partial e_G} \right)^2 \right]} > 0 \tag{32}$$

From the second order conditions (14) and (15) , we know that $\partial^2 p / \partial^2 e_G < 0$ and $\partial^2 p / \partial^2 e_I > 0$. And since $\partial N / \partial q = -\partial U_I / \partial q > 0$, then equation (32) must be positive.

A.3 Proof of proposition 2

We can get to this result by showing that $\partial EU_G / \partial a_g^* > 0$ and $\partial EU_G / \partial q_g^* < 0$.

In the first stage, G maximizes his expected utility with respect to a and q. The first order conditions are

$$\frac{\partial EU_G}{\partial a} = p \frac{\partial U_G}{\partial a} + U_G \left(\frac{\partial p}{\partial e_G} \frac{\partial e_G}{\partial a} + \frac{\partial p}{\partial e_I} \frac{\partial e_I}{\partial a} \right) - \frac{\partial e_G}{\partial a} = 0 \tag{33}$$

$$\frac{\partial EU_G}{\partial q} = p \frac{\partial U_G}{\partial q} + U_G \left(\frac{\partial p}{\partial e_G} \frac{\partial e_G}{\partial q} + \frac{\partial p}{\partial e_I} \frac{\partial e_I}{\partial q} \right) - \frac{\partial e_G}{\partial q} = 0, \tag{34}$$

which by (12) and (13) can be simplified to

$$\frac{\partial EU_G}{\partial a} = p\frac{\partial U_G}{\partial a} - \frac{U_G}{N}\frac{\partial e_I}{\partial a} = 0$$

$$\frac{\partial EU_G}{\partial q} = p\frac{\partial U_G}{\partial q} - \frac{U_G}{N}\frac{\partial e_I}{\partial q} = 0.$$

Since (a_g^*, q_g^*) maximizes UG by assumption, $\frac{\partial U_G}{\partial a} \mid_{(a,q)=(a_g^*,q_g^*)} = 0$ and $\frac{\partial U_G}{\partial q} \mid_{(a,q)=(a_g^*,q_g^*)} = 0$, so that the above conditions are reduced to

$$\frac{\partial EU_G}{\partial a_g^*} = \left.\frac{\partial EU_G}{\partial a}\right|_{(a,q)=(a_g^*,q_g^*)} = -\frac{U_G}{N}\frac{\partial e_I}{\partial a} \tag{35}$$

$$\frac{\partial EU_G}{\partial q_g^*} = \left.\frac{\partial EU_G}{\partial q}\right|_{(a,q)=(a_g^*,q_g^*)} = -\frac{U_G}{N}\frac{\partial e_I}{\partial q}. \tag{36}$$

To get the signs of $\partial EU_G/\partial a_g^*$ and $\partial EU_G/\partial q_g^*$, we need the signs of $\partial e_I/\partial a$ and $\partial e_I/\partial q$. Totally differentiating (10) and (11) ,

$$\frac{\partial \Delta_G}{\partial e_G}de_G + \frac{\partial \Delta_G}{\partial e_I}de_I + \frac{\partial \Delta_G}{\partial a}da + \frac{\partial \Delta_G}{\partial q}dq = 0 \tag{37}$$

$$\frac{\partial \Delta_I}{\partial e_G}de_G + \frac{\partial \Delta_I}{\partial e_I}de_I + \frac{\partial \Delta_I}{\partial a}da + \frac{\partial \Delta_I}{\partial q}dq = 0 \tag{38}$$

Dividing (37) and (38) by da and isolating $\partial e_G/\partial a$ to one side yields,

$$\frac{\partial \Delta_G}{\partial e_I}\frac{de_I}{da} + \frac{\partial \Delta_G}{\partial a} + \frac{\partial \Delta_G}{\partial q}\frac{dq}{da} = -\frac{\partial \Delta_G}{\partial e_G}\frac{de_G}{da} \tag{39}$$

$$\frac{\partial \Delta_I}{\partial e_I}\frac{de_I}{da} + \frac{\partial \Delta_I}{\partial a} + \frac{\partial \Delta_I}{\partial q}\frac{dq}{da} = -\frac{\partial \Delta_I}{\partial e_G}\frac{de_G}{da} \tag{40}$$

Dividing equation (39) by (40) and cross multiplying yields,

$$\frac{\partial \Delta_I}{\partial e_G}\left(\frac{\partial \Delta_G}{\partial e_I}\frac{de_I}{da} + \frac{\partial \Delta_G}{\partial a} + \frac{\partial \Delta_G}{\partial q}\frac{dq}{da}\right) = \frac{\partial \Delta_G}{\partial e_G}\left(\frac{\partial \Delta_I}{\partial e_I}\frac{de_I}{da} + \frac{\partial \Delta_I}{\partial a} + \frac{\partial \Delta_I}{\partial q}\frac{dq}{da}\right)$$

Finally, combining like terms and isolating $\partial e_I/\partial a$ to one side results in,

$$\frac{de_I}{da} = \frac{\frac{\partial \Delta_G}{\partial e_G}\frac{\partial \Delta_I}{\partial a} + \frac{\partial \Delta_I}{\partial q}\frac{dq}{da} - \frac{\partial \Delta_G}{\partial e_G}\frac{\partial \Delta_G}{\partial a} + \frac{\partial \Delta_G}{\partial q}\frac{dq}{da}}{\frac{\partial \Delta_I}{\partial e_G}\frac{\partial \Delta_G}{\partial e_I} - \frac{\partial \Delta_G}{\partial e_G}\frac{\partial \Delta_I}{\partial e_I}} \tag{41}$$

Therefore, using equations (14) , (15) and (25) - (30) , the expression in (35) becomes

$$\frac{de_I}{da} = \frac{\frac{\partial^2 p}{\partial e_G^2} U_G \frac{1}{N} \frac{\partial N}{\partial a} + \frac{1}{N} \frac{\partial N}{\partial q} \frac{dq}{da} + \frac{\partial^2 p}{\partial e_I \partial e_G} N \frac{1}{U_G} \frac{\partial U_G}{\partial a} + \frac{1}{U_G} \frac{\partial U_G}{\partial q} \frac{dq}{da}}{\left(-\frac{\partial^2 p}{\partial e_I \partial e_G} N\right) \frac{\partial^2 p}{\partial e_I \partial e_G} U_G + \frac{\partial^2 p}{\partial e_G^2} U_G \frac{\partial^2 p}{\partial e_I^2} N} \tag{42}$$

Recall that at the government's ideal point (a_g^*, q_g^*), $\partial U_G / \partial q_g^* = 0$ and $\partial U_G / \partial a_g^* = 0$, hence the second term in the numerator drops out and we get

$$\left.\frac{de_I}{da}\right|_{(a,q)=(a_g^*, q_g^*)} = \frac{\frac{\partial^2 p}{\partial e_G^2} \frac{\partial N}{\partial a} + \frac{\partial N}{\partial q} \frac{dq}{da}}{N^2 \frac{\partial^2 p}{\partial e_G^2} \frac{\partial^2 p}{\partial e_I^2} - \frac{\partial^2 p}{\partial e_I \partial e_G}^2} < 0 \tag{43}$$

Note that $\partial^2 p / \partial e_G^2 < 0$, $\partial^2 p / \partial e_I^2 > 0$, making the denominator of (43) negative. Thus, the sign of the numerator comes from the fact that the expression in parenthesis is equal to $dN/da = -dU_I/da$, which is negative, by construction of $U_I(a, q)$. Equation (43) tells us that at G's desired reform point, the lobbying effort of the interest group decreases with more access-broadening reforms proposed. This implies that the sign of (35) becomes

$$\frac{\partial EU_G}{\partial a_g^*} = -\frac{U_G}{N} \frac{\partial e_I}{\partial a_g^*} > 0 \tag{44}$$

The expression in (44) means that the proposed access reform $\bar{a}$ is somewhere to the right of the optimum level without opposition, a_g^*. Using the same procedure, we derive $\partial e_I / \partial q$ to get the sign of $\partial EU_G / \partial q_g^*$. From (20) and (21) , divide both equations by dq and isolate de_G/dq.

$$\frac{\partial \Delta_G}{\partial e_I} \frac{de_I}{dq} + \frac{\partial \Delta_G}{\partial q} + \frac{\partial \Delta_G}{\partial a} \frac{da}{dq} = -\frac{\partial \Delta_G}{\partial e_G} \frac{de_G}{dq} \tag{45}$$

$$\frac{\partial \Delta_I}{\partial e_I} \frac{de_I}{dq} + \frac{\partial \Delta_I}{\partial q} + \frac{\partial \Delta_I}{\partial a} \frac{da}{dq} = -\frac{\partial \Delta_I}{\partial e_G} \frac{de_G}{dq} \tag{46}$$

Dividing the (45) by (46) and cross multiplying,

$$\frac{\partial \Delta_I}{\partial e_G} \left(\frac{\partial \Delta_G}{\partial e_I} \frac{de_I}{dq} + \frac{\partial \Delta_G}{\partial q} + \frac{\partial \Delta_G}{\partial a} \frac{da}{dq} \right) = \frac{\partial \Delta_G}{\partial e_G} \left(\frac{\partial \Delta_I}{\partial e_I} \frac{de_I}{dq} + \frac{\partial \Delta_I}{\partial q} + \frac{\partial \Delta_I}{\partial a} \frac{da}{dq} \right)$$

Isolating de_I/dq to one side yields

$$\frac{de_I}{dq} = \frac{\frac{\partial \Delta_G}{\partial e_G} \frac{\partial \Delta_I}{dq} + \frac{\partial \Delta_I}{\partial a} \frac{da}{dq} - \frac{\partial \Delta_G}{\partial e_G} \frac{\partial \Delta_G}{dq} + \frac{\partial \Delta_G}{\partial a} \frac{da}{dq}}{\frac{\partial \Delta_I}{\partial e_G} \frac{\partial \Delta_G}{\partial e_I} - \frac{\partial \Delta_G}{\partial e_G} \frac{\partial \Delta_I}{\partial e_I}} \tag{47}$$

Again, using (14) , (15) and (25) to (30) , we get

$$\frac{de_I}{dq} = \frac{\dfrac{\partial^2 p}{\partial e_G^2} U_G \dfrac{1}{N}\dfrac{\partial N}{\partial q} + \dfrac{1}{N}\dfrac{\partial N}{\partial a}\dfrac{\partial a}{dq} + \dfrac{\partial^2 p}{\partial e_I \partial e_G} N \dfrac{1}{U_G}\dfrac{\partial U_G}{\partial q} + \dfrac{1}{U_G}\dfrac{\partial U_G}{\partial a}\dfrac{da}{dq}}{\left(-\dfrac{\partial^2 p}{\partial e_I \partial e_G} N\right)\dfrac{\partial^2 p}{\partial e_I \partial e_G} U_G + \dfrac{\partial^2 p}{\partial e_G^2} U_G \dfrac{\partial^2 p}{\partial e_I^2} N} \tag{48}$$

At G's ideal point (a_g^*, q_g^*), the second term in the numerator drops out and we are left with

$$\left.\frac{de_I}{dq}\right|_{(a,q)=(a_g^*,q_g^*)} = \frac{\dfrac{\partial^2 p}{\partial e_G^2}\dfrac{\partial N}{\partial q} + \dfrac{\partial N}{\partial a}\dfrac{da}{dq}}{N^2 \dfrac{\partial^2 p}{\partial e_G^2}\dfrac{\partial^2 p}{\partial e_I^2} - \dfrac{\partial^2 p}{\partial e_I \partial e_G}^2} > 0 \tag{49}$$

The denominator of (49) is negative, as in equation (43) . The positive sign therefore comes from the fact that $\partial^2 p/\partial e_G^2 < 0$ and the expression in parenthesis in the numerator is equal to $dN/dq = -dU_I/dq$, which is positive given the assumptions made for $U_I(a,q)$. This implies that the sign of (36) is

$$\frac{\partial EU_G}{\partial q_g^*} = -\frac{U_G}{N}\frac{\partial e_I}{\partial q} < 0. \tag{50}$$