The Housing Cost Disease *

Nicola Borri — LUISS

Pietro Reichlin — LUISS & CEPR

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Abstract

We use a simple two-sector life cycle economy with bequests to explain the rising trends in wealth-to-income ratios, housing wealth and wealth inequality that have been documented in most advanced countries at least since the '70s, as a consequence of a rising labor efficiency in manufacturing (*housing cost disease*). When consumption inequality across households is sufficiently large, the housing cost disease has adverse effects on a measure of social welfare based on an egalitarian principle: the higher the housing's value appreciation, the lower the welfare benefit of a rising labor efficiency in manufacturing.

KEYWORDS: Housing Wealth, Cost Disease, Overlapping Generations, Wealth Inequality.

JEL CODES: D91, O11, H2, G1.

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*Borri: Department of Economics and Finance, LUISS University, Viale Romania 32, 00197 Rome, Italy; nborri@luiss.it; Tel: +39 06 85225959; http://docenti.luiss.it/borri. Reichlin: Department of Economics and Finance, LUISS University, Viale Romania 32, 00197 Rome, Italy; preichlin@luiss.it; Tel: +39 06 85225554; http://docenti.luiss.it/reichlin. We thank seminar participants at Barcelona GSE, University of Tokyo, Bank of Japan, Kyoto University. A previous version of this paper circulated as "Economic expansions and housing booms." Support of a grant from MURST, PRIN 2010, is gratefully acknowledged. We thank Alberto Martin, Ingvar Strid, Gregory Thwaites and Thomas Steger for helpful comments.

1 Introduction

Wealth-to-income ratios have been increasing in most advanced economies at least since the 1970s. Since wealth is unevenly distributed, a concern for a widening inequality gap is emerging in academic and non-academic circles. In this paper, we build a frictionless two-sector life-cycle model with bequests able to replicate most of the stylized facts concerning the dynamic of total wealth, housing wealth, and wealth inequality, as a consequence of an improvement in the efficiency of labor in the general economy relative to the construction sector.

In a series of influential papers, Piketty (2014), Piketty and Zucman (2014) and Piketty and Saez (2014), attribute the rising wealth-to-income ratios to the falling income growth rates and the long-run stability of the saving rate[1], and claim that these trends are responsible for the rising income and wealth inequality. Piketty (2014) has famously recommended a worldwide tax on capital as a way to curb this phenomenon. This interpretation of the historical trends in wealth and inequality is problematic for at least two reasons. First, the joint behavior of the saving and growth rates in advanced economies since 1970 does not explain the rising wealth-to-income ratios on the basis of a simple application of the Solow model (*i.e.*, on average, net saving rates fell by as much, if not more, than income growth rates). Second, the existing trends in wealth-to-income ratios and income shares are strongly determined by the dynamic of housing wealth and capital gains, as documented in recent research by Bonnet et al. (2014), Rognlie (2014) and Weil (2015).

In this paper we show that a sort of Baumol's cost disease may be responsible for the increase in the housing share of wealth, wealth-to-income ratios and wealth inequality. In the seminal work by Baumol (1967), a market economy has two sectors producing two goods using labor as the only input and enjoying different patterns of

[1] However, the theoretical underpinning of this theory has been challenged by Krusell and Smith (2015).

technological progress. Under perfect labor mobility and wage equalization, a rising labor productivity in the dynamic sector generates a higher production cost and, then, a rising relative output price in the stagnant sector (for example, to use one of Baumol's examples, music played by a horn quintet). If the demand of the stagnant sector output is sufficiently inelastic, labor will move to this sector and aggregate output growth may decline. We extend Baumol's analysis to a model with capital and land, where construction plays the role of the stagnant sector, while manufacturing experiences labor-augmenting technological progress, and we define the conditions for a rise in the efficiency of labor in manufacturing to generate a strong housing price appreciation, a rise in the wealth-to-income ratio (mostly driven by higher housing wealth and a weak dynamic in average labor productivity) and in the size of bequests (*i.e.*, a rising wealth inequality). We refer to this set of results as *the housing cost disease*.

We show that the empirical evidence supports the housing cost disease interpretation. First, we document the existence of a positive and large correlation between a labor-augmenting productivity residual in the manufacturing sector, relative to the one in construction, and the total and housing wealth-to-income ratios for a sample of the 8 largest economies over the 1970-2007 period. Second, contrary to the prediction of the Solow growth model and to Piketty's own interpretation, total and housing wealth-to-income ratios appear to be poorly correlated with income growth and saving rates since 1970. In particular, the fall in saving rates experienced by many advanced economies in the last forty years more than compensate for the fall in growth rates.

With this paper, we build a bridge between two different strands of the literature. The first, attributes most of the the long-run increasing trend in wealth and inequality to saving induced growth (for example, Piketty (2014), Piketty and Zucman (2014) and Piketty and Saez (2014)). The second, explains movements in housing prices as a result of sectoral changes in productivity (for example, Kahn (2008), Iacoviello and Neri (2010) and Moro and Nuno (2012)). We are the first, to the best of our knowledge, to develop a general equilibrium model that relates long-run trends in wealth and inequality to relative growth in labor efficiency and housing prices.

We consider an economy with overlapping generations of heterogeneous altruistic households living for two periods, supplying their labor inelastically when young, and deriving utility from housing services. The only source of heterogeneity between individuals is the degree of parental altruism, represented by a discount rate applied to the next generation's utility. Consistently with the assumption of one-sided altruism we assume that parents cannot force gifts on their children. Hence, this heterogeneity generates a partition of the set of households at steady states into a subset of *rich* individuals receiving bequests from their parents and a subset of *poor* individuals receiving (and giving) no bequests. A key assumption is that housing is a produced good (with labor, capital and land as inputs) and the economy has two sectors: manufacturing and construction. A multi-sector approach is important to study the evolution of wealth's composition in advanced economies (vs. economies in the early stage of the development process) since housing replaced land in households' assets and because factor price equalization across sectors generates interesting linkages between the dynamics of productivity and asset prices.

We show that the housing cost disease is most likely under the assumptions that manufacturing is more capital intensive than construction, the land share of income is sufficiently small, housing demand is sufficiently inelastic with respect to its own price, and the elasticity of substitution between capital and labor in the construction sector is close to one. These two elasticities play an important role in our model. In particular, with unitary elasticity of substitution between capital and labor in construction and between goods (consumption and housing services) in a CES representation of preferences, a rising productivity in manufacturing is allocation neutral, in the sense that total and housing wealth-to-income ratios, as well as the shares of labor across sectors, remain unchanged, whereas bequests (of rich households) increase on a one to one basis.

When, instead, housing demand is sufficiently inelastic, the housing cost disease holds and bequests increase more than proportionally with manufacturing productivity.

We use our model to evaluate the effect of the housing cost disease on social welfare. Piketty (2014) advocates the institution of a wealth tax on the assumption that the increase in wealth-to-income is not desirable because of the implications in terms of unequal distribution of wealth across households. However, if the increase in housing prices depends on the rising relative productivity of non-construction sectors, then policies targeting specifically the housing sector are probably more appropriate, as noted for example by Auerbach and Hasset (2015)[2]. We leave for future research the evaluation of such policies, and use, instead, the model to see if a change in the composition of wealth toward housing, following a rise of efficiency in manufacturing, is desirable in terms of an egalitarian welfare criterion. Deaton and Laroque (2001) investigated a similar question and found that the presence of a market for housing determines a portfolio reallocation away from capital towards housing, causing the accumulation of capital to fall short of the *Golden Rule* level. Our welfare criterion is based on an egalitarian welfare function that takes into account the households heterogeneous degrees of altruism with respect to the next generations and allows for unrestricted transfers across generations. We conclude that, when housing appreciation is sufficiently strong and consumption inequality sufficiently large, the steady state welfare benefit of a rising labor efficiency in manufacturing is lower than it would be in the planning optimum. In principle, a housing appreciation has two opposite effects on welfare. First, it raises the wealth of the poor old households so as to relax the non-negativity constraint on bequest values. Second, it makes housing less affordable. The last effect appears to be stronger than the former and detrimental to an egalitarian social welfare measure when poor households' consumption is too low. Our results cast

[2]Note that housing taxation is, in any case, very controversial, since housing is a consumption good, as well as an asset, and home ownership is much more evenly distributed across individuals than stocks and other financial assets.

some doubts on the benefits of a wealth tax under a (non-paternalistic) egalitarian perspective, as a wealth tax is likely to raise housing prices.

The remainder of the paper is organized as follows. Section 2 present some stylized facts on wealth ratios and inequality for eight advanced economies. Section 4 introduces the model and characterizes steady states with bequests. Section **??** shows the effects of improvements in relative labor productivity and the conditions for a housing cost disease. Section 6 discusses the welfare implications. Section 7 concludes.

2 Stylized Facts

In this section, we combine the empirical evidence from different sets of stylized facts supporting the *housing cost disease*. First, we look at the evolution of national wealth and of one of its main components, housing wealth, using data from Piketty and Zucman (2014). Second, we look at the evolution of wealth and income inequality, using data from Piketty and Saez (2014)'s Top World Income Database. Third, we show the existence of a positive relationship between long-run changes in wealth-to-income ratios and relative productivity improvements in manufacturing, with respect to construction, estimated starting from O'Mahony and Timmer (2009)'s KLEMS data.

Piketty and Zucman (2014) have put together an incredibly rich dataset on wealth and income, starting from national accounts data, for the period 1970-2010, for the largest eight developed economies: the United States, Germany, the United Kingdom, Canada, Japan, France, Italy and Australia[3]. All assets and liabilities are valued at prevailing market prices, rather than estimated starting from the sums of previous investment flows. Private wealth is net wealth of households, and assets include all non-financial and financial assets. Public wealth is net wealth of public administrations

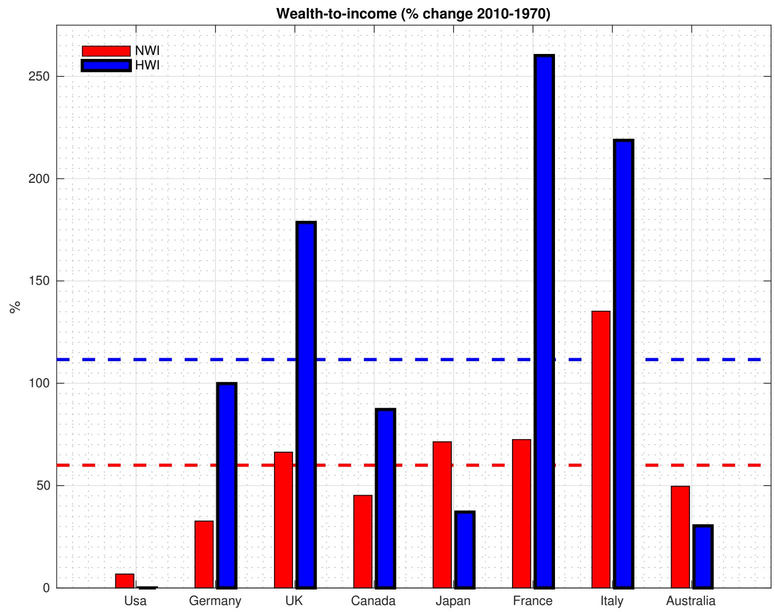
[3]For a smaller subset of countries, Piketty and Zucman (2014) provide longer time-series. However, we choose to restrict our focus on a time-period for which we could maximize the number of countries in the sample and with more reliable data. For a detailed description of the data refer to section A in the Appendix or directly to Piketty and Zucman (2014).

and government agencies. National wealth is the sum of private and public wealth. Note that the financial component of private wealth includes households' holdings of domestic public debt, even though, at the national level, holdings of domestic debt are netted out, as they are a liability of the public sector. Housing wealth is one of the components of total wealth, and it measures the net value of households' real-estate holdings. In this section we focus on national, rather than private, and housing wealth, while we present data on households' private wealth in a separate appendix, as public debt should not be part of individuals' net (of the present value of future taxes) wealth over the long run. In this case, housing wealth increased also in the US.

In the eight largest economies, national wealth increased substantially more than income over the period 1970 to 2010. Figure 1 shows that, on average, the national wealth-to-income ratio increased by about 60 percent (red horizontal dashed-line). Interestingly, housing wealth increased even more, on average by about 112 percent (blue horizontal dashed-line). There exist important cross-country differences. For example, Italy is the country with the largest increase in the ratio between national and housing wealth-to-income: 135 and 218 percent, respectively. On the contrary, in the US the national wealth-to-income ratio increased only by 6 percent, while housing wealth-to-income decreased by about 19 percent. In a separate appendix we show that these figures are robust to ending the sample in 2007, before the Great Recession, and to computing percentage changes using five-year averages at the beginning and end of the sample.

Since wealth is typically unevenly distributed, it is not surprising that, over the same 1970 to 2010 period, income and wealth inequality increased along with the wealth-to-income ratio. Figure 2 shows that, on average, the shares going to the top 1 and 10 percent of the income distribution increased, respectively, by 46 and 20 percent. Similarly, also the shares going to the top 1 and 10 percent of the wealth distribution increased, by approximately 18 and 7 percent. While the US is the country with the

Figure 1: Households Wealth-to-Income Ratios (percentage changes 2010-1970)

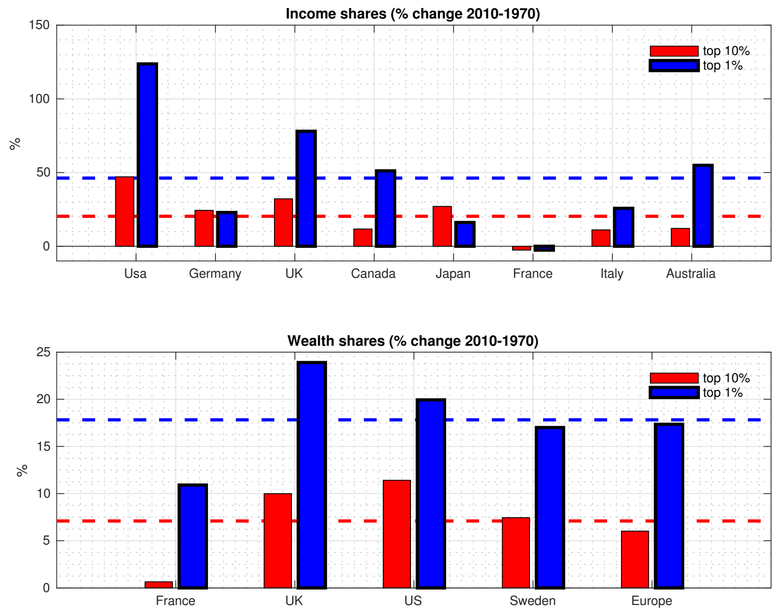


Notes: This figure plots the percentage changes in national (NWI) and housing (HWI) wealth-to-income ratios over the sample 2010-1970. For Australia the sample is 2010-1978. Horizontal dashed-lines correspond to the cross-sectional averages (60 and 112 percent respectively). Both national and housing wealth are annual, at market prices, from Piketty and Zucman (2014). Additional details on the data are available in the Appendix (section A).

smallest increase in the wealth-to-income ratio, it is also the country with the largest increase in both income and wealth inequality. In fact, the shares going to the top 1 and 10 percent of the income distribution increased, respectively, by 123 and 47 percent; those going to the top 1 and 10 percent of the wealth distribution increased, respectively, by 20 and 11 percent.

The reported large increase in wealth ratios and inequality is not a novel result. In fact, it is at the center not only of the academic, but also of the political debate. We are rather interested in understanding the potential explanations behind these facts and the conditions that make them more likely to occur. The main conjecture investigated in this paper is the positive effect of relative productivity improvements in manufacturing on the wealth ratios and inequality, with a strong effect on housing wealth: this is what we refer to as the *housing cost disease*. To provide some initial empirical support to our claim, we combine the data on wealth ratios, for the eight largest advanced

Figure 2: Income and Wealth Shares (percentage changes 2010-1970)

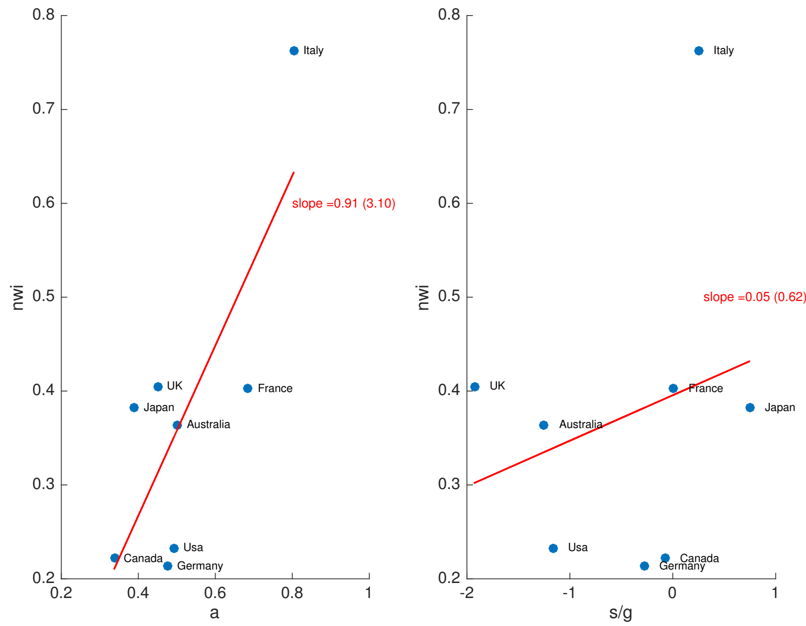


Notes: This figure plots the percentage changes, over the sample 2010-1970, of the income (top panel) and wealth (bottom panel) shares of the the top 10 (red bars) and 1 (blue bars) percent of the distribution. The income shares are for the US, Germany, the UK, Japan, France, Italy and Australia. The wealth shares are for France, the UK, the US, Sweden and Europe. Data on income (wealth) shares are annual (decennial) from Piketty and Saez (2014)'s World Top Income Database. For the UK the sample is 2008-1971; for Sweden 2000-1970; for Italy 2005-1970. Additional details on the series are available in the Appendix (section A).

economies, with data on relative productivity in manufacturing, estimated from the O'Mahony and Timmer (2009)'s EU KLEMS Growth and Productivity Accounts. In particular, we estimate relative productivity improvements by feeding sectoral data on gross value added, capital and labor inputs to production functions following the technological assumptions described in details in the sections that introduce the model. In particular, relative labor efficiency in manufacturing is estimated under the assumption of Cobb-Douglas production function in manufacturing, and CES with elasticity smaller than unity in construction. In the left panel of figure 3, we plot long-run log changes in national wealth-to-income against long-run log changes in relative productivity improvements in manufacturing. The sample starts in 1970 and ends in 2007, as the latter is the last year of coverage for the data from KLEMS. There exists a clear positive relationship between relative productivity and national wealth, with an estimated elasticity of 0.91 percent. One alternative explanation discussed in the literature, suggested by Piketty and Zucman (2014) on the basis on the classic Solow model, is that, in the long-run, the wealth-to-income ratio should be equal to the ratio between the (net of depreciation) saving and income growth rate. However, for the eight largest advanced economies, the relationship between long-run log changes in national wealth and the ratio between saving and income growth rates is weak at best. In fact, the estimated elasticity is equal to 0.05 percent and not statistically significant at conventional levels. In the appendix to this paper, we show that the relationship between housing wealth, one of the main component of total wealth, and relative productivity improvements is even stronger, with an estimated elasticity of 1.8 percent. On the contrary, there is no clear relationship between long-run changes in housing wealth and the saving to income growth ratio.

With this section we presented empirical regularities supporting our theoretical conjecture on the housing cost disease. However, our empirical findings do not prove any causal link, but rather the existence of co-movements, and are subject to several issues. First, the small number of countries in the sample. Second, the limited time-length of the sample as the objective is to analyze long-run trends. Third, possible measurement errors in both Piketty and Zucman (2014) and O'Mahony and Timmer (2009)'s data. In a separate appendix we address some of these issues, for example by considering the relationship between long-run changes in housing prices, a key driver of housing wealth, and relative productivity improvements, for a larger sample of sixteen OECD countries. In the next sections, we present a model that shows under what conditions the *housing cost disease* is more likely.

Figure 3: Housing Cost Disease vs. Piketty's View



Notes: This figure plots long-run log changes in national wealth-to-income against long-run log changes in relative labor efficiency in manufacturing (left panel, a) and in the ratio between national saving and income growth rates (right panel, s/g) for the US, Germany, the UK, Japan, France, Italy and Australia. The log changes are computed between the average values of each variable for the period 2007-2004 and 1974-1970. Relative labor efficiency is estimated from O'Mahony and Timmer (2009)'s KLEMS data assuming Cobb-Douglas technology for the manufacturing sector and CES technology with elasticity of substitution equal to $\sigma^h = 0.6$ for the construction sector. The red lines correspond to OLS fitted values. We also report estimates for the OLS slopes (t-stats in brackets). Data for national wealth-to-income, saving and income growth rates are from Piketty and Zucman (2014). Additional details on the series are available in the Appendix (section A).

3 A Simplified Housing Cost Disease

To explain these stylized facts it is useful to begin with a simplified model with no capital along the lines of Baumol (1967) to lay down the intuition behind our results. Denote with a the exogenous labor productivity in manufacturing and assume that both the manufacturing and the housing sectors use labor as the only input, *i.e.*,

$$Y^m = aL^m, \qquad Y^h = L^h,$$

where Y^j denotes the sector-specific output and L^j the sector specific labor for $j = h, m$. By perfect competition, profit maximization and perfect labor mobility, it is straightforward to derive:

$$W = q^h = a,$$

where W is the market wage and q^h the (relative) price of a unit of housing. Denoting with L the total workforce, per-capita income is simply:

$$y = (Y^m + q^h Y^h)/L = (aL^m + aL^h)/L = a.$$

Now assume that the economy is in a steady state with a constant per-capita demand of the housing stock, h^d, and denote with n and δ the per-period population growth and housing depreciation rates. Then,

$$Y^h/L = (n + \delta)h^d.$$

A natural assumption is that $h^d = h^d(\pi, W)$, with π denoting the user cost of housing, *i.e.*, the cost of a unit of housing net of the discounted (un-depreciated) resale value. Since we are at steady state, we have

$$\pi = q^h - (1 - \delta)q^h/(1 + r) = q^h \left(\frac{\delta + r}{1 + r}\right)$$

for some exogenously fixed rate of interest, r. Then, recalling that $W = q^h = a$,

$$h^d = h^d \left(a \left(\frac{\delta + r}{1 + r}\right), a\right).$$

Since housing is the only source of households' wealth, the wealth-to-income ratio is:

$$\beta \equiv \frac{q^h h^d}{y} = h^d.$$

Then, the effect of a rising productivity on the wealth-to-income ratio depends on the elasticities of housing demand with respect to its own price ($\hat{h}^d_\pi$) and wage income ($\hat{h}^d_W$). In particular, the percentage change in β generated by a one percent increase in a is equal to the sum of these two elasticities, *i.e.*,

$$\frac{\partial \beta/\beta}{\partial a/a} = \hat{h}^d_\pi + \hat{h}^d_W.$$

Observe that, with an homothetic representation of preferences, β increases with a if and only if housing demand is inelastic with respect to its own price, *i.e.*, $-\hat{h}^d_\pi < 1$. This condition squares with most empirical estimations. In fact, the general consensus in the existing literature on housing demand is that both income and price elasticities are relatively small in absolute value, the first ranging between 0.5 to 1, and the second between -1 and and -0.5 (for example, Mayo (1981), Hansen et al. (1996)). In what follows, we show that this basic intuition holds in a more sophisticated life-cycle economy with capital and land. In particular, we show that, under some general assumptions on preferences and technology, the wealth-to-income ratio does not depend on the relative labor productivity of manufacturing when both the housing demand own price elasticity and the elasticity of substitution between factor inputs in construction are equal to one. On the other hand, a relatively inelastic housing demand, paired with an increase in relative labor productivity, generates a rise in the wealth-to-income ratio and an increase in wealth inequality.

4 The Model

In this section we present a simple life-cycle model with bequests, two sectors (construction and manufacturing), three assets (business capital, housing and land), and exogenous technical progress. Both sectors use labor and capital and land is used in construction only. Assuming that capital and labor are perfectly mobile across sectors and firms are competitive, we identify conditions on technologies and households' preferences compatible with the stylized facts characterizing the largest advanced economies presented in section 2. We show that, when housing demand is sufficiently inelastic and the land share of output is sufficiently small, a rise in labor efficiency in manufacturing generates a set of phenomena that we label *housing cost disease*. Namely, for a plausible parametrization of the model, there is a sizable increase in the steady state values of total and housing wealth-to-income ratios, bequests (*i.e.*, wealth inequality), housing and land prices, share of labor in construction. It is worth noticing that housing wealth increases much more than total wealth and this is mainly due to a price effect. Furthermore, bequests rise substantially (more than a 100% for any 100% rise in productivity) and that, contrary to the model with labor as the only input, average labor productivity increases less than proportionally with productivity (*i.e.*, the housing cost disease generates a stagnant measured overall productivity). Finally, these phenomena are all compatible with a modest increase in the labor share employed in the construction sector.

4.1 Production

We now introduce a more sophisticated life-cycle model with capital and land starting with the description of the production sector. Manufacturing output (Y^m) can be consumed or used as capital in both sectors and construction output (Y^h) corresponds to investment in new housing. Technology is represented by the production functions

$$Y_t^h = F_h(K_t^h, A_t^h L_t^h, Z_t), \quad Y_t^m = F_m(K_t^m, A_t^m L_t^m),$$

where, for $j = h, m$, K^j and L^j are the amounts of capital and labor employed in the two sectors, A^j is a labor-augmenting technological level and Z represents the stock of land available for residential development. We let this stock be time-dependent because the government, or some other public authority, may decide to change the amount of land available for construction to enact specific policies or respond to demographic variables. To simplify the exposition, we assume that the proceeds from selling land permits are used by the government to finance wasteful government spending. Note that we are implicitly assuming that past flows of new housing do not reduce the land stock available for the current production of housing. Therefore, our definition of housing production is not limited to new buildings on previously not used land, but includes rebuilding, renovation and construction of new floors of existing houses. Alternatively, Davis and Heathcote (2005) and Favilukis et al. (2015) consider a production function for new housing where the contribution of land depends on the flow of new permits generated by the government at any period (and sold at the competitive rental rate).

For analytical convenience, we restrict F_h to the class of CES productions functions and impose more standard restrictions on F_m.

Assumption 1. *Both production functions display constant returns to scale, are strictly increasing, strictly concave, continuously differentiable and such that*

$$\lim_{K^j/A^jL^j \to 0} F_{j,2}/F_{j,1} = 0, \qquad \lim_{K^j/A^jL^j \to \infty} F_{j,2}/F_{j,1} = \infty,$$

where $F_{j,i}$ denote the partial derivatives of F_j with respect to the i-th argument. More specifically, F_h belongs to the class of CES productions functions with elasticity of substitution between inputs defined by σ^h.

Importantly, the CES representation for F_h implies that the profit maximizing values of the capital-labor ratios under factor price equalization are independent of the amount of land, Z.

It is convenient to provide a more compact notation by normalizing variables with respect to the level of labor efficiency. In particular, for $j = h, m$, let $k^j = K^j/A^jL^j$ be the sector-specific capital intensities, $z = Z/A^hL^h$ the available land per unit of labor efficiency in construction and y^j the sector-specific labor productivities in efficiency units. By constant returns to scale

$$y^h = F_h(k^h, 1, z) \equiv f_h(k^h, z) \qquad y^m = F_m(k^m, 1) \equiv f_m(k^m),$$

where f_j denotes the intensive-form production functions.

We further assume that firms in construction and manufacturing are price-takers and labor and capital are fully mobile across the two sectors. Let $a = A^m/A^h$ be the labor augmenting efficiency in manufacturing relative to construction (henceforth *relative productivity*); r the real interest rate; $w = W/A^m$ the wage rate per units of efficiency in the manufacturing sector; and denote with $f_{h,k}$ and $f_{h,z}$ the partial derivatives of $f_h(k^k, z)$ with respect to k^h and z, respectively. Then, letting q^h be the price of a unit of new housing, profit maximization at any interior solution (*i.e.*, strictly positive (k_t^j, L_t^j, y_t^j) for $j = h, m$) implies

$$1 + r = f_{m,k} = q^h f_{h,k}, \tag{1}$$

$$w = f_m - k^m f_{m,k} = (q^h/a)[f_h - k^h f_{h,k} - z f_{h,z}]. \tag{2}$$

By the properties of the production functions, the above two equations provide a well defined map from (r, a) into (w, k^h, k^m) for all $a > 0$ and r in a suitable interval. In particular, we can state the following.

Proposition 1. *For any given strictly positive* (r, a)*, with* $r \in \mathcal{A} = [\underline{r}, \bar{r}]$*, there is a unique solution,* $(w(r), k^h(r, a), k^m(r))$*, to equations (1)-(2), as a differentiable function of* (r, a)*, such that* w*,* k^h *and* k^m *are all decreasing in* r *and*[4]

$$\hat{k}_a^h \equiv \frac{\partial k^h/k^h}{\partial a/a} = \sigma^h. \tag{3}$$

[4]From now on, to simplify the notation, we use a "hat" to denote partial elasticities, *i.e.*, letting $h(x)$ be any differentiable function in $\mathbb{R}^n$, we let $\hat{h}_{x_i} = \partial \log h(x)/\partial \log x_i$.

A sketch of the proof is the following. Since $f_{m,k}(k^m)$ is decreasing in k^m, the marginal productivity of capital in manufacturing is locally invertible in some interval $\mathcal{A} = [\underline{r}, \bar{r}]$. Then, by the profit maximization condition (1) we obtain $k^m = k^m(r)$, with $k^m(.)$ decreasing in r and such that $k^m(\underline{r}) = \infty$, $k^m(\bar{r}) = 0$. Existence of the function $w(r)$ follows from (1) and $k^m = k^m(r)$, which proves also $w'(r) < 0$. Now observe that, since the CES production function is homothetic, the ratio between the marginal product of labor and the marginal product of capital in the construction sector depends on k^h only and

$$\frac{f_h(k^h, z) - k^h f_{h,k}(k^h, z) - z f_{h,z}(k^h, z)}{f_{h,k}(k^h, z)} = \omega_h(k^h),$$

where $\omega_h(k^h)$ is increasing and invertible in $\mathbb{R}_+$. By factor price equalization,

$$\omega_h(k^h) = aw(r)/r.$$

and, then, $k^h = \omega_h^{-1}(aw(r)/(1 + r)) \equiv k^h(r, a)$. By taking the derivative, we obtain (3). Notice that the fact that (w, k^h, k^m) are independent of z is a consequence of the CES specification of F_h. By equations (1)-(2) and the above findings we can write the housing price as

$$q^h = \frac{(1 + r)}{f_{h,k}\left(k^h(r, a), z\right)} \tag{4}$$

for all (r, z, a) in $\mathcal{A} \times \mathbb{R}_+^2$. In section **??** we discuss how r, z and a affect the housing price q^h.

4.2 Households

A set L_t of households, growing at a rate $n \geq 0$, is born every period $t = 0, 1, 2, \ldots$ Households live for two periods, supply labor time inelastically, in young age only, and have identical time-invariant preferences for manufacturing consumption and housing services, the latter being measured by the housing stock. Households are characterized by some degree of altruism with respect to their offsprings defined by an individual specific discount rate of the next generation's utility. In particular, households born at time t belong to different types, indexed by i, with i in a finite set $\mathcal{I}$, and each type i composed of a mass m_i of individuals (*i.e.*, a collection of positive numbers, $(m_i)_{i\in\mathcal{I}}$, such that $\sum_{i\in\mathcal{I}} m_i = 1$), with life-time utility defined by:

$$V^{t,i} = u(c_t^{t,i}, c_{t+1}^{t,i}, h_{t+1}^i) + \theta_i(1+n)V^{t+1,i},$$

for all $t \geq 0$, where $(c_t^{i,t}, c_{t+1}^{i,t})$ are age-contingent consumptions, h_{t+1}^i is the housing stock acquired by the household in young age. Preferences satisfy the following assumption.

Assumption 2. *The (inter-generational) discount factors satisfy $\theta_i(1+n) < 1$ for all $i \in \mathcal{I}$ and the utility function belongs to the CES class with elasticity of substitution between goods denoted by γ.*

The upper bound on the discount rates, θ_i, insures convergence of each dynasty's long-run utility function. Importantly, the CES hypothesis guarantees normality and the "law of demand" (in particular, the demand for housing is decreasing in its own price).

We assume perfect financial markets allowing for unlimited lending and borrowing and, for simplicity, we ignore the housing rental market[5]. Any household born at time t acquires land and residential property when young, enjoys the housing services generated by it, resells the property when old and leaves some bequests to the offsprings. Due to the absence of financial frictions, we can write the households' inter-temporal

[5]Households are assumed to derive some, however small, satisfaction from ownership, so that, in the absence of market frictions, ownership is a dominant choice relative to renting. The average home ownership rate across OECD countries is approximately 67%. At the top of the distribution are countries like Greece (87%) and Spain with high rates of more than 80%; while at the bottom countries like Germany (43%), Japan (36%) and Switzerland (35%). Data are from the OECD (2012).

budget constraints as:

$$c_t^{t,i} + \frac{c_{t+1}^{t,i}}{1+r_{t+1}} + \pi_t h_{t+1}^i + \frac{(1+n)b_{t+1}^i}{1+r_{t+1}} = W_t + b_t^i, \tag{5}$$

where b_t^i denotes bequests, W_t is the time-t real wage and

$$\pi_t = q_t^h - (1-\delta)q_{t+1}^h/(1+r_{t+1})$$

denotes the user cost of housing, *i.e.*, the cost of a unit of housing net of the present value of selling the same un-depreciated unit the next period and $\delta \in (0,1)$ is the housing depreciation rate. Notice that the above representation of the inter-temporal budget constraint takes into account the absence of arbitrage opportunities related to the assets included in households' portfolios, such as bonds, housing and land. Since parental altruism is one-sided, we rule out forced gifts from children to parents, and impose the non-negativity constraint

$$b_{t+1}^i \geq 0.$$

Denoting with $u_{j,t}^i$ the partial derivative of $u(c_t^{t,i}, c_{t+1}^{t,i}, h_{t+1}^i)$ with respect to the j-th argument and with q^z the price of a unit of land, the first order conditions characterizing the young household's optimal consumption choice subject to the budget constraint, (5), are

$$u_{1,t}^i = (1+r_{t+1})u_{2,t}^i, \qquad u_{1,t}^i \pi_t = u_{3,t}^i, \qquad u_{2,t}^i \geq \theta_i u_{1,t+1}^i, \tag{6}$$

together with the complementary slackness condition

$$b_{t+1}^i \left(u_{2,t}^i - \theta_i u_{1,t+1}^i \right) = 0 \tag{7}$$

and the no-arbitrage condition related to land investment,

$$q_t^z = (q_{t+1}^h f_{h,z}(k_{t+1}^h, z_{t+1}) + q_{t+1}^z)/(1 + r_{t+1}), \tag{8}$$

where q^z denotes the unit price of land. From the above first order conditions we derive the time-t households saving, s_t^i, the demand for housing, h_{t+1}^i, and the supply of bequests, b_{t+1}^i. These are specified as:

$$\begin{aligned} s_t^i &= S^i(\pi_t, r_{t+1}, W_t + b_t^i), & (9)\\ h_{t+1}^i &= H^{d,i}(\pi_t, r_{t+1}, W_t + b_t^i), & (10)\\ b_{t+1}^i &= B^i(\pi_t, r_{t+1}, W_t + b_t^i). & (11) \end{aligned}$$

4.3 Equilibrium

Without loss of generality, we normalize $A^h = 1$ and assume that the government adjusts the amount of land available for construction to the changing population in such a way as to generate a time-invariant land per unit of labor. In particular, we restrict the land policy as follows

$$Z_t = \xi L_t. \tag{12}$$

This assumption allows us to normalize the equilibrium variables and have a dynamic system that is neutral with respect to the level of population.

Let K, L and H be the total stock of business capital, labor and housing. Full employment implies

$$\begin{aligned} L_t &= L_t^h + L_t^m, & (13)\\ K_t &= K_t^h + K_t^m, & (14)\\ H_{t+1} &= Y_t^h + (1-\delta)H_t. & (15) \end{aligned}$$

Now we express all equilibrium restrictions and the relevant variables in per-capita units. In particular, letting

$$k_t = K_t/L_t, \quad h_t = H_t/L_t, \quad \lambda_t = L_t^h/L_t,$$

we replace the full employment conditions (13), (14) and (15) with

$$k_t = \lambda_t k_t^h + (1-\lambda_t)a_t k_t^m, \tag{16}$$

$$(1+n)h_{t+1} = \lambda_t y_t^h + (1-\delta)h_t \tag{17}$$

and $\lambda_t \in [0,1]$. To close the model we define the per capita aggregate saving and housing demand

$$s_t \equiv \sum_i m_i S^i(\pi_t, r_{t+1}, W_t + b_t^i),$$

$$h_{t+1}^d \equiv \sum_i m_i H^{d,i}(\pi_t, r_{t+1}, W_t + b_t^i)/(1+n),$$

and impose market clearing in the the housing and capital markets, *i.e.*,

$$h_{t+1} = h_{t+1}^d, \tag{18}$$

$$s_t = (1+n)(k_{t+1} + q_t^h h_{t+1} + q_t^z \xi). \tag{19}$$

By profit maximization and factor price equalization,

$$k_t^m = k^m(r_t), \quad k_t^h = k^h(r_t, a_t), \quad W_t = a_t w(r_t), \quad q_t^h = (1+r_t)/f_{h,k}(k_t^h, z_t), \tag{20}$$

where, because of the land policy (12),

$$z_t = \xi/\lambda_t. \tag{21}$$

Finally, by the government balanced budget condition, the flow of per-capita public spending at all $t \geq 0$ is

$$g_t = q_t^z(1+n)n\xi. \tag{22}$$

Remember that the functions in (20) are positive and continuous and guarantee an interior allocation of factors across sectors for r in $\mathcal{A} = [\underline{r}, \bar{r})$ and all $a > 0$. Then, an *interior competitive equilibrium* is a positive sequence,

$$\{k_t, k_t^h, k_t^m, \lambda_t, h_t, b_{t+1}, r_{t+1}, w_t, q_t^h, q_t^z\}_{t=0}^{\infty},$$

with $r_{t+1} \in \mathcal{A}$ and $\pi_t = q_t^h - (1-\delta)q_{t+1}^h/(1+r_{t+1}) > 0$ for all $t \geq 0$, satisfying the optimality conditions (6)-(8), the market clearing conditions (16), (17), (19), (18), the factor price equalization conditions (20), the land policy (21) and the balanced budget condition (22), for all $t \geq 0$, for a given sequence of relative productivities, $\{a_t\}_{t=0}^{\infty}$, and some initial conditions, $(k_0, h_0, b_0, r_0) > 0$.

4.4 Steady States with Bequests

From now on we concentrate on a steady state equilibrium with two types of households. In particular, letting $\mathcal{I} = \{p, r\}$ and $\theta_r > \theta_p$, we say that household type r is *rich* and household type p is *poor*, although we could as well say that the former is more altruistic than the latter with respect to their own children. Under this specification, and by the first order conditions (6)-(7), it is clear that we may have two type of steady states. In the first, $r \leq (1-\theta_r)/\theta_r$ and no individual leaves any bequests, so that the resulting equilibrium is equivalent to the one that would take place in a *canonical* overlapping generations economy. In the second, the rich individuals leave positive bequests, whereas the poor leave zero bequests at any time. In this case we have

$$r = (1-\theta_r)/\theta_r \equiv r^* < (1-\theta_p)/\theta_p.$$

We refer to the first type of equilibrium as a *zero bequests steady state* (ZBSS) and to the second type as a *positive bequests steady state* (PBSS). In what follows, we focus exclusively on PBSS, mostly because this type of equilibrium allows for a sharp characterization of intra-generational inequality. It is understood that a PBSS is assumed to be *interior*, *i.e.*, to imply that both sectors are active. In particular, we impose the following assumption.

Assumption 3. $\lim_{k^m \to 0} f_{m,k} > \min_{i\in\mathcal{I}} 1/\theta_i > \lim_{k^m \to \infty} f_{m,k}$.

The above implies that a PBSS exists and it is such that the wage rate per unit of efficiency is uniquely fixed at $w^* = w(r^*)$, whereas $w \geq w^*$ at a steady state with zero bequests. Recalling that any interior allocation provides $k^m = k^m(r)$, $k^h = k^h(r,a)$, the allocation of the capital-labor ratios is uniquely determined by (r^*, a). Hence, there are six remaining equilibrium variables to be determined by the steady state equilibrium conditions characterizing a PBSS: the average capital stock, k, the share of labor in construction, λ, the housing stock, h, the asset prices, q^h, q^z, and the steady state bequest of the rich household, b^r.

Dropping the superscript "$*$" on the PBSS equilibrium values of the interest rate, wage and capital, equation (17) provides the following relation

$$(\delta + n)h = f_h(k^h(r,a))\lambda, \tag{23}$$

and, by profit maximization, the no-arbitrage condition (8) and the land policy (land-policy1), steady state asset prices are

$$\begin{aligned} q^h &\equiv (1+r)/f_{h,k}(k^h(r,a), \xi/\lambda), && (24)\\ q^z &\equiv q^h f_{h,z}(k^h(r,a), \xi/\lambda)/r, && (25)\\ \pi &= q^h(\delta + r)/(1+r). && (26) \end{aligned}$$

Regarding households' demand of consumption and housing at a PBSS, we observe that these are the solutions to the maximization of $u(c^{y,i}, c^{o,i}, h^i)$ subject to the (present value) budget constraint

$$c^{y,i} + \frac{c^{o,i}}{1+r} + \pi h^i = aw + \frac{(r-n)b^i}{1+r} \equiv I^i, \tag{27}$$

to be specified as $(c^{y,i}(\pi, aw, I^i), c^{o,i}(\pi, aw, I^i), h^i(\pi, aw, I^i))$.

The CES utility specification has the important implication that these demands are homothetic and satisfy the law of demand (*i.e.*, housing and age-contingent consumptions are decreasing in their own price). In particular, they are specify these as

$$c^{y,i} = \phi^y I^i, \qquad \frac{1}{1+r}c^{o,i} = \phi^o I^i, \qquad \pi h^i = \phi^h I^i,$$

where ϕ^j (for $j = y, o, h$) are the expenditure shares, all positive and continuous functions of π (see appendix B for details). A further important property is that the elasticities of consumption and housing demand with respect to the user cost of housing are independent of individuals' wealth and given by

$$\hat{h}^i_\pi = \hat{c}^{y,i}_\pi = \hat{c}^{y,i}_\pi/(1+r) = -(1-\gamma)\phi^h,$$

where γ is the elasticity of substitution between goods. Finally, since consumption and housing are normal goods, we conclude that the rich households' housing demand, h^r, and saving,

$$s^r \equiv aw + b^r - c^{y,r}(\pi, aw, I^r) = (1-\phi^y)aw + \left(\frac{1+(1-\phi^y)r + \phi^y n}{1+r}\right) b^r,$$

are both increasing in bequests. Hence, higher bequests generate more housing demand and more saving for the r-type individual relative to the p-type at the PBSS. An important related property that we will exploit to derive the comparative statics of the model, is that the impact of a rising bequest on saving is greater than the impact on the money spent on housing, *i.e.*,

$$\partial s^i/\partial b^i > q^h \partial h^i/\partial b^i > 0. \tag{28}$$

Notice that this property holds more generally than for CES utility representations and it just requires normality and $r > n \geq 0$.

The CES specification allows for an easy aggregation of individuals' demands and savings elasticities with respect to housing prices. In particular, we define the aggregate demand for housing and aggregate saving as

$$h^d(\pi(v,a), aw, b) \equiv \sum_i m_i h^i(\pi, aw, b^i)/(1+n), \tag{29}$$

$$s(\pi(v,a), aw, b) \equiv \sum_i m_i s^i(\pi, aw, b^i), \tag{30}$$

where, to simplify the notation, we have set $b = b^r$ (as $b^p = 0$ at the PBSS, there is no confusion regarding the identity of bequests).

Observe that the effect of a rising user cost of housing on the aggregate demand for housing is

$$\hat{h}^d_\pi = \frac{1}{1+n} \sum_i \left(\frac{m_i h^i}{h^d} \right) \hat{h}^i_\pi, \qquad \hat{s}_\pi = \sum_i \left(\frac{m_i s^i}{s} \right) \hat{s}^i_\pi,$$

and, then, by the CES specification,

$$1 + \hat{h}^d_\pi = (1-\gamma)(1-\phi^h), \qquad \hat{s}_\pi = (1-\gamma)\phi^h c^y/s, \tag{31}$$

where $c^y = \sum_i m_i c^{y,i}$ is the aggregate young age consumption. Hence, if $\gamma < 1$, a rise in the user cost of housing generates, *ceteris paribus*, a rise in the demand for housing wealth and saving. On the contrary, if $\gamma = 1$, then $\hat{h}^d_\pi = -1$ and $\hat{s}_\pi = 0$.

5 The Housing Cost Disease

Using the results obtained in the last section, we can now explore under what conditions our model replicates some of the features of the two-sector economy studied by Baumol (1967), with construction of housing playing the role of the *stagnant sector* and manufacturing the role of the *progressive sector*.

We recall that the *Baumol's cost disease* holds if, following a rise in productivity in the dynamic sector, (a) the relative price of the stagnant sector output increases (*price increase*); (b) the stagnant industry takes a rising share of nominal output (*unbalanced growth*); and (c) the changing composition of output across stagnant and dynamic industries reduces the effect of the productivity improvement on the average productivity (*adverse effect on productivity*). We restate the Baumol's cost disease result in the present framework as follows.

Define *housing wealth* as $v \equiv q^h h$ and the average income per-capita as

$$y = ay^m(1-\lambda) + q^h y^h \lambda = aw + (1+r)k + rq^z\xi. \tag{32}$$

Then, the *wealth-to income-ratio*,

$$\beta = \frac{k + v + q^z\xi}{y} = \beta^k + \beta^h + \beta^z,$$

where $\beta^k = k/y$ is the business capital, $\beta^h = v/y$ the housing and $\beta^z = q^z\xi/y$ the land components. Then, we say that there exists a *housing cost disease* if, at an equilibrium PBSS,

(HA)	$\partial q^h/\partial a > 0$		(housing appreciation),
(IW)	$\partial \beta^h/\partial a > 0,$	$\partial \beta/\partial a > 0$	(increasing wealth-to-income ratios),
(IN)	$\partial b/\partial a > 0$		(increasing wealth inequality),
(SP)	$\partial y/\partial a < y/a$		(stagnant labor productivity).

Observe that (IW) may be defined, alternatively, as an increase in the share of labor employed in construction. However, the two phenomena are strictly related by (**??**).

5.1 Assumptions

Before we analyze the impact of a, we should introduce some key variables playing an important role for the comparative statics of our model. First, we define the sector-specific *factor shares* (or output elasticities): The first set is given by

$$\mathcal{S}_k^j = f_{j,k}k^j/f_j, \qquad \mathcal{S}_z^h = f_{h,z}z/f_h, \qquad \mathcal{S}_l^h = 1 - \mathcal{S}_k^h - \mathcal{S}_z^h.$$

Then, we let the *capital intensity differential* across the two sectors be defined as

$$\Delta = \left(\frac{\delta + n}{1 + r}\right)\mathcal{S}_k^h\left(\frac{ak^m - k^h}{k^h}\right).$$

Observe that Δ is a function of factor shares. In particular,

$$\frac{ak^m - k^h}{k^h} = \frac{1}{1 - \mathcal{S}_k^h}\left(\frac{\mathcal{S}_l^h(\mathcal{S}_k^m - \mathcal{S}_k^h)}{\mathcal{S}_k^h(1 - \mathcal{S}_k^m)} - \frac{\mathcal{S}_z^h(1 - \mathcal{S}_k^m)}{\mathcal{S}_k^m}\right). \tag{33}$$

Then, $\mathcal{S}_k^m > \mathcal{S}_k^h$ is a necessary condition for the manufacturing sector to exhibit a higher capital intensity, *i.e.*, $\Delta > 0$[6]. In general, factor shares in the construction sector and Δ are functions of λ and a. However, in the special case of Cobb-Douglas production in housing (*i.e.*, $\sigma^h = 1$), Δ is a constant since both $\mathcal{S}_k^h$ and $\mathcal{S}_z^h$ are constant.

To appreciate the role of factor shares and capital intensities, observe that, by profit maximization and the supply of housing, (23), we derive

$$\lambda = ((\delta + n)\mathcal{S}_k^h/((1 + r)k^h))v, \tag{34}$$

$$q^z\xi = ((\delta + n)\mathcal{S}_z^h/r)v. \tag{35}$$

[6]Note that $\mathcal{S}_k^m > \mathcal{S}_k^h$ if and only if the housing price, q^h, is decreasing in the interest rate, r.

Then, using the above with (16) and (32),

$$k = ak^m - \Delta v, \tag{36}$$

$$y = ay^m - ((1+r)\Delta - (\delta+n)\mathcal{S}_z^h)v, \tag{37}$$

$$\beta = (ak^m + ((\delta+n)\mathcal{S}_z^h/r + 1 - \Delta)v)/y. \tag{38}$$

Observe that, when Δ is positive and $\mathcal{S}_z^h$ is sufficiently small, all else unchanged, k and y are decreased and β^h is increased by a larger housing wealth, v, both directly and thorough a fall in y. Provided that Δ is also not too large, a higher v may, in turn, generate a larger wealth-to-output ratio.

These preliminary observations provide some intuition as to why the housing cost disease requires the construction sector to exhibit a smaller capital intensity than manufacturing and a relatively small land share of output. In particular, we now introduce the following assumptions.

Therefore, under (39), both the average capital and output per worker fall short of their respective values in the manufacturing sector by a margin that depends on the size of the housing wealth. This observation provides some intuition about the role of the assumed restrictions on the relative capital intensity for the housing cost disease.

Assumption 4. $\sigma^h = 1$ *and*

$$1 + (\delta+n)\mathcal{S}_z^h/r > \Delta > (\delta+n)\mathcal{S}_z^h/r. \tag{39}$$

The restriction $\sigma^h = 1$ is mostly a simplifying assumption, as it implies that factor shares and the capital intensity differential, Δ, are parametric. However, we should mention that $\sigma^h < 1$ implies that the direct effect of a on the land price is negative and that, for $\mathcal{S}_z^h$ close to zero, Δ is increasing in a if and only if $\sigma^h < 1$. The upper bound on Δ in (39) is a requirement for a meaningful steady state equilibrium when bequests are not too large. In appendix D (proposition 4) we show, in fact, that this upper bound is necessary for generating $\lambda \leq 1$ at equilibrium, *i.e.*, non-negative shares of labor in the two sectors, under the assumption that aggregate saving falls short of the wage bill. This is a natural restriction, which is clearly satisfied when total bequests are not too large (and, *a fortiori*, in the canonical overlapping generations model). The lower bound on Δ is, instead, a key restriction for generating the rising wealth-to-income ratios that are part of the housing cost disease defined in this paper and requires a sufficiently large capital intensity in manufacturing relative to construction and a sufficiently low land share of income.

5.2 Impacts of Productivity Improvements

A further motivation for concentrating on economies satisfying assumption 4 is that these properties imply a "well behaved" comparative statics for the system of equations defining a PBSS when the own price elasticity of housing demand is sufficiently close to one ($i.e.$, $\gamma \sim 1$). In particular, in appendix D we prove that the whole equilibrium system reduces to two equations and two unknowns. The two equations are the market clearing condition in the housing market and the capital market equilibrium; the two unknowns are the rich households' bequests, b, and the value of housing wealth, v. This is accomplished by using (23) to derive a mapping

$$(v, a) \to (\lambda, k, q^h, q^z, \pi)$$

and using the market clearing conditions for housing and capital markets (18)-(19) at steady state to derive the excess demand for housing wealth and the excess supply of saving (over investment) as

$$G^d(b,v,a) \equiv q^h(v,a)h^d(\pi(v,a),aw,b) - v, \quad (40)$$

$$G^s(b,v,a) \equiv \frac{1}{1+n}s(\pi(v,a),aw,b) - k(v,a) - v - q^z(v,a)\xi. \quad (41)$$

Then, the steady state specification of the equilibrium conditions (18)-(19) define the reduced form equilibrium steady state conditions for any given a as

$$G^d(b,v,a) = 0, \quad (42)$$

$$G^s(b,v,a) = 0, \quad (43)$$

and a PBSS is a positive pair, $(b^*(a), v^*(a))$, such that

$$0 = G^d(b^*(a), v^*(a), a) = G^s(b^*(a), v^*(a), a). \quad (44)$$

Then, letting $v^d(b,a)$ and $v^s(b,a)$ be the solutions for v to (42) and (43), respectively, for a given pair (b,a), we say that v^d is the demand and v^s the supply of housing wealth. Intuitively, v^d is the households' real expenditure for the stock of available housing and v^s defines the amount of housing wealth that is consistent with a capital market equilibrium, *i.e.*, with the available amount of savings, business capital and land value. It turns out that, if the economy is *unit elastic*, *i.e.*, $\sigma^h = \gamma = 1$, and (39) is verified, the demand and supply schedules, *i.e.*, $v^d(b,a)$ and $v^s(b,a)$ in the space (b,v), are well defined, have a unique intersection at $b^*(a)$, where v^s is steeper than v^d (*i.e.*, $v^s_b > v^d_b$). The case of a demand schedule less steep than the supply schedule, which we refer to as a *regular intersection*, and follows from the assumption (39) and the property (28), *i.e.*, the fact that a rise in bequests has a greater impact on households' saving than on their expenditure on housing. The unit-elastic economy appears, then, to be a natural benchmark or reference case because, coupled with the capital intensity assumption 4, is well behaved (*i.e.*, it allows for meaningful comparative statics results) and, as it will be shown below in proposition 2, it implies that both $b^*(a)$ and $v^*(a)$ are unit-elastic with respect to a and that changes in a are neutral with respect to the shares of labor across sectors as well as on the wealth-to-income ratios.

Proposition 2. *Consider a PBSS of an economy satisfying assumption 4. If* $\gamma = 1$,

$$\hat{b}^*_a = \hat{v}^*_a = \hat{y}^*_a = 1, \quad \hat{\beta}^*_a = \hat{\beta}^{h,*}_a = \hat{\lambda}^*_a = 0, \quad \hat{q}^{h,*}_a = 1 - \mathcal{S}^h_k, \quad \hat{q}^{z,*}_a = 1.$$

If, on the other hand, γ *is sufficiently close to one and in the range defined by*

$$1 > \gamma > 1 - \frac{1}{1-\phi^h}\left(\frac{\mathcal{S}^h_l - \mathcal{S}^h_z}{2\mathcal{S}^h_l \mathcal{S}^h_z}\right), \tag{45}$$

we have $\hat{b}^*_a > 1$, $\hat{v}^*_a > 1$, $\hat{y}^*_a < 1$, $\hat{\beta}^*_a > 0$, $\hat{\beta}^{h,*}_a > 0$, $\hat{\lambda}^*_a > 0$, $\hat{q}^{h,*}_a > 1 - \mathcal{S}^h_k$, $\hat{q}^{z,*}_a > 1$.

More specifically, when γ satisfies (45), we provide the following lower bounds for asset price appreciations (cf. appendix D):

$$\begin{aligned}
\hat{q}^{h,*}_a &> (1 - \mathcal{S}^h_k)\left(1 + \frac{(1-\gamma)(1-\phi^h)\mathcal{S}^h_z}{1-(1-\gamma)(1-\phi^h)\mathcal{S}^h_z}\right), \\
\hat{q}^{z,*}_a &> 1 + (1 - \mathcal{S}^h_k)\frac{(1-\gamma)(1-\phi^h)\mathcal{S}^h_z}{1-(1-\gamma)(1-\phi^h)\mathcal{S}^h_z}.
\end{aligned}$$

What is the empirical support for the assumptions that we impose to prove the generate a housing cost disease? In the literature, there is consistent support for the assumptions that the construction sector is less capital intensive than manufacturing and little consensus on the most plausible values for the elasticity of substitution between capital and labor. In particular, Valentinyi and Herrendorf (2008) set the capital share in manufacturing and construction respectively to 0.4 and 0.2. Moreover, although the lower bound on Δ defined in (39) is higher than zero, this is defined in terms of two variables, $(n + \delta)$ and $\mathcal{S}^h_z$, that are likely to be small. In particular, Neels

(1982) provide an estimate of the output elasticity of land (*i.e.*, our measure of $\mathcal{S}_z^h$) between 0.03 and 0.06, and Davis and Heathcote (2005) set this value at 0.106 for their own calibration[7]. Regarding the elasticity of substitution between capital and labor, Piketty (2014) assumes this to be greater than one (an assumption that justifies his belief that a rising capital-output ratio generates a rising capital share), Chirinko (2008) provides a comprehensive survey of the available empirical evidence about the elasticity of substitution between capital and labor, as well as his own estimates, and puts the most likely range for σ^j between 0.5 and 0.6. Finally, we observe that there exists strong evidence that housing demand responds less than proportionally to a rise in price. In particular, Hanushek and Quigley (1980), Mayo (1981) and Ermisch et al. (1996) provide estimates of the housing demand elasticity in the range $(-0.8, -0.5)$.

In any case, it is important to stress that the above assumptions are not necessary, bur rather they provide with the most favorable environment for the housing cost disease. In particular, the low price elasticity in the demand for the output of the stagnant sector is one of the key assumption in Baumol (1967)'s model. Similarly, Ngai and Pissarides (2007) assumes a low (*i.e.*, below one) elasticity of substitution across final goods in order to show that employment is gradually shifting to sectors with low productivity growth. In our model, a rise in relative efficiency in manufacturing determines an increase in the relative price of housing with a small effect on the demand of this good. Therefore, the demand for housing wealth ($v = qh$) increases with a and we observe a reallocation of production and labor to the less productive sector. The assumption that $\Delta > 0$ has no analogous counterpart in the literature following the Baumol's cost disease proposition.

[7]This value is based on an unpublished 2000 memo from Dennis Duke to Paul L. Hsen entitled "Summary of the One-Family Construction Cost Study".

5.3 Income Shares

In a series of influential papers, Piketty and Zucman (2014), Piketty (2015a) and Piketty (2015b) have argued that the rising capital share of income is a consequence of the joint hypothesis of a steadily rising capital-output ratio and low diminishing returns to capital. Most of the evidence on the rising capital-output ratios and capital shares provided in this literature reflects the role of housing. For example, Rognlie (2014) claims that housing "accounts for nearly 100% of the long-term increase in the capital/income ratio, and more than 100% of the long-term increase in the net capital share of income." According to Bonnet et al. (2014), the capital income ratio has dropped or remained roughly constant, when we take housing capital aside. Therefore, it is interesting to evaluate whether the housing cost disease may be responsible for a higher capital share of income. We find that this is the case, but only when we consider a definition of capital share that includes imputed rents. In our model, the most natural definition of the capital share of income is

$$\zeta(a, v) = 1 - aw/y.$$

Then, quite trivially,

$$\hat{\zeta}_a^* = \frac{aw}{y - aw}(\hat{y}_a - 1),$$

i.e., the average capital share of income always falls in the presence of a housing cost disease, as wages grow proportionally with a and average labor productivity grows by less than proportionally with a. However, if, following the prevailing national accounting practice, we include imputed rents in the definition of GDP the definition of capital share becomes:

$$\zeta^h = 1 - \frac{aw}{y + \pi h} = 1 - \frac{1 - \zeta}{1 + \left(\frac{\delta + r}{1 + r}\right) \beta^h}.$$

Hence, ζ^h is typically larger than ζ^k and it is increasing in β^h for any given y. By straightforward computations we derive that, when $\hat{v}_a^* > 1$,

$$\hat{\zeta}_a^{h,*} \geq 0 \qquad \Leftrightarrow \qquad \Delta(1+r) \leq \left(\frac{\delta + r}{1+r}\right) + (\delta + n)\mathcal{S}_z^h.$$

By (39), the above condition requires $\Delta < (\delta + r)/(1 + r)$. Hence, when the definition of income includes imputed rents, a rising relative productivity may generate a rising capital share if the growth in housing wealth is sufficiently strong.

5.4 Numerical Simulation

In the previous sections we showed analytically the effects of productivity improvements under the special assumptions of unit-elastic housing construction technology ($\sigma^h = 1$), and considering both the case of unit-elastic preferences, $\gamma = 1$, and of $\gamma < 1$ (cf. proposition 2). However, there is no consensus in the empirical literature on the value of the elasticity of substitution between capital and labor. For example, while Piketty (2014) assumes a value greater then one, Chirinko (2008) points to values between 0.5 and 0.6. In this section, we evaluate numerically the implications of our model under the more general assumption of σ^h possibly smaller than unity. In particular, we evaluate the consequences of a rising relative efficiency in manufacturing on stationary equilibrium variables and show the impact of the housing cost disease on housing prices, wealth ratios, average productivity and inequality. We report in table 1 all the parameters used in the numerical simulations. We set preferences' parameters such that the weights on consumption when young and old is the same, and equal to 0.4, while the weight on housing services is equal to 0.2. The elasticity of substitution for the households' preferences, γ, is smaller than unity and equal to 0.5. The real interest rate, for a holding period equal to a generation, is set to 0.33. Recall that in the PBSS, for the two-type of households' case (*i.e.*, rich and poor), described in section 4.3, the real interest rate is pinned down by the preference for altruism of the rich household that in our case is then equal to $\theta_r = 0.75$[8]. The depreciation of the housing stock is set equal to $\delta = 20\%$, implying complete depletion over five generations as in Deaton and Laroque (2001). Technology is Cobb-Douglas in the manufacturing sector, while we let the production function of the housing sector be of the more general CES specification (when $\sigma^h = 1$, technology is Cobb-Douglas also in the housing sector). We also assume that the manufacturing sector is more capital intensive than the housing sector, and set accordingly $\alpha^m = 0.4$ and $\alpha^h = 0.1$. The land share in housing is small and equal to $\eta^h = 0.10$. Finally, we set the population's growth rate to $n = 0\%$, the fraction of rich households to $m_r = 10\%$ and the parameter ξ in the land policy equation (12), to 1, so that the amount of available land for construction adjusts proportionally to any, eventual, change in the population.

We are interested in evaluating the long-run effects of productivity improvements and figures 4 and 5 summarize our main results. According to our estimates using O'Mahony and Timmer (2009)'s KLEMS data, on average, across the eight largest developed economies, relative labor efficiency in manufacturing increased approximately by 50 percent between 1970 to 2007. Therefore, we plot the percentage changes in the steady-state values of the main variables of the model for different levels of the relative efficiency in manufacturing $a = 1, \ldots, 1.5$ with respect to their values when $a = 1$. Note that our goal is not to match the levels of the main variables (for example, the wealth-to-income ratios), as the OLG model is quite stylized and not equipped to generate, for example, the large levels of wealth ratios we observe in the data. In fact, in our simulations wealth-to-income ratios are never larger than 1, while in the data they are a multiple of 1 in all countries. Rather, we are interesting in evaluating how far can a simple model go, without financial frictions, in explaining the long-run

[8] As a back of the envelope calculation, assume that a generation lasts twenty-five years. In this case, the (net) annual real interest rate is equal to $\bar{r}_a = (1+r)^{1/25} - 1$. If $\theta_r = 0.75 = 1/(1+r)$, then $\bar{r}_a = 1.16\%$.

Table 1: Model's Parameters

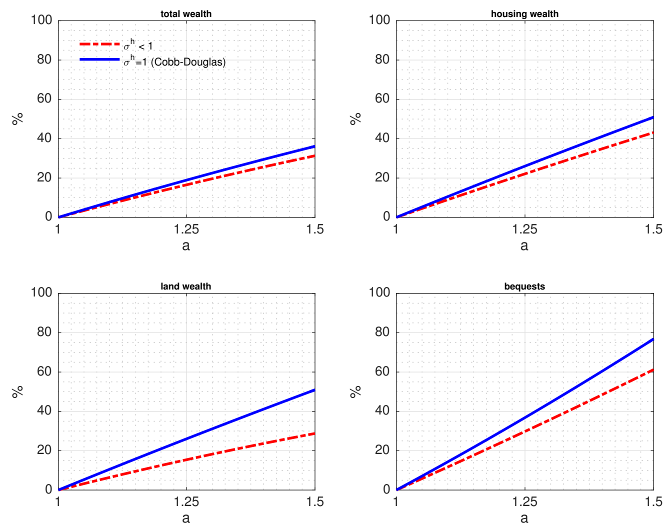
Preferences		
Weight consumption young:	χ^y	0.40
Weight consumption old:	χ^o	0.40
Weight housing services:	χ^h	0.20
Elasticity of substitution preferences:	γ	0.50
Interest rate:	r	0.33
Technology		
Housing depreciation (%):	δ	0.20
Capital share in housing:	α^h	0.10
Land share in housing:	η^h	0.10
Capital share in manufacturing:	α^m	0.40
Elasticity of substitution housing:	σ^h	0.60
Elasticity of substitution manufacturing:	σ^m	1.00
Economy structure		
Population growth rate (%):	n	0.00
Fraction of rich households (%):	mr	0.10
Land policy rate:	ξ	1.00

Notes: This table reports the parameters used to simulate the model for different steady-states corresponding to different values of the exogenous relative efficiency in manufacturing a. For robustness, we also run simulations for values of $\sigma^h = 1$ (*i.e.*, Cobb-Douglas technology). The real interest rate r corresponds to a holding period equal to a generation.

trends discussed in section 2. Also, while our numerical exercise compares different steady-states, the long-run trends in the data might combine different steady states and transitions within a given steady-state. Figure 4 plots the changes in total, housing and land wealth ratios as well as in bequests, our proxy for wealth inequality. We consider two different cases for the elasticity between inputs in the construction sector: in our baseline specification σ^h is smaller than one and equal to 0.6 (red-dashed line); alternatively, we consider the case of $\sigma^h = 1$, *i.e.*, Cobb-Douglas (blue-solid line). All wealth ratios increase with the improvements in relative productivity, and the increase is somewhat stronger the larger the elasticity of substitution. For an increase of 50 percent of a (*i.e.*, from 1 to 1.5), in the baseline specification of $\sigma^h < 1$, total wealth increases by about 40 percent, housing wealth by about 50 percent, land wealth by a 30 percent (for $\sigma^h < 1$). Bequests, our measure of inequality, increase by approximately

60 percent.

Figure 4: Wealth Ratios and Inequality

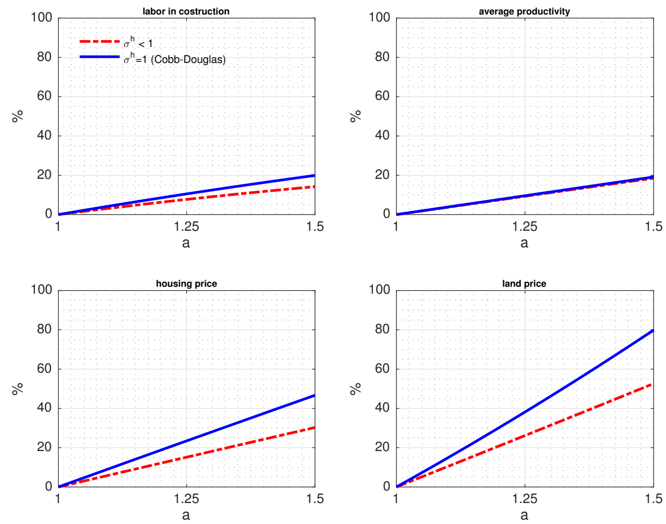


Notes: This figure plots the percentage changes in the steady-state values of total (β), housing (β^h) and land (β^z) wealth-to-income ratios and the level of bequests (b), for different values of the relative productivity sector $a = 1, \ldots, 1.5$, with respect to their value when $a = 1$. The red-dashed line is for the baseline value of the elasticity of substitution in the housing sector, $\sigma^h = 0.6$. The blue solid lines corresponds to Cobb-Douglas technology in the housing sector ($\sigma^h = 1$).

Figure 5 documents additional characteristics associated with a housing cost disease. First, we observe a re-allocation of labor toward the stagnant sector: the share of workers in construction (λ) increases by about 10 percent, with the increase stronger for larger values of σ^h. Despite the strong rise in λ, the share of workers in construction stays below 10 percent in all cases. Second, average productivity increases less than proportionally exactly because of the shifts of workers to the less productive sector. Third, the housing price increases by a minimum of 30 percent ($\sigma^h < 1$) to a maximum of about 50 percent ($\sigma^h > 1$). Fourth, the land price responds strongly to the increase in relative productivity in manufacturing: it increased by about 50 percent for $\sigma^h < 1$ and by about 80 percent for $\sigma^h = 1$.

In this section, we presented results of numerical simulations of the model showing that, under our specification, it generates a large *housing cost disease*. In the next section, we evaluates the implications of the *housing cost disease* in terms of welfare.

Figure 5: Asset Prices and Re-allocation of Inputs



Notes: This figure plots the percentage changes in the steady-state values of the labor share in construction (λ), the average productivity (y), and the prices of houses (q^h) and land (q^z) for different values of the relative productivity sector $a = 1, \ldots, 1.5$, with respect to their value when $a = 1$. The red-dashed line is for the baseline value of the elasticity of substitution in the housing sector, $\sigma^h = 0.6$. The blue solid lines corresponds to Cobb-Douglas technology in the housing sector ($\sigma^h = 1$).

6 Welfare

In this section we address the housing cost disease problem from a welfare point of view and try to evaluate whether, according to our model, the fact that housing takes a large share of private wealth is undesirable.

Within a similar overlapping generations model, Deaton and Laroque (2001) find that the presence of a demand for housing generates a portfolio reallocation away from capital and towards housing, causing the accumulation of capital to fall short of the *Golden Rule* level. Deaton and Laroque take this result as a possible reason for confiscating property and giving it to consumers at no charge. An additional reason to argue in favor of a tax on housing wealth may be based on Piketty's argument that a rising wealth to income ratio, especially when caused by a rising housing stock appreciation, may lead to increasing inequality, in particular when housing takes a sizeable share of intergenerational bequests. However, Auerbach and Hasset (2015) note that reducing the tax benefits for owner-occupied housing in a progressive manner (or a deregulation in land use) may be more effective than a wealth tax in addressing the inequality problem. All of these conclusions must be taken with some caution. Regarding Deaton and Laroque's analysis, it should be noted that allocations departing from the Golden Rule are inconsistent with a social optimum only if we endorse a specific social welfare criterion, such as a weighted sum of all generations' utilities with rate of time preference equal to the population growth rate. In fact, any market allocation at which the rate of interest is larger than the population growth rate is Pareto optimal and, in these cases, reducing the value of the housing stock may have adverse effects on some generation's welfare. Conversely, when the real interest rate falls short of the population growth rate, a case that, in our model, can only occur with zero bequests, Pareto improvements can be obtained by decreasing investment in housing as well as in the capital stock. In other words, the crowding-out of capital induced by housing demand and the inter-generational transfers may, in fact, be desirable to avoid an over-accumulation of capital. With reference to the effects of a wealth tax to reduce wealth inequality, we should recall that housing is a consumption good as well as an asset, and a rise in after tax housing prices may reduce the poor households welfare.

In this section, we examine how a change in the reallocation of resources and prices induced by a rise in relative productivity affects social welfare in a market economy, under an egalitarian and non-paternalistic criterion. This means that we consider a social welfare function biased against inequality (of individuals' utilities) and such that future generations' utilities for their own consumption are discounted with individuals' discount rates. There are a number of reasons why this question may be interesting. First, since rising housing prices generate more bequests and wealth inequality, the housing cost disease may be social welfare diminishing from an egalitarian perspective (although, due to heterogeneous discount rates, some inequality is compatible with an egalitarian planning optimum). Second, since, in a competitive equilibrium, poor households are unable to leave any bequests to their children because of one-sided altruism, raising the poor old individuals' wealth may improve social welfare. Then, a rise in housing prices may relax the non-negativity constraint on bequest values and generate the increase in old age consumption that the market is preventing under one-sided altruism. Third, housing is a consumption good as well as an asset. Therefore, a housing appreciation may decrease welfare as it makes housing less affordable.

As a first step in our discussion about the welfare implications of a rising labor efficiency in manufacturing, we compute the effect of an unanticipated rise in the level of the relative labor efficiency, $a = A^m/A^h$, at $t = 0$, for a constant labor efficiency in construction, $A_t^h = 1$ at a competitive equilibrium such that the poor-type households leave zero bequests and the rich-type leave positive bequests at all periods (*i.e.*, PBSS). Recall that, by forward iteration, the initial old type-i household's utility is

$$V^{-1,i} = u(c_{-1}^{-1,i}, c_0^{-1,i}, h_0^{-1,i}) + \sum_{t=0}^{\infty}(\theta_i(1+n))^{t+1}u(c_t^{t,i}, c_{t+1}^{t,i}, h_{t+1}^i).$$

The above functional is assumed to be well defined and finite. By exploiting the envelope theorem, indirect effects on individuals decisions can be shown to be irrelevant, and we derive

$$\begin{aligned}\frac{\partial V^{-1,i}}{\partial a} &= u_{2,-1}^i\left((1-\delta)h_0^i\frac{\partial q_0^h}{\partial a} + z_0^i\frac{\partial q_0^h}{\partial a}\right)\\ &+ \sum_{t=0}^{\infty}\rho_t^i\left(\frac{\partial W_t}{\partial a} + \frac{c_{t+1}^{t,i}}{(1+r_{t+1})^2}\frac{\partial r_{t+1}}{\partial a} - h_{t+1}^i\frac{\partial \pi_t}{\partial a}\right), \qquad (46)\end{aligned}$$

where z_0^i is the individuals' initial endowment of land and

$$\rho_t^i \equiv (\theta_i(1+n))^{t+1}u_{1,t}^i,$$

are the time-t subjective prices of households of type $i \in \{p, r\}$.

Now consider a steady state, where $c_t^{t,i} = c^{y,i}$, $c_t^{t-1,i} = c^{o,i}$ for all $t \geq 0$ are the stationary young and old age consumptions of the two types, and define

$$\mu^i \equiv \sum_{t=0}^{\infty} \rho_t^i = \frac{\theta_i(1+n)}{1-\theta_i(1+n)} u_1^i.$$

Then, we can decompose the impact of a rising a on households (dynastic) welfare into two parts: the effect arising from a change in the wage rate, W, and the effect arising from a change in asset prices. In particular, recall that, at a PBSS, $r_t = r = 1/\theta_p - 1$ for all $t \geq 0$ and $W_t = aw(r)$. Then,

$$\frac{\partial V^{-1,i}}{\partial a} = \frac{\mu^i}{a}\left[W + \left(\frac{1-\theta_i(1+n)}{\theta_i(1+n)}\right)\frac{q^z z^i}{(1+r)}\hat{q}_a^z + \left(\frac{(1-\delta)}{\theta_i(1+n)} - (1+r)\right)\frac{q^h h^i}{(1+r)}\hat{q}_a^h\right].$$

Notice that households' welfare is unambiguously enhanced by a land price appreciation, whereas the impact of a housing appreciation depends on subjective discount rates. Since $\theta_r = (1+r)^{-1}$,

$$\frac{(1-\delta)}{\theta_r(1+n)} - (1+r) = -(1+r)\left(\frac{\delta+n}{1+n}\right) < 0$$

and

$$\frac{(1-\delta)}{\theta_p(1+n)} - (1+r) \geq 0 \quad \Leftrightarrow \quad \theta_p < \frac{1-\delta}{(1+r)(1+n)}.$$

Therefore, if subjective discount rates are sufficiently similar, the housing price appreciation following an improvement in manufacturing productivity is welfare reducing for all households. In this case, the positive impact of a higher housing wealth on the initial old's utility is more than compensated by the negative impact on future generations' welfare due to the fact that housing becomes less affordable. The case $\theta_p \sim \theta_r$ appears to be significant if we assume that wealth inequality is mostly deriving from the initial distribution of assets across households, instead of the different degrees of altruism with respect to the offsprings. For example, assuming that the poor households own zero land, we derive

$$\frac{\partial V^{-1,p}}{\partial a} \leq 0 \qquad \Leftrightarrow \qquad \hat{q}_a^h \geq \left(\frac{1+n}{\delta+n}\right)\frac{W}{q^h h^p}.$$

In other words, an improvement in manufacturing productivity is welfare reducing for the poor households when the housing price elasticity with respect to a is relatively large and the wage to housing wealth ratio is relatively small.

What are the implications of the above findings on social welfare and government policies? To answer this questions we consider the following egalitarian welfare function:

$$\mathcal{U} = \sum_i m_i \psi(V^{-1,i}),$$

where $\psi(.)$ is a positive concave function. It is straightforward to verify that, for some given amount of land and public spending, a competitive equilibrium corresponds to a social optimum (conditional on the given amount of land and social spending) provided that the non-negativity constraint on bequests is not binding and

$$\psi'(V^{-1,i})\rho_t^i = \frac{(1+n)^t}{\Pi_{j=1}^t(1+r_j)}\mu_0 \tag{47}$$

for all $i \in \mathcal{I}$ and some scalar $\mu_0 > 0$. In other words, sub-optimality may only occur because the Planner would not generate wasteful spending and, for some households, she may provide negative bequests, which is impossible in the market economy. Regarding bequests, we have

$$u_{2,t}^i = \theta_i u_{1,t+1}^i$$

for all i at a planning optimum. Notice, also, that, although the Planner is egalitarian, she takes into account the subjective discount factors, θ_i, representing their degree of altruism with respect to the offsprings, in allocating resources. Using (47) and the remaining first order characterization of the planning optimum for a CES specification of individuals' preferences, one can easily show that, under the assumption $\theta_r > \theta_p$,

$$\lim_{t\to\infty} \frac{c_t^{t,p}}{c_t^{t,r}} = 0,$$

i.e., the Planner's allocation is such that the poor (less altruistic) type young households end up with a lower consumption than the rich (more altruistic) type.

Now we consider the effect on the Planner's welfare function, $\mathcal{U}$, of a once and for all rise in a at a PBSS, assuming that poor households own zero land (*i.e.*, $z^p = 0$). By exploiting the equilibrium conditions (*i.e.*, resource feasibility, first order conditions for individual optimality at equilibrium and budget constraints) and letting $\eta^i \equiv \psi'(V^{i,-1})$, we derive

$$a\frac{\partial \mathcal{U}}{\partial a} = \eta^r \mu^r \left(W(1-\lambda) - a\frac{\partial g}{\partial a} \right) + m_p \Psi, \tag{48}$$

where

$$\Psi = (\eta^p \mu^p - \eta^r \mu^r) W + \hat{q}_a^h \frac{q^h h^p}{1+r} \left(\left(\frac{\delta + n}{1+n} \right) \left(\frac{\eta^r \mu^r}{\theta_r} - \frac{\eta^p \mu^p}{\theta_p} \right) + (1+r) \left(\frac{\theta_r - \theta_p}{\theta_p} \right) \right).$$

Since a First Best allocation is such that $\eta^p \mu^p = \eta^r \mu^r$, the first summation on the right hand side of (48) represents the *undistorted* component of the welfare effect of a rising a conditional on the wasteful government spending of the revenues from land sale, whereas Ψ represents two possible distortions: the first one arising from the possibility that consumption is not allocated as dictated by the Planner across individuals of the same generation, and the second from the fact that the poor-type households are unable to leave positive bequests to their offsprings. Observe that, at equilibrium, the sign of $\eta^p \mu^p - \eta^r \mu^r$ is ambiguous, since it depends on the differences between consumptions across the two type of households (in the same age and time), the dynastic-specific discount factors and utilities. If the discount factors are very similar and the rich households have a much larger consumption compared with the poor household, we have $\eta^p \mu^p - \eta^r \mu^r > 0$. We say that the impact of a on social welfare falls short of the first best level net of wasteful government spending if $\Psi < 0$.

To evaluate the sign of Ψ we concentrate on a case that appears to be particularly relevant in our model. Namely, we assume CES utility function (as in the previous sections) and we evaluate Ψ for θ_p converging to θ_r. With a CES representation of preferences, utility is linear in households' income and the marginal utilities of age-contingent consumptions and housing are identical across households at steady states, so that $u_1^p = u_1^r$. Then, due to the concavity of ψ, when subjective discount rates are sufficiently similar across households, we get $\mu^p > \mu^r$ (since $b^r > b^p = 0$ implies $V^{-1,p} < V^{-1,r}$) and, then,

$$\Psi < 0 \qquad \Leftrightarrow \qquad \hat{q}_a^h > \left(\frac{1+n}{\delta+n}\right)\frac{W}{q^h h^p}.$$

The intuition is the following. When discount rates are sufficiently similar across households, the impact of a rising productivity in manufacturing on social welfare falls short of the first best level if the poor (less altruistic) households are penalized. As we know from the analysis in the previous sections, a rise in manufacturing productivity brings about a higher wage and a higher housing price. The former has a positive effect on poor households welfare, whereas the latter has two distinct and opposing effects. On the one hand, a higher housing price increases the old individual's wealth thereby increasing her utility. On the other hand, the higher housing price makes this good less affordable. The above analysis shows that this last effect of a housing appreciation exceeds the former, and, if wages are sufficiently low relative to the value of housing, the negative welfare impact of the housing appreciation may overcome the positive impact due to higher wages.

7 Conclusions

We have shown that a Baumol's cost disease can explain the increase in total and housing wealth-to-income ratios and wealth inequality that took place in the eight largest advanced economies in the last forty years. To show this, we have employed a simple life-cycle model with no financial frictions, two sectors (construction and manufacturing) and one-sided parental altruism. Key assumptions are that the construction sector is less capital intensive than manufacturing and housing demand sufficiently inelastic. Under these assumptions, a rise in labor efficiency in manufacturing produces a strong upward pressure on housing prices, a rise in the total and housing wealth-to-income ratios and a rise in bequests. The increase in housing valuations can possibly mitigate (relative to the First Best level) the beneficial effects of a rising productivity in manufacturing under an egalitarian welfare criterion when market allocations imply high enough consumption inequality and low enough heterogeneity in parental altruism.

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A Data

In the paper we use two main source of data. The first is Piketty and Zucman (2014), which provide data on wealth-to-income ratios, as well as saving and income growth rates, for the eight largest advanced economies (*i.e.*, the US, Germany, the UK, Canada, Japan, France, Italy and Australia). The second is the KLEMS database (O'Mahony and Timmer, 2009), from which we are able to build a measure of relative labor efficiency in manufacturing. For most countries, data from Piketty and Zucman (2014) are annual and covers the 1970-2010 sample; data from KLEMS are also annual, and covers the 1970-2007 sample. However, in both databases, the available samples for some variables and countries are shorter. As a general rule, for each country we use the longest available series and fill missing values with the closest available observation. For example, if for a given series data start only in 1973, we set the values for 1970 to 1972 to the first value of the series in 1973. This section of the appendix presents additional details on the data, as well as robustness checks on the stylized facts and empirical relations discussed in the paper. In addition, we use data from Piketty and Saez (2014) to capture the increase in income and wealth inequality in our sample of advanced economies.

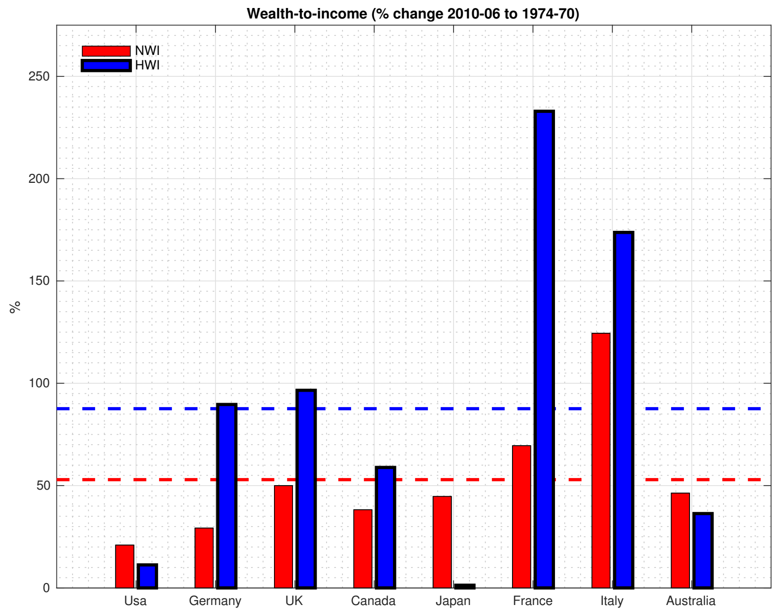
A.1 Data on Wealth Ratios

Wealth ratios are from Piketty and Zucman (2014), who collect an incredibly rich dataset on wealth and income for the eight largest developed economies (the US, Germany, the UK, Canada, Japan, France, Italy and Australia) over the 1970-2010 period. Since the dataset is built according to the UN System of National Accounts (SNA), data are, to the best of our knowledge, homogeneous across countries. In the paper, we use "national", as opposed to "private", wealth if not otherwise specified. Private wealth is the net wealth of households and non-profit institutions serving households. Assets include both non-financial assets (*i.e.*, land, buildings, machines, etc.) and financial assets (*i.e.*, stocks, bonds, shares in pension funds, etc.). Note that two types of assets are not included in the definition of wealth: pay-as-you-go public pensions and durable goods. Assets and liabilities are evaluated at market prices and this is one important difference between Piketty and Zucman (2014)'s and prior datasets built on the basis of aggregation of saving flows. National wealth is instead the sum of private and public wealth, where the latter is the net wealth of public administrations and government agencies. Note that national wealth can be also decomposed as the sum of domestic capital and net foreign assets so that in a closed economy it is exactly equal to the market value of domestic capital. The income definition used to build the wealth ratios is "net-of-depreciation". Therefore, whenever we refer to saving or growth rates we implicitly consider their "net" definitions. In section 2 we documented the large average increase in national wealth, and in particular housing wealth, over the period 1970 to 2010. In this section we provide additional details on the behavior of wealth ratios. First, for robustness, we report in figure A1 the percentage changes in national and housing wealth-to-income using five year averages at the end-points of the sample (*i.e.*, the mean values for the years 2010 to 2006 and 1974 to 1970) to confirm that the figures reported in the paper do not depend only on some very low or high value in the years 1970 and 2010. The mean percentage changes are, respectively, 52 and 87 percent and very close to those reported in the section 2 (*i.e.*, 60 and 112 percent). Second, in figure A2 we plot the time-series for national (top panel) and housing (middle panel) wealth-to-income as well as for housing as a share of national wealth. For all countries national and housing wealth is a multiple of net income. In 1970 the average value of the national wealth-to-income ratio was approximately 3 and since then it has been increasing and has reached an average value of close to 5 in 2010. Housing wealth is, for most countries, close to one-half of national wealth and it has been trending upwards since 1970 and up to the Great Recession. Third, private, rather than national, wealth follows a similar pattern. In figure A3 we plot the time-series for the ratio between private wealth and net income (top panel) and disposable income (bottom panel), where the latter is after taxes and government transfers. All the series share a similar empirical regularities for all the countries in the sample: a general upward trend in wealth.

A.2 Data on Wealth and Income Inequality

We collect data on wealth and income inequality from Piketty and Saez (2014)'s World Top Income Database. Data on wealth inequality are decennial, for the period 1950 to 2010, for France, the UK, the US and Europe. In figure A4 we plot the time-series for the shares of the top 10 percent (top panel) and 1 percent (bottom panel) of the wealth distribution. In both cases, wealth inequality is high at the beginning of the sample; it drops in all countries, with the exception of the US, up until 1970; and then it increases gradually back to the values of the 1950. The US is the country with the highest level of wealth inequality, with 70 percent (35 percent) of the total wealth going to the top 10 percent (1 percent) of the wealth distribution. The UK has recently reached levels of wealth inequality similar to those of the US, while France, and Europe more

Figure A1: National Wealth (percentage changes 2010-06 to 1974-70)

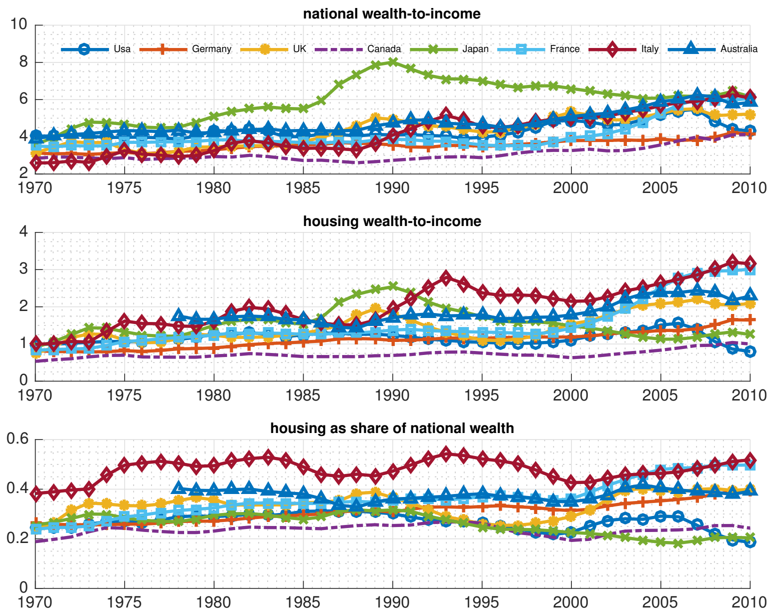


Notes: This figure plots the percentage changes in national (NWI) and housing (HWI) wealth-to-income ratios between the average values for the years 2010 to 2006 and 1974 to 1970. For Australia we use average values for the years 2010 to 2006 and 1982 to 1978. Horizontal dashed-lines correspond to the cross-sectional averages (52 and 87 percent respectively). Both national and housing wealth are annual, at market prices, from Piketty and Zucman (2014). Additional details on the data are available in the Appendix (section A).

in general, show a significant lower degree of inequality. Note that measuring wealth is hard and likely subject to measurement errors. In addition, several authors (for example, Auerbach and Hasset (2015) and references therein) have argued that Piketty and Saez (2014)'s data contains inconsistencies with respect to the description of the data construction and aggregation. While we acknowledge the possible limitations of Piketty and Saez (2014)'s data, it is outside the scope of our paper to build an alternative dataset.

Data on income inequality are annual for the period 1970 to 2010 for the eight largest advanced economies. In figure A5 we plot the time-series for the shares of the top 10 percent (top panel) and 1 percent (bottom panel) of the income distribution. While data on wealth is to some extent controversial, data on income inequality is less subject to measurement errors. For all countries, income inequality trends up, with the US being the country with the largest increase. In the US, approximately 45 percent (17 percent) of the aggregate net income goes to the top 10 percent (1 percent) of the income distribution. As for wealth, also for income European countries show a lower, even though increasing, degree of inequality.

Figure A2: National Wealth-to-Income Ratios



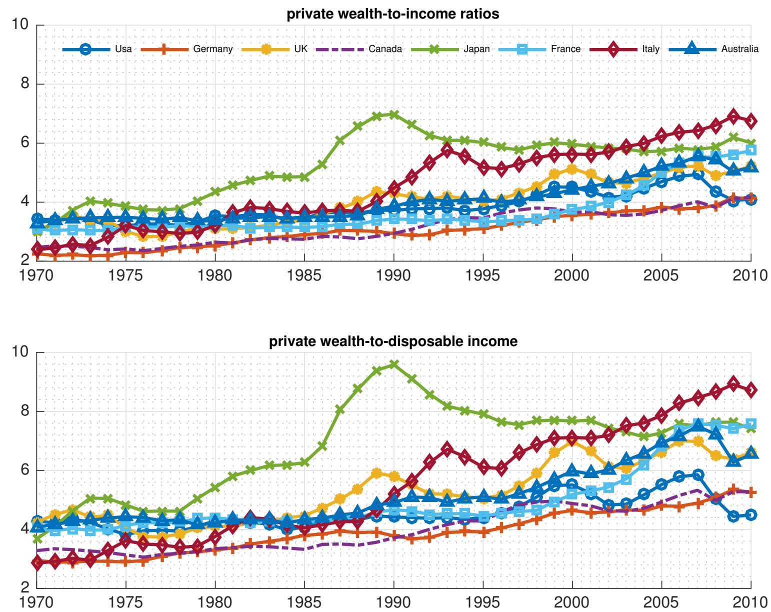
Notes: This figure plots the evolution of national wealth-to-income ratios. Income is net of depreciation and wealth is at market prices. The countries in the sample are the United States, Germany, the UK, Canada, Japan, France, Italy and Australia. The top panel plots the national wealth-to-income ratios; the middle panel the housing wealth-to-income ratios; the bottom panel housing wealth as a share of national wealth. Data are annual for the period 1970-2010 from Piketty and Zucman (2014).

Therefore, data on wealth and income inequality show that in all countries in the sample inequality has been trending up since 1970.

A.3 Data on Relative Productivity

We use the KLEMS database, available at http://www.euklems.net/, and organized by O'Mahony and Timmer (2009), to construct a measure of relative labor efficiency in manufacturing. In particular, we use the ISIC Rev. 3 version of the KLEMS database (March, 2011), which reports data from 1970 up to 2007. For each of the country, and for different sectors, we collect data on gross value added at current prices (VA); gross value added volume index (VA_QI); capital compensation (CAP); capital services volume index (CAP_QI) and labor services volume index (LAB_QI). In particular, we select sectors "total industry" (TOT) as proxy for the manufacturing sector, and "construction" (F). The countries in the sample are: the US (USA), Japan (JPN), Italy (ITA), Germany (DEU), France (FRA), the UK (GBR), Canada (CAN), Australia (AUS), Spain (ESP), Ireland (IRL), Austria (AUT), Belgium (BEL), Denmark (DNK), Finland (FIN), the Netherlands (NLD), Sweden (SWE). For the US, data on VA, VA_QI

Figure A3: Private Wealth-to-Income Ratios



Notes: This figure plots the evolution of private wealth-to-income ratios. In the top panel income is net of depreciation and in the bottom panel is net of depreciation and after taxes and government transfers. Wealth is at market prices. The countries in the sample are the United States, Germany, the UK, Canada, Japan, France, Italy and Australia. Data are annual for the period 1970-2010 from Piketty and Zucman (2014).

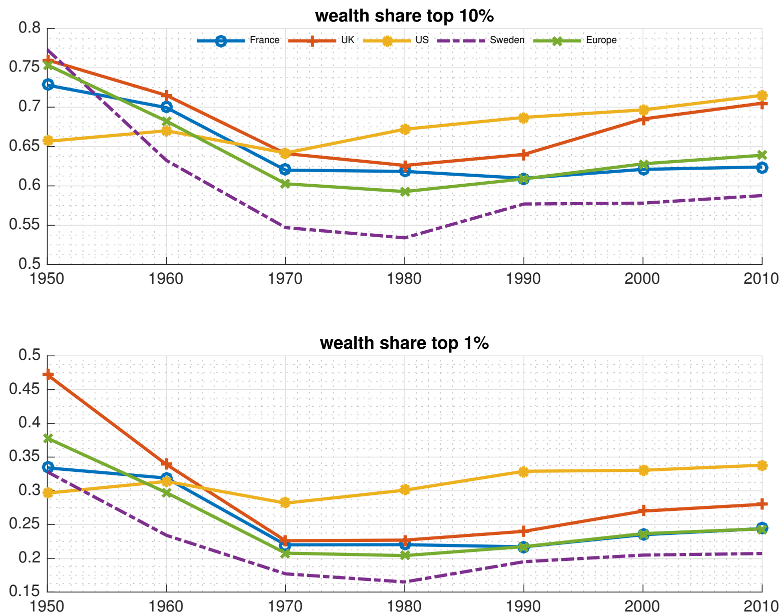
and CAP start in 1977; for Japan data on VA, VA_QI, CAP, LAB_QI start in 1973 and end in 2006; for Spain data on LAB_QI start in 1980; for Ireland data on CAP_QI and LAB_QI start in 1988; for France data on LAB_QI start in 1980; for Australia data on CAP_QI and LAB_QI start in 1982; for Austria data on LAB_QI start in 1980 and for CAP_QI in 1976; for Belgium data on LAB_QI start in 1980 and end in 2006; for Denmark data on LAB_QI start in 1980; for the Netherlands data on LAB_QI start in 1979; for Sweden data on LAB_QI start in 1981; for Canada, KLEMS does not contain data on the "total industry" sector, so that we proxy manufacturing with "wood and product of wood" and all series end in 2004.

We estimate labor augmenting productivity residuals starting from our assumptions on the technology in the two sectors. For the manufacturing sector the labor-augmented productivity A^m is:

$$A^m = [Y^m (K^m)^{-\alpha^m} (L^m)^{\alpha^m - 1}]^{1-\alpha^m}, \tag{A1}$$

where, for each country, we estimate α^m as the ratio between CAP and VA: *i.e.*, as the capital share of income in manufacturing. For the construction sector, we first assume $\eta^h = 0$, as we do not have data on the land input; second, we assume that α^h is the

Figure A4: Wealth Distribution



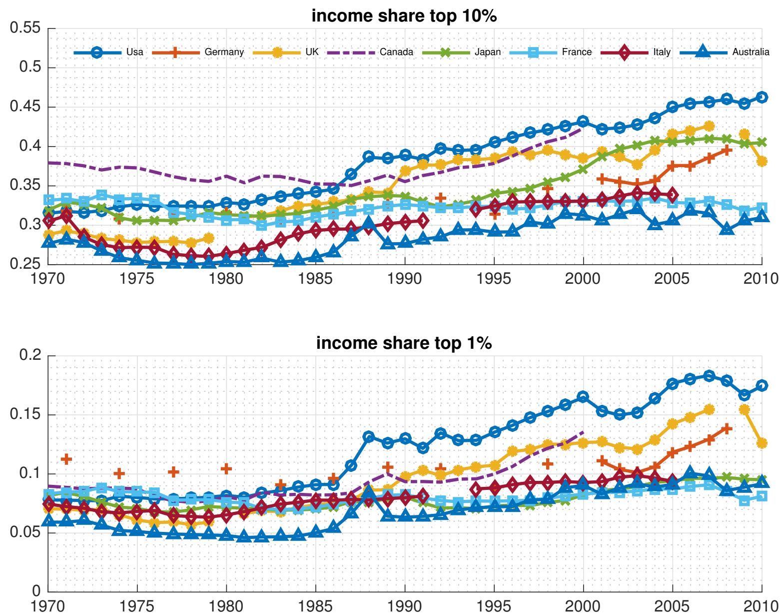
Notes: The top (bottom) panel of this figure plots the evolution of the shares of the top 10 (top 1) percent of the wealth distribution. The countries, or economic regions, in the sample are the France, the UK, the US, Sweden and Europe. Data are decennial from the World Top Income Database for the sample 1950-2010.

capital share of income in construction, as for the Cobb-Douglas case; third, we set the elasticity of substitution between capital and labor to $\sigma^h = 0.6$, as in the model. The labor-augmented productivity A^h is then:

$$A^h = \frac{1}{L^h}\left[\frac{(Y^h)^{\frac{\sigma^h-1}{\sigma^h}} - \alpha^h (K^h)^{\frac{\sigma^h-1}{\sigma^h}}}{1-\alpha^h}\right]^{\frac{\sigma^h}{\sigma^h-1}} \tag{A2}$$

Relative labor efficiency in manufacturing in then simply equal to $a_t = A_t^m / A_t^h$. In figure A6 we plot the time-series of the estimated relative labor efficiency in manufacturing for the countries in the Piketty and Zucman (2014)'s sample. The black lines correspond to the baseline case of $\sigma^h = 0.6 < 1$, while the green lines correspond to the case of $\sigma^h = 1$ (*i.e.*, Cobb-Douglas production also in construction). In all countries we observe unbalanced productivity improvements, with productivity growing faster in the manufacturing sector. In figure A7 we plot the percentage changes in relative productivity over the 1970 to 2007 period. On average, productivity in manufacturing has been growing faster than in construction in all countries, and regardless of the assumption on σ^h. The mean increase is larger when we assume $\sigma^h < 1$ (*i.e.*, approx-

Figure A5: Income Distribution



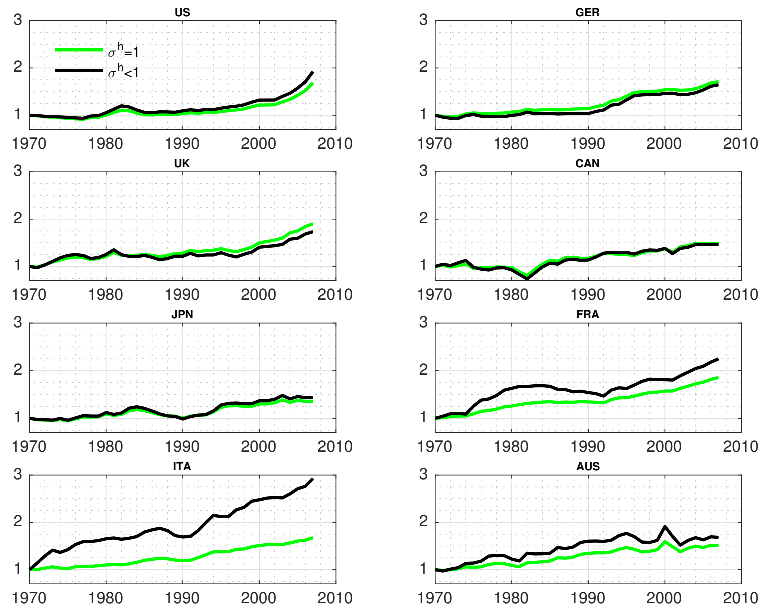
Notes: The top (bottom) panel of this figure plots the evolution of the shares of the top 10 (top 1) percent of the income distribution. The countries in the sample are the United States, Germany, the UK, Canada, Japan, France, Italy and Australia. Data are annual from the World Top Income Database for the sample 1970-2010.

imately 88 percent), than when we set $\sigma^h = 1$ (65 percent). The difference between the two measures is particularly striking for Italy, where relative productivity increases by twice as much with elasticity of substitution smaller than unity. Countries where the increase in relative productivity is larger for $\sigma^h < 1$ tend to have a larger capital share of income in construction. For example, the latter is equal to approximately 0.3 in Italy and only 0.17 in Germany.

A.4 Data on Housing Prices

The large increase in housing wealth, with respect to income, that we documented in the paper is to a large extent driven by the increase in housing prices. We collect quarterly real housing price data from the OECD (2012) for the period 1970:I to 2013:IV. In figure A8 we plot the long-run changes in real housing prices from 1970 to 2010. For the eight largest advanced economies of the Piketty and Zucman (2014)'s sample, the mean increase in real housing prices is approximately 136 percent. Germany is the only country with a drop in real housing prices over the sample, while the UK is the country with the largest increase (approximately 350 percent). In some of the countries, real housing prices have an inverse u-shape. For example, starting in 1970, the maximum

Figure A6: Relative Labor Efficiency in Manufacturing



Notes: This figure plots two measures of relative labor efficiency in manufacturing for the US, Germany, the UK, Canada, Japan, France, Italy and Australia for the period 1970-2007. Relative labor efficiency is estimated as residual assuming Cobb-Douglas technology in manufacturing, and either Cobb-Douglas (green lines) or CES (black lines) technology in housing construction. The elasticity of substitution for the CES technology is set to $\sigma^h = 0.6$. All series are normalized to 1 in 1970. Data are annual from O'Mahony and Timmer (2009). For each country we use the longest available series and extend it to the 1970-2007 sample setting any missing values to the closest available observation.

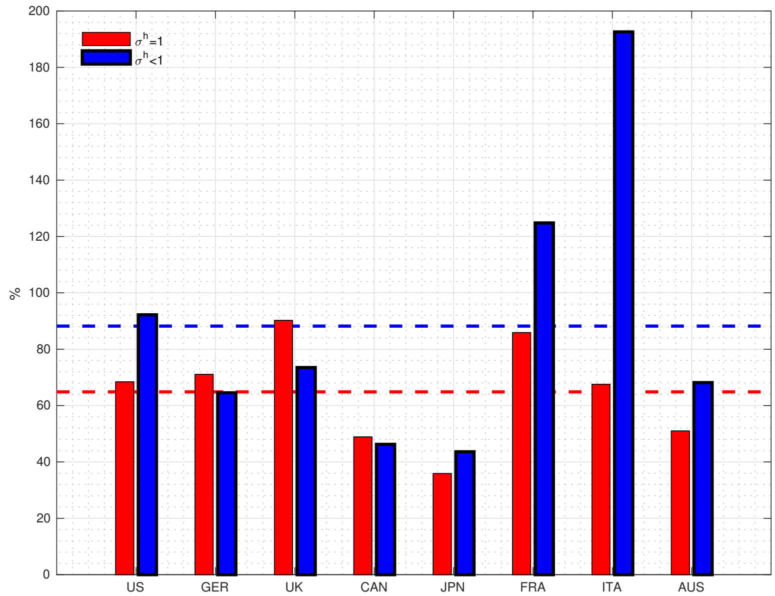
increase in real housing prices for Japan is 100 percent, compared to a more modest 12 percent increase with respect to the values in 2010. In the US real housing prices dropped significantly during the Great Recession: for example, the maximum increase is over the period 1970:I to 2006:IV and is equal to 117 percent. In fact, the mean maximum increase in real housing prices starting from 1970 is 170 percent.

In figure A9, for the same eight countries, we plot the time-series of real housing prices. Japan, and to some extent the US, are the countries where real housing prices, over the sample 1970:I-2013:IV, have a u-shape. On the contrary, Germany is the only country where prices started started to fall in the mid-90's and then recovered only after the Great Recession. For all the remaining countries real housing prices trend upward, despite some variability.

A.5 Robustness on Styilized Facts

In section 2 we documented the existence of a positive, and statistically significant, relationship between long-run log changes in national wealth-to-income and relative

Figure A7: Relative Labor Efficiency in Manufacturing (percentage changes 2007-1970)

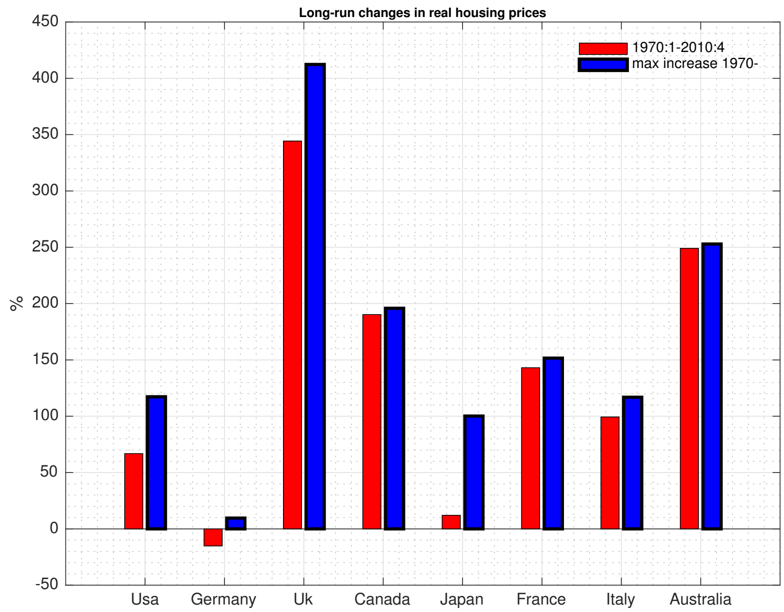


Notes: This figure plots the percentage changes in two measures of relative labor efficiency in manufacturing for the US, Germany, the UK, Canada, Japan, France, Italy and Australia for the period 1970-2007. Relative labor efficiency is estimated as residual assuming Cobb-Douglas technology in manufacturing, and either Cobb-Douglas (red bars) or CES (blue bars) technology in housing construction. The elasticity of substitution for the CES technology is set to $\sigma^h = 0.6$. The horizontal dashed-lines correspond to the cross-country averages and are, respectively, equal to 65 and 88 percent. Data are annual from O'Mahony and Timmer (2009). For each country we use the longest available series and extend it to the 1970-2007 sample setting any missing values to the closest available observation.

productivity in manufacturing. On the contrary, the relationship between the saving to income growth ratio, based on the prediction of the Solow's model, is weak at best. In the section, we show that these results are confirmed if we consider long-run changes housing wealth and prices. In figure **??** we plot long-run changes in housing wealth-to-income ratios and relative labor efficiency in manufacturing (left panel) and saving to income growth ratio (right panel), for the eight largest economies of our sample. The long-run changes are for the period 1970 to 2007. We plot with a red lines the OLS fitted residuals, and report OLS slope and its t-stat (in brackets). The relationship between changes in productivity (a) and housing wealth(hwi) is positive and significant: for a 1 percent increase in a, hwi increases by 1.8 percent. On the contrary, the effect of an increase in the ratio between saving and income growth rate (s/g) is not significant, and slightly negative.

Since an important component of the increase in housing wealth is the large increase in housing prices, in figure **??** we also show that, for a large sample of 16 OECD countries, over the 1970 to 2007 period, there exists a positive, even though only

Figure A8: Real Housing Prices (percentage changes 2010-1970)



Notes: This figure plots the percentage changes in real housing prices for the US, Germany, the UK, Canada, Japan, France, Italy and Australia. The red bars correspond to the the percentage changes over the period 1970:I-2010:IV. The blue bars correspond, for each country, to the largest percentage increase in housing prices starting in 1970: for the US 2006:IV; Germany 1981:II; the UK 2007:IV; Canada 2010:III; Japan 1991:I; France 2007:IV; Italy 2008:I; Australia 2010:II. Data are quarterly from the OECD (2012) for the period 1970:I - 2010:IV.

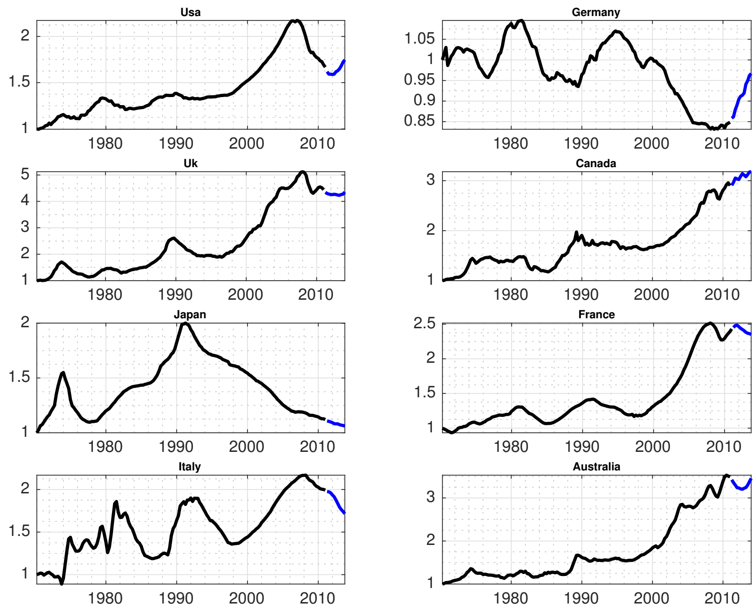
weakly significant, relationship between long-run log changes in real housing prices and relative productivity improvements in manufacturing, with an estimated elasticity of 0.42 percent.

B CES Utility Specification

We specify utility as:

$$u(c^{y,i}, c^{o,i}, h^i) = \begin{cases} \left[\chi^y (c^{y,i})^{\frac{\gamma-1}{\gamma}} + \chi^o (c^{o,i})^{\frac{\gamma-1}{\gamma}} + \chi^h (h^i)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}} & \text{if } \gamma \neq 1, \\ \chi^y \log c^{y,i} + \chi^o \log c^{i,o} + \chi^h \log h^i & \text{otherwise,} \end{cases} \tag{A3}$$

Figure A9: Real Housing Prices



Notes: This figure plots quarterly real housing prices for the US, Germany, the UK, Canada, Japan, France, Italy and Australia for the period 1970:I to 2010:IV. Blue lines denotes observations after 2010:IV. Data are quarterly from the OECD (2012) for the period 1970:I - 2013:IV.

where $\chi^j > 0$ for $j = y, o, h$ and $\sum_{j=y,o,h} \chi^j = 1$. Then, the expenditure shares are defined by

$$\phi^y = \frac{(\chi^y)^\gamma}{(\chi^y)^\gamma + (\chi^o)^\gamma(1+r)^{\gamma-1} + (\chi^h)^\gamma \pi^{1-\gamma}}, \tag{A4}$$

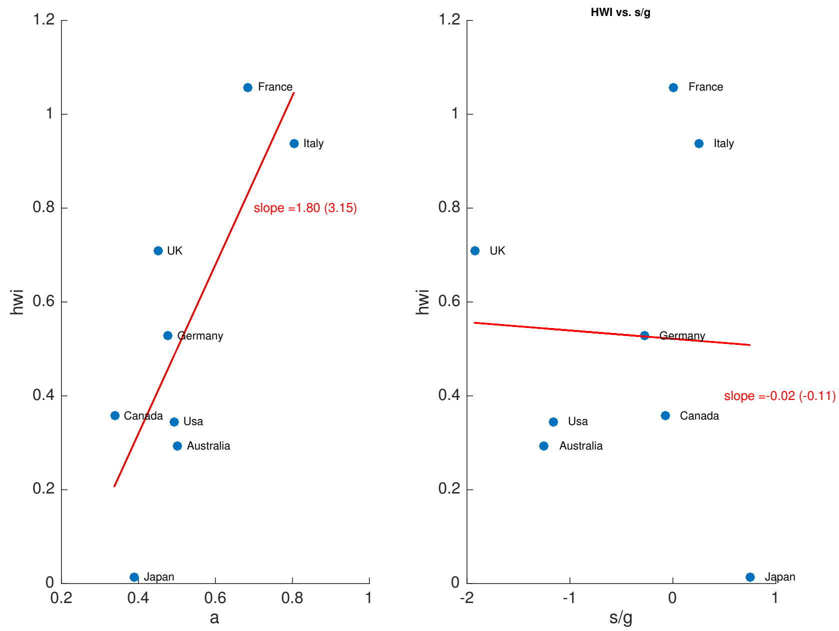
$$\phi^o = \frac{(\chi^o)^\gamma(1+r)^{\gamma-1}}{(\chi^y)^\gamma + (\chi^o)^\gamma(1+r)^{\gamma-1} + (\chi^h)^\gamma \pi^{1-\gamma}}, \tag{A5}$$

$$\phi^h = \frac{(\chi^h)^\gamma \pi^{1-\gamma}}{(\chi^y)^\gamma + (\chi^o)^\gamma(1+r)^{\gamma-1} + (\chi^h)^\gamma \pi^{1-\gamma}}. \tag{A6}$$

C Numerical Solution of Steady-State

The model is simulated numerically using the parameters presented in table 1 and discussed in section 5.4. The utility function and the corresponding expenditure shares are those specified in section B and equation A3. We assume Cobb-Douglas technology in manufacturing, and CES with elasticity of substitution σ^h in construction. In what follows we express all variables in intensive form following the notation presented in the paper. The production functions for the two sectors are:

Figure A10: Housing Cost Disease vs. Piketty's view (Housing Wealth-to-Income)

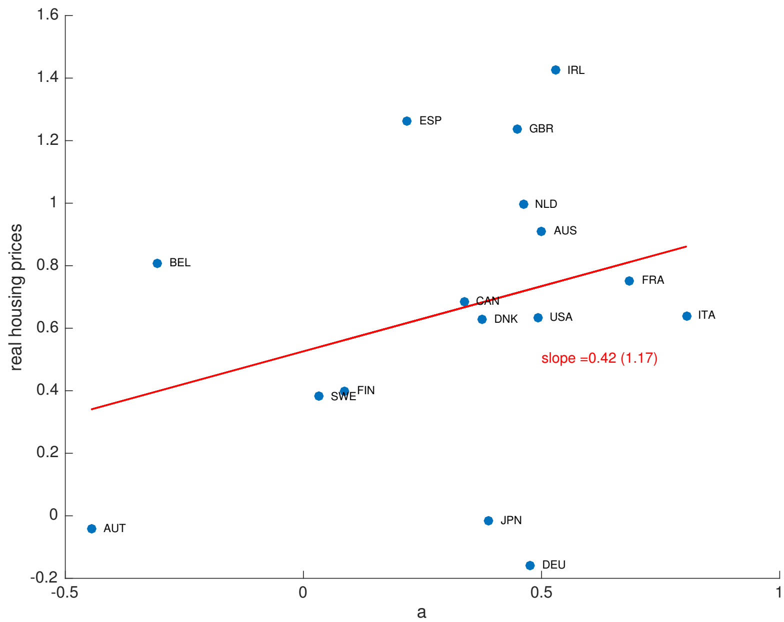


Notes: This figure plots long-run log changes in housing wealth-to-income against long-run log changes in relative labor efficiency in manufacturing (left panel, a) and in the ratio between national saving and income growth rates (right panel, s/g) for the US, Germany, the UK, Japan, France, Italy and Australia. The log changes are computed between the average values of each variable for the period 2007-2004 and 1974-1970. Relative labor efficiency is estimated from O'Mahony and Timmer (2009)'s data assuming Cobb-Douglas technology for the manufacturing sector and CES technology with elasticity of substitution equal to 0.6 for the construction sector. The red lines correspond to OLS fitted values. We also report estimates for the OLS slopes (t-stats in brackets). Data for national wealth-to-income, saving and income growth rates are from Piketty and Zucman (2014). Additional details on the series are available in the Appendix (section A).

$$
\begin{aligned}
f_m &= (k^m)^{\alpha^m}, \\
f_h &= [\alpha^h (k^h)^{\frac{\sigma^h - 1}{\sigma^h}} + \eta^h (z^h)^{\frac{\sigma^h - 1}{\sigma^h}} + (1 - \alpha^h - \eta^h)]^{\frac{\sigma^h}{\sigma^h - 1}}.
\end{aligned}
$$

We consider the two-type case (*i.e.*, rich and poor) and the PBSS, so that in the steady-state $b^r > b^p = 0$ and the interest rate is pinned down by the discount rate that rich households attach to the utility of their offsprings. For a given level of the relative productivity in manufacturing, a, several variables are directly pinned down by the exogenous parameters:

Figure A11: Housing Prices and Relative Labor Efficiency in Manufacturing



Notes: This figure plots long-run log changes in housing prices against long-run log changes in relative labor efficiency in manufacturing (left panel, a) for the US (USA), Japan (JPN), Italy (ITA), Germany (DEU), France (FRA), the UK (GBR), Canada (CAN), Australia (AUS), Spain (ESP), Ireland (IRL), Austria (AUT), Belgium (BEL), Denmark (DNK), Finland (FIN), the Netherlands (NLD), Sweden (SWE). The log changes are computed between the average values of each variable for the period 2007-2004 and 1974-1970. Relative labor efficiency is estimated from O'Mahony and Timmer (2009)'s data assuming Cobb-Douglas technology for the manufacturing sector and CES technology with elasticity of substitution equal to 0.6 for the construction sector. The red lines correspond to OLS fitted values. We also report estimates for the OLS slopes (t-stats in brackets). Data for housing prices are real and from the OECD. Additional details on the series are available in the Appendix (section A).

$$
\begin{aligned}
r &= 1/\theta^r - 1 \\
k^m &= (\frac{\alpha^m}{1+r})^{\frac{1}{1-\alpha^m}} \\
w &= (1-\alpha^m)(k^m)^{\alpha^m}
\end{aligned}
$$

Our algorithm numerically solves the following system of equations[9]:

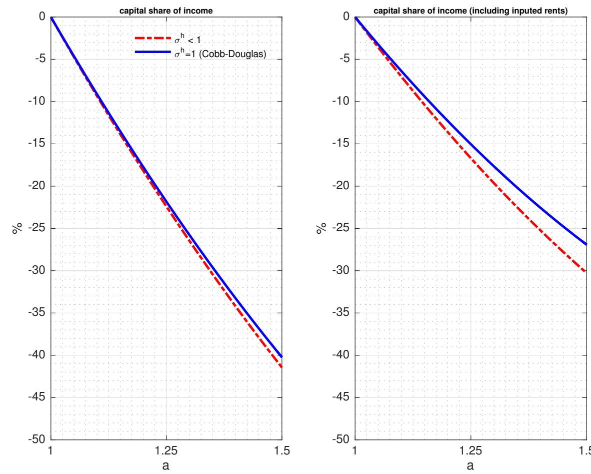
[9] We make available on our personal web pages the Matlab code that solves the model and generates all figures and results presented in the paper and this appendix.

$$
\begin{aligned}
q^h &= (\frac{1+r}{\alpha^h})(k^h)^{1/\sigma^h}[\alpha^h(k^h)^{\frac{\sigma^h-1}{\sigma^h}} + \eta^h(z^h)^{\frac{\sigma^h-1}{\sigma^h}} + (1-\alpha^h-\eta^h)]^{1/(1-\sigma^h)}, && \text{(A7)}\\
\mathcal{S}^h_k &= \alpha^h(k^h)^{\frac{\sigma^h-1}{\sigma^h}}[\alpha^h(k^h)^{\frac{\sigma^h-1}{\sigma^h}} + \eta^h(z^h)^{\frac{\sigma^h-1}{\sigma^h}} + (1-\alpha^h-\eta^h)]^{-1}, && \text{(A8)}\\
\Delta &= (\frac{n+\delta}{1+r})\mathcal{S}^h_k(\frac{ak^m-k^h}{k^h}), && \text{(A9)}\\
\pi &= q^h(\frac{\delta+r}{1+r}) && \text{(A10)}\\
h &= (\frac{1}{1+n})(\frac{\phi^h}{\pi})(aw+m^r(1-(1+n)\theta^r)b), && \text{(A11)}\\
s &= (1-\phi^y)aw+m^rb((1-\phi^y)+\phi^y(1+n)\theta^r), && \text{(A12)}\\
\lambda &= \frac{(n+\delta)\mathcal{S}^h_k v^d}{(1+r)k^h}, && \text{(A13)}\\
v^d &= q^h h, && \text{(A14)}\\
v^s &= \frac{\frac{s}{1+n}-ak^m-q^z\xi}{1-\Delta}, && \text{(A15)}\\
\frac{aw}{1+r} &= \frac{1-\alpha^h-\eta^h}{\alpha^h}\frac{[\alpha^h(k^h)^{\frac{\sigma^h-1}{\sigma^h}} + \eta^h(z^h)^{\frac{\sigma^h-1}{\sigma^h}} + (1-\alpha^h-\eta^h)]^{\frac{\sigma^h}{\sigma^h-1}}}{(k^h)^{\alpha^h-1}}(\frac{\xi}{\lambda})^{\eta^h} && \text{(A16)}
\end{aligned}
$$

where ϕ^y and ϕ^o are defined,respectively, by equations A4 and A6. In the simulations the assumption 4, regarding the value of the relative capital intensity in manufacturing, is always verified. Note that when $\sigma^h = 1$, the production function in construction is Cobb-Douglas and the system is simplified as the capital share, $\mathcal{S}^h_k$, and the capital in the housing sector, k^h, are directly pinned down by the exogenous parameters.

In section 5.4 we presented the effects of relative productivity improvements in manufacturing on wealth ratios, inequality, asset prices and the allocations of factors. In this section we present some additional results. Recall that the capital share is defined as $\zeta = 1 - aw/y$. Since aw respond proportionally to an increase in a, while y increases less than proportionally, we expect the capital share to decline. This is confirmed by our simulation results in figure A12. The left panel plots ζ and shows that it decreases by about 40 percent for a 50 percent increase in relative productivity in manufacturing. In section 5 of the main paper, we also showed that an alternative measure of capital share, that includes imputed rents (ζ^h), can theoretically increase if the increase in housing wealth is large enough. However, under our parametrization, also this alternative definition of capital share declines with a, but by less than ζ.

Figure A12: Capital Shares



Notes: This figure plots the percentage changes in the steady-state values of the capital share of income (ζ) for different values of the relative productivity sector $a = 1, \ldots, 1.5$, with respect to their value for $a = 1$. We consider two alternative definitions: the left panel corresponds to the standard definition of capital share $\zeta = 1 - aw/y$; the right panel corresponds instead to a definition that includes imputed rents as in $\zeta^h = (y + \pi h - aw)/(y + \pi h)$. The red-dashed line is for the baseline value of the elasticity of substitution in the housing sector, $\sigma^h = 0.6$. The blue solid lines corresponds to Cobb-Douglas technology in the housing sector ($\sigma^h = 1$).

D Comparative Statics

D.1 Labor Allocation and Asset prices

Since the land policy (21) implies $z = \xi/\lambda$, equation (23) implies that the share of labor in construction is determined implicitly by v (and a) through the following equation

$$\frac{\lambda f_h(k^h(r,a), \xi/\lambda)}{f_{h,k}(k^h(r,a), \xi/\lambda)} = \left(\frac{\delta + n}{1 + r}\right) v. \tag{A17}$$

It is possible to show that the left hand side of (A17) is increasing in λ, so that following proposition holds.

Proposition 3. *For all $v \in [0, v^m(a)]$, with*

$$v^m(a) = \frac{f_{h,k}(k^h(r,a), \xi)}{f_h(k^h(r,a), \xi)} \left(\frac{1 + r}{\delta + n}\right),$$

there exists a value $\lambda(v, a) \in [0, 1]$ solving equation (A17), such that $\lambda_v(v, a) > 0$, $\lambda_a(v, a) < 0$ and $\lambda(0, a) = 0$, $\lambda(v^m(a), a) = 1$.

Proof. Letting

$$\phi(\lambda, a) = \lambda f_h(k^h(r,a), \xi/\lambda)/f_{h,k}(k^h(r,a), \xi/\lambda),$$

direct computation shows that

$$\phi_\lambda = (k^h/\mathcal{S}_k^h)\left(1-\mathcal{S}_z^h+(\mathcal{S}_k^h/\sigma^h)(k^h/\mathcal{S}_k^h)^{\frac{\sigma^h-1}{\sigma^h}}\right), \tag{A18}$$

$$\phi_a = \lambda(k^h/a)(\sigma^h+(1-\mathcal{S}_k^h)/\mathcal{S}_k^h), \tag{A19}$$

so that

$$\lambda_v(v,a)=\frac{(\delta+n)}{(1+r)\phi_\lambda}>0, \qquad \lambda_a(v,a)=-\frac{\phi_a}{\phi_\lambda}<0.$$

□

Quite intuitively, a rise in housing wealth generates a reallocation of labor toward the construction sector and a rise in a a reallocation of labor away from this sector. Note that these are only partial effects: in equilibrium, housing wealth, v, is affected by a, so that a rise in productivity in the manufacturing sector may shift labor to the construction sector if this has a positive and large enough effect on v.

By the mapping $\lambda = \lambda(v,a)$ and the land policy (21), we derive the steady state asset prices defined in (24), (25), (26) as functions of (v,a) as well. Observe that the effect of a higher housing wealth on asset prices is unambiguously positive. In fact, since $\lambda_v(v,a) > 0$, a rise in housing wealth generates a larger labor share in construction, reduces the land-to-labor ratio in the construction sector and, then, it reduces the marginal product of capital and it increases the marginal product of land. Since r is pinned down by θ_r, all asset prices increase. On the other hand, the impact of a on asset prices for given v is ambiguous. When land is totally unproductive, q^h is independent of v and, by differentiation of q^h with respect to a, we derive $\partial q^h/\partial a > 0$. Hence, similarly to the Baumol's prediction, a rise in productivity in manufacturing would generate a rise in the relative price of the output of the stagnant sector. In more general cases, a affects λ (for any given v) and the latter affects the land-labor ratio, z. In particular, since $z = \xi/\lambda(v,a)$, $\lambda_a < 0$, and the CES specification implies $f_{h,k,z} > 0$, we have that q^h and q^z are increasing in a for given z and decreasing in z for given a. Hence, for given v, a rise in a has two effects on asset prices. The first is a positive direct effect because of a higher capital labor ratio in the construction sector. The second is a negative indirect effect through a higher land-labor ratio induced by a lower share of labor in this sector. We can provide an analytical representation of these effects in terms of factor shares. In particular, the partial elasticities of $q^h(v,a)$

and $q^z(v,a)$ are

$$\hat{q}^h_v = \frac{\mathcal{S}^h_z (q^h y^h/(1+r))^{\frac{\sigma_h - 1}{\sigma_h}}}{\sigma^h(1-\mathcal{S}^h_z) + \mathcal{S}^h_z (q^h y^h/(1+r))^{\frac{\sigma_h - 1}{\sigma_h}}}, \qquad \hat{q}^z_v = \frac{1-(1-\sigma^h)\hat{q}^h_v}{\sigma^h} \tag{A20}$$

and

$$\hat{q}^h_a = (1-\mathcal{S}^h_k) - ((1-\mathcal{S}^h_k) + \sigma^h \mathcal{S}^h_k)\hat{q}^h_v, \qquad \hat{q}^z_a = -\left(\frac{1-\sigma^h}{\sigma^h}\right)\hat{q}^h_a. \tag{A21}$$

The basic insights are that the partial elasticities of q^h and q^z with respect to v are positive, with $\hat{q}^h_v$ in $[0,1]$ and $\hat{q}^z_v \geq \hat{q}^h_v$, that q^z is increasing in a if and only if $\sigma^h \geq 1$ and that

$$\hat{q}^h_a \geq 0 \qquad \Leftrightarrow \qquad \hat{q}^h_v \leq \frac{(1-\mathcal{S}^h_k)}{(1-\mathcal{S}^h_k) + \sigma^h \mathcal{S}^h_k}.$$

Note that the condition above holds when $\mathcal{S}^h_z$ is not too large or $\mathcal{S}^h_k$ is sufficiently close to zero. However, more general conditions may also generate a positive relation between q^h and a. In the specific case of a Cobb-Douglas production function in the construction sector,

$$\hat{q}^h_v = \mathcal{S}^h_z, \qquad \hat{q}^z_v = 1, \qquad \hat{q}^h_a = \mathcal{S}^h_l, \qquad \hat{q}^z_a = 0. \tag{A22}$$

D.2 Upper Bound on Δ with Zero Bequests

Proposition 4. *Assume that $b^r = 0$. Then, $\Delta < 1$ at equilibrium.*

Proof. Since $\Delta = (\delta + n)(ak^m - k^h)/q^h y^h$, we have

$$1 - \Delta = \frac{q^h y^h - (\delta+n)(ak^m - k^h)}{q^h y^h}. \tag{A23}$$

Since $v = \lambda q^h y^h/(\delta + n)$, $G^s(b,v,a) = 0$ implies

$$\lambda(q^h y^h - (\delta+n)(ak^m - k^h)) = \frac{\delta+n}{1+n}(s - (1+n)(ak^m + q^z\xi)).$$

Now let $1 - \Delta + q^z\xi/v < 0$. Then, $\lambda \in [0,1]$ if and only if $s < (1+n)(ak^m + q^z\xi)$ and

$$s > \left(\frac{1+n}{\delta+n}\right) q^h y^h + (1+n)(k^h + q^z\xi).$$

Since $\delta \leq 1$, the above requires

$$s > q^h y^h \geq W.$$

□

D.3 Proof of proposition 2

By (31), $\gamma = 1$ implies $1 + \hat{h}^d_\pi = \hat{s}_\pi = 0$ and, because $\sigma^h = 1$, $\Delta_v = \mathcal{S}^h_{z,v} = 0$. Then, recalling (40), (41), it follows that, when $\gamma = \sigma^h = 1$,

$$G^d_v = -1, \qquad G^s_v = -(1 - \Delta + (\delta + n)\mathcal{S}^h_z / r),$$

and, hence, (39) and $\sigma^h = \gamma = 1$ imply

$$-G^d_v = 1 > -G^s_v > 0. \tag{A24}$$

Lemma 1. *For any given (π, aw), and $b^i > 0$,*

$$\partial s^i(\pi, aw, b^i)/\partial b^i > \partial q^h h^i(\pi, aw, b^i)/\partial b^i > 0.$$

Proof. From the individuals' budget constraints, and for any demand function, $(c^{y,i}, c^{o,i}, h^i)$, we have

$$s^i = W + b^i - c^{y,i}(.), \qquad c^{y,i}_I(.) + c^{o,i}_I(.)/(1+r) + \pi h^i_I(.) = 1,$$

where the subscript I on each demand function denotes the partial derivative with respect to I^i. Then,

$$\frac{\partial s^r(.)}{\partial b} = 1 - \left(\frac{r-n}{1+r}\right) c^{y,r}_I(.) = \left(\frac{r-n}{1+r}\right) c^{y,r}_I(.) + \frac{c^{o,r}_I(.)}{1+r} + \pi h^r_I(.)$$

and

$$\frac{\partial q^h h^r(.)}{\partial b} = \left(\frac{r-n}{1+r}\right) q^h h^r_I.$$

Since $q^h = (1+r)\pi/(r+\delta)$,

$$\frac{\partial q^h h^r(.)}{\partial b} = \left(\frac{r-n}{\delta+r}\right) \pi h^r < \pi h^r < \frac{\partial s^r(.)}{\partial b}.$$

Since $\delta + n > 0$, we get the proposition. □

By the above lemma we have $G^s_b > G^d_b > 0$, so that, by (A24), the assumption $\sigma^h = \gamma = 1$, implies

$$v^s_b = \frac{G^s_b}{-G^s_v} > \frac{G^d_b}{-G^d_v} = v^d_b.$$

Now let $\sigma^h = 1$ and assume that the above inequality is verified. Defining the partial elasticities of housing demand as

$$\hat{v}_a^j = v_a^j(b,a)a/v, \qquad \hat{v}_b^j = v_b^j(b,a)b/v,$$

for $j = d, s$, we obtain

$$\hat{b}_a = \frac{\hat{v}_a^d - \hat{v}_a^s}{\hat{v}_b^s - \hat{v}_b^d}. \tag{A25}$$

Since demand functions are homothetic, we get

$$aG_a^j + bG_b^j = \Sigma^j, \qquad \text{for } j = d, s,$$

where, by the assumption $\sigma^h = 1$,

$$\Sigma^d = v(\mathcal{S}_l^h(1 + \hat{h}_\pi^d) + 1), \tag{A26}$$

$$\Sigma^s = v(1 - \Delta + (\delta + n)\mathcal{S}_z^h/r) - s\hat{s}_\pi\mathcal{S}_l^h. \tag{A27}$$

Using the above in (A25) we get

$$\hat{b}_a^* = 1 + \Gamma, \qquad \hat{v}_a^* = 1 + \frac{(1 - \mathcal{S}_k^h)(1 + \hat{h}_\pi^d)}{1 - \mathcal{S}_z^h(1 + \hat{h}_\pi^d)} + \hat{v}_b^d\Gamma, \tag{A28}$$

where

$$\Gamma = \frac{1}{\hat{v}_b^s - \hat{v}_b^d}\left(\frac{\Sigma^d}{-vG_v^d} - \frac{\Sigma^s}{-vG_v^s}\right).$$

By rearranging terms,

$$\Gamma = \frac{(1-\gamma)}{(\hat{v}_b^s - \hat{v}_b^d)G_v^d G_v^s}M,$$

where

$$M = (1 - \Delta + (\delta + n)\mathcal{S}_z^h/r)v(\mathcal{S}_l^h + \mathcal{S}_z^h) + \frac{\phi^h c^y}{1+n}(\mathcal{S}_l^h - \mathcal{S}_z^h - 2(1-\gamma)(1-\phi^h)\mathcal{S}_l^h\mathcal{S}_z^h).$$

Then, $\gamma = 1$ implies $\hat{b}_a^* = \hat{v}_a^* = 1$.

Using (37) and (38), the equilibrium elasticities under the assumption $\sigma^h = 1$ are given by

$$\hat{y}_a^* = 1 - \beta^h((1+r)\Delta - (\delta+n)\mathcal{S}_z^h)(\hat{v}_a^* - 1), \tag{A29}$$
$$\hat{\beta}_a^{h,*} = (1 + \beta^h((1+r)\Delta - (\delta+n)\mathcal{S}_z^h))(\hat{v}_a^* - 1), \tag{A30}$$
$$\hat{\beta}_a^* = \frac{\beta^{h,*}}{\beta}\left[\beta((1+r)\Delta - (\delta+n)\mathcal{S}_z^h) + \left(1 - \Delta + (\delta+n)\frac{\mathcal{S}_z^h}{r}\right)\right](\hat{v}_a^* - 1). \tag{A31}$$

Then, for the unit-elastic economy $\hat{y}_a^* = 1$ and

$$\hat{q}_a^{h,*} = 1 - \mathcal{S}_k^h, \quad \hat{q}_a^{z,*} = 1 - \mathcal{S}_z^h, \quad \hat{\beta}_a^* = \hat{\beta}_a^{h,*} = 0.$$

Now recall that (39) implies

$$(1+r)\Delta - (\delta+n)\mathcal{S}_z^h > \Delta > (\delta+n)\mathcal{S}_z^h \geq 0, \qquad 1 - \Delta + (\delta+n)\mathcal{S}_z^h/r > 0.$$

Then, inspection of (A29), (A30), (A31) reveals that average labor productivity increases by less than proportionally and wealth-to-income ratios increase with a whenever the model provides $\hat{v}_a^* > 1$. In turn, recalling that condition (45) implies $M > 0$, we derive that (45) implies $\hat{b}_a^* > 1$ and

$$\hat{v}_a^* > 1 + \frac{(1-\mathcal{S}_k^h)(1+\hat{h}_\pi^d)}{1-\mathcal{S}_z^h(1+\hat{h}_\pi^d)}.$$

Finally, from equations (A18), (A19), for $\sigma^h = 1$ we obtain

$$\hat{\lambda}_a^* = \frac{\hat{v}_a^* - 1}{1 - \mathcal{S}_z^h},$$

i.e., $\hat{\lambda}_a^* = 0$ for $\gamma = 1$ and

$$\hat{\lambda}_a^* > \frac{(1-\mathcal{S}_k^h)(1+\hat{h}_\pi^d)}{1-\mathcal{S}_z^h(1+\hat{h}_\pi^d)}\frac{1}{1-\mathcal{S}_z^h}$$

for γ satisfying condition (45). This completes the proof of proposition 2.