Workers' Remittances, Capital Accumulation and Efficiency in Developing Countries *

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Abstract

This paper studies the impact of workers' remittances on capital accumulation. We consider two overlapping generations economies: a recipient country - in which labor is endogenous and children education is paid by parents - and an emitter country - in which migrants supply labor inelastically and send altruistically remittances to family. In the recipient country, remittances reduce labor supply, domestic savings and capital accumulation with mixed and country-specific impacts on efficiency. Appropriate lump-sump taxes and subsidies allows to bring economies to the optimal steady-state in term of saving, education and labor supply. We calibrate the model for 11 recipient countries to quantify impacts and policy recommendation.

Keywords: Remittances; Overlapping generations; Endogenous labor supply; Capital accumulation; Golden rule; Optimal taxation.

JEL classification: O11; F24; C62; H21

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1 Introduction

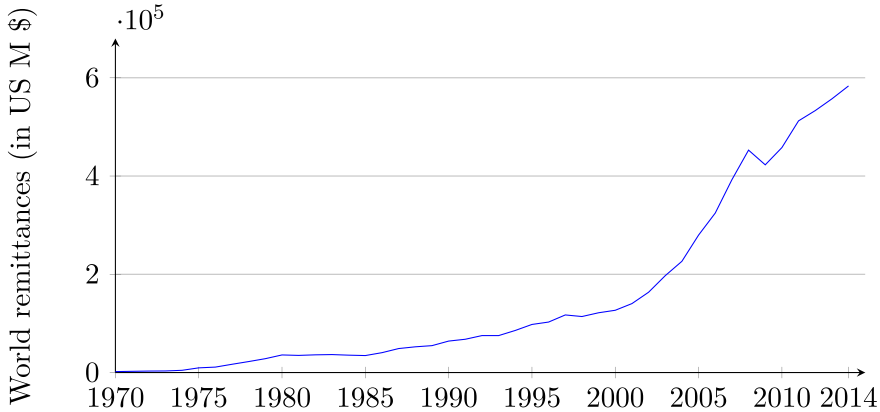
Since globalization, workers' remittances flows are increasing. These currency transfers are of some hundreds dollars per sending but represent at the world level huge amount of money. Some empirical studies, Docquier and Rapoport (2006) [18], Barajas *et al.* (2008) [8], and more recently, Chami and Fullenkamp (2013) [15], show the exponential growth of workers' remittances flows. This global exponential growth is represented by the figure 1.1 using World Bank data. Flows of remittances in the world each year are picked from 1970 to 2014. We can easily see the weak and constant growth during the seventies followed by the huge acceleration since the last decade of the twentieth century. Global remittances did not represent more than 10 billions of US dollars during the middle of seventies and represented more than 100 billions in the beginning of the new century. Currently, they are estimated to more than 500 billions dollars. This huge increase is due to the rise of world migratory flows since the fifties. By the way, the OECD recorded in 2013 more than 230 millions migrants in the world with an increase of 4 millions per year during the last decade. Among these migrants, 60% live in developed countries. The exponential growth of remittances flows is also due to the rise of countries exporting labor force sending money back home and to the decrease of sending costs. Acosta *et al.* [2] argued that the amount of remittances is equivalent to a significant part of foreign direct investments. These flows can exceed the global financial development assistance program, and even FDI for some countries. Furthermore real amounts are probably higher due to unofficial flows.

According to Barajas *et al.* [8], 91% of the current workers' remittances are directed to developing countries. It is a significant phenomenon, particularly at the developing countries GDP scale. The relative amount attains almost one quarter of GDP in several countries, which is a huge part, but even nearly half of GDP as it is the case in Tajikistan according to the World Bank. However workers' remittances are unequally allocated between developing regions. Contrary to Asian countries, African countries receive less remittances. Furthermore growth of these inflows is weaker in Sub-Saharan Africa compared to Northern Africa and Asia, implying an increase of disparities across time.

Given the scale of facts, and particularly the importance of relative amounts, remittances have huge impacts in recipient economies. The first thing to notice is that they have large effects on poverty reduction (affecting essential commodities like food, clothes...), but also on health, mortality and particularly on child mortality (remittances are also used to health expenditures). These financial flows have also impacts on education, with an increase of children attending school, and the length of studies. A positive impact on development is therefore expected, with various effects.

Behavioral studies are unanimous about uses of worker's remittances. They are mainly used to consume and not much injected in productive capital. Less than 10% are used to invest according studies[1]. Impacts of remittances on saving, are ambiguous, they are country-specific. Barajas *et al.* [8] explained that households tend to save less when inflows are expected to be permanent. Nevertheless, if they are expected to be transitory, households will save more and consume less. According to them, the official development aid is 3 times more volatile than remittances, foreign direct investments are 22 times more volatile and exports, 74 times. The expected impact is that as remittances appear to be very stable, therefore they serve to consumption and less to saving. Furthermore, under the income effect, we could have negative correlation between remittances and saving. This negative effect is found by some empirical studies. This is indeed

[1] The Inter-American Development Bank's Multilateral Investment Fund (2004); Facility for Euro-Mediterranean Investment and Partnership-Study on improving the efficiency of workers' remittances in Mediterranean countries, European Investment Bank.



Source: World Bank database (2015)
Note: Amounts are in current US millions dollars, and data for the year 2014 are estimations.
In 1970, amount of remittances in the World was around 2 billions dollars whereas they represented more than four hundred of billions since the beginning of 2010's.

Figure 1.1: Evolution of migrants' remittances since 1970

the result of Morton *et. Al.* (2010) [26]. By Using World Bank data (2008) they only found 1 country (Bangladesh) over 17 with a positive and significant correlation. For all others like El Salvador, Guatemala, Haiti, Jamaica, Lebanon or Philippines for instance, the correlation is negative. By focusing on econometric studies, a negative impact of remittances on saving is also observed by Athukorala and Sen (2004) [6] for India. They based their analysis on the determinants of saving, using data covering the period 1954-1998. The used framework is the life-cycle model, being according to them *"the standard theory for the explanation of changes in private saving over time and across countries"*. In this framework, the retirement is the main motive for saving, and therefore workers are net savers and and then consume this saving in the retirement period (smooth of consumption). India has had according to them a high saving rate using various sources of data as national accounts, economic surveys and Reserve Bank of India. The estimation method is the general to specific modeling procedure. Authors estimated a unrestricted equation before progressively simplify it. They found that remittances relative to the Gross National Disposable Income negatively affect the private saving. For instance, according to them, an increase of 1 percentage point of these inflows is correlated with a decrease of 0.71 percentage point in the long run. Nevertheless, this result is only significant at the 10-percent level. More recently, Hossain (2014) [22] also found negative impact of remittances on domestic savings for 63 developing countries by studying the impact of each Foreing Capital Inflow (FCI). The 63 countries dataset covered the period 1973-2010, with an unbalanced macro panel analysis. Data mainly comes from World Development Indicators and Global Development Finance provided, by the World Bank, but also from International Monetary Fund. The framework is again based on the life-cycle model and estimations rest on Common Correlated Effects Mean Group Estimator (CCEMG) defined as a mean of CCE estimator. He found a negative and statistically impact of remittances on domestic saving. More precisely, a rise of 1 percentage point decreases saving by 1.215 percentage point ceteris paribus. Nevetheless, other inflows have not a statistically significance in the considered sample. Therefore, remittances seems to decline saving. Nevertheless, results tend to be country-specific as other studies show positive effects. Indeed, according to Baldé (2011) [7], remittances enhance saving in Sub-Saharan Africa. His study is based on data covering the period 1980-2004 for 37 countries given by the World Development Indicators and the Center for Global Development. As Hossain, Baldé exploited an unbalanced panel. The estimation was based on the Two-Stage Least Square instrumental variable method in order to overcome the endogeneity problem. The main result is that an increase of 10% in these sub-Saharan country raises saving by 7%. Furthermore, remittances heighten more saving than official aid. Finally, Ziesemer (2012) [33] noted that the total effect of remittances on investment is positive. Data also comes from FDI, but here for 52 countries. Dynamic panel data methods are used with GMM estimators and fixed effects. Worker's remittances have positive but decreasing effect on saving over GDP in this study.

Another effect described by the literature is the decline of labor supply: workers need to escape from the harshness of work. This phenomenon is due to the fact that households spend more time consuming leisure and goods. It is obvious that if the harshness of work is important, an increase in wealth reduces the labor supply. This negative relation occurs for Instance in Amuedo-Dorantes and Pozo (2012) [4] for Mexico. They exploited data from the Mexican Statistical institute covering the 2000-2008 period and used Instrumental Variable regression due to endogeneity (average wage rate in Mexican emigrants destination with their volatility as instrument). Their main result is that an increase of 1000 peso reduces the employment likelihood by 9.7% percentage point for the men and by 3.6% percentage point for the women. Furthermore, this raise of remittances lowers worked hours for employees (estimated to 6 hours per month for men and 7.8 hours per month for women). In Jamaica, Kim (2007) [23] also found negative effect on labor supply with a cross-sectional and pseudo-panel analysis. Data comes from the Survey of Living Conditions and the Labor Force Survey. Kim argued that in 2002, remittances had negative effects on the labor market participation both with cross-sectional and panel analysis. With the first, the studied inflows lower by 3.6% the participation at the labor market. Nevertheless, coefficient of hours worked was not significant. Hence, the main result of this study is that remittances negatively affect labor participation (higher reservation wage), but not necessarily worked hours. Finally, in 2006, Acosta [1] focused on remittances and labor supply for El Salvador by using a cross-sectional household survey (no availability of panel) and Instrumental Variables techniques (the instruments are the migration network at the village level and the history of household migration). The covered period is 1998-2000. After controlling for endogeneity, a negative result is highlighted but only for female labor supply.

Remittances have microeconomic impacts on consumption, saving and labor. Following these empirical studies, we will theoretically focus on repercussions of remittances in recipient country on capital accumulation and on labor supply. It is expected that agent will decrease labor time and saving to increase investment in education.

Therefore, in order to analyze both remittance decision in foreign country where migrant live and the impact of this decision in the recipient country, we consider two overlapping economies which are the home and the foreign country. The target is to understand how workers' remittances affect, in the migrant's home country the main economic variables. Economies are composed by households and firms. The firsts are modeled by a representative agent who lives for 3 periods (childhood[2], adulthood and then a retirement period), and firms are modeled by a representative firm producing, with a neoclassical production function, a unique output good which can be consumed or invested as physical capital. The main assumption of our framework is that parents educate their children in order to obtain inflows from them when they have migrated in another country with more favorable economic conditions. Remittances depend therefore on investment on education. In the home country, an agent can migrate after the childhood in the foreign country or remain to supply endogenous labor at a competitive wage rate. We assume that an exogenous number of children are successfully migrating in each period. An agent, who decided to not migrate, draws utility[3] from consumption in adulthood,

[2]This first period is implicit, all decisions are taken by parents.

[3]We assume life-cycle utility function

consumption at retirement and disutility from labor while in activity due to tough working conditions. The received wage is dedicated to consumption, saving and education of children who will emigrate in another country. Consumption in last period of life is allowed by the return of saving and the amount of received remittances. Overseas preferences are based on consumption in both periods of life and on the amount of remittances. A pure altruistic motive to remit is consistent with empirical studies as shown by Barajas *et al.* [8]. An immigrated agent supplies inelastically labor when middle aged. The obtained income only depends on education financed by parents in home country, and allows to consume, save and remit. In the last period, return of saving allows to consume. The main objective is to determine impact on capital accumulation in the recipient country, through impact on saving, education and labor supply but particularly the role played by remittances on the efficiency of this capital accumulation. The literature on the Golden rule of capital accumulation is abundant in Overlapping Generation model, but less with endogenous labor supply. Following Grigorian and Melkonyan [21], the two programs are solved by backward induction as parents integrate child's decisions in their owns. The more parents educate their child, the more remittances they receive.

We show that, in this framework, education of children can be perceived as a substitute of saving, because future remittances will go for the last period of consumption. The main consequence is thus a decrease of saving with negative impacts on capital accumulation. We also explain a weakening of labor supply through remittances income effect. We make comparison between economic variables of recipient countries, and their level if the country did not receive remittances. Our analysis is also focused on stability to analyze potential macroeconomic fluctuations. Under a local stability analysis, it is shown that remittances flows, or even potential regulation by authorities do not affect stability of the unique positive equilibrium. Finally, analyzing the golden rule, we show that studied financial flows, have impacts on efficiency. Indeed, according to initial conditions framing parameters, efficiency can be improved or worsen through workers' remittances mechanisms. This technical part is based on Michel and Pestieau [25], but adapted for our framework. It is shown that under some conditions, it could exist an optimal amount of remittances which allows the economy to restore efficiency. We then demonstrate that an appropriate lump-sump taxation, with taxes and transfers is sufficient to lead the recipient economies to their optimal steady-state in term of saving, labor supply and capital accumulation. As previously, a comparison of the scale of taxation is made between a recipient countries and this same countries if remittances were removed. To quantify the theoretical analysis, we calibrate our model for 11 recipient countries (Algeria, Bolivia, Colombia, Egypt, El Salvador, Morocco, Mexico, Peru, The Philippines, Sri-Lanka and finally Tunisia). We describe impacts and the scale of policy recommendation but also detail what would be the optimal policy in the absence of remittances. These calibrations also show the country-specific effects of remittances usually described by empirical literature on worker's remittances inflows.

This paper is organized as follows: the next section sets up the simple model based on the home and foreign economies. Section 3 proves the existence of a unique steady-state and shows impacts of entering remittances (through education-saving trade-off) on capital accumulation. In section 4, we point out the golden rule conditions and draw up a paralell between workers' remittances flows and efficiency of capital accumulation. Section 5 presents an empirical illustration of our model, based on some developing countries from different continents by underlining how a simple taxation can allow to get back to the golden rule. Finally the last section contains concluding remarks.

2 The model

In order to set up a framework to analyze consequences of remittance flows, we consider two overlapping economies which are the home and the foreign country, where households live for 3 periods. The target is to understand how workers' remittances affect, in the migrants' home country, capital accumulation in the long run through changes of consumption, saving and labor supply.

Time is considered here as discrete and goes, as usually, from 0 to ∞. Economies are composed by households (modeled by a representative agent) and firms (modeled by a representative firm). Decisions are taken in each point of discrete time $t = 1, 2, ...$, with an initial condition in period $t = 0$. Economies produce a unique output good which can be consumed or invested as physical capital.

2.1 The firms in home country

Only firms in home country are studied in our model since the target is to analyze impacts of remittances on the macroeconomic equilibrium in the recipient countries. Overseas, we only focus on sending decision.

We consider a competitive economy, with a representative firm where the unique output is produced at each period using physical capital (K) and labor (L) with a neoclassical production function $F(K_t, L_t)$. Capital stock is, in each period, the result of saving accumulated during the previous period. By assuming "long" period, as it is usual with discrete overlapping generations model, we assume a fully capital depreciation.

Assumption 1. *The production function is supposed to be Cobb-Douglass, defined by* $F(K_t, L_t) = AK_t^s L_t^{1-s} \Leftrightarrow f(k_t) = Ak_t^s$ *with* $s < 1/2$*, expressing the elasticity of revenue with respect to the capital stock (or also the share of capital in the income).*

This production function which depends on capital and labor is increasing, concave over $\mathbb{R}_{++}$ and homogeneous of degree one. By the way, we define $k_t = \frac{K_t}{L_t}$ due to homogeneity of degree one, and $f(k_t)$ depicts the production function expressed in its intensive form. Trough a competitive framework, firms maximize their profits which can be written by the following way by normalizing price to one:

$$\Pi = F(K_t, L_t) - w_t L_t - R_t K_t$$

The maximization of profit with respect to labor and capital per period determines the competitive wage and the interest return in the domestic country satisfying:

$$w_t = (1-s) Ak_t^s \tag{1}$$

$$R_t = sAk_t^{s-1} \tag{2}$$

2.2 The Households

In both countries, agents are young, then workers and ultimately old. Nevertheless, during the first period education is the unique variable, controlled by the parents. Labor is endogenous in home country and for simplicity of algebra, agents supply inelastically labor in foreign country. Preferences are represented by a life-cycle utility function.

Assumption 2. *We assume logarithmic utility functions in each country which are additively separable, increasing and concave for each argument. The marginal utility of zero consumption is infinite.*

First of all, we describe our framework in home country where agents born. In period t, we consider that N_t individuals born and the growth of births is assumed to be constant and represented by $1+n$ with $n>0$. Before becoming worker, an agent can migrate in the foreign country. If not, he remains in the home country to supply endogenous labor at the competitive wage rate. We suppose that an exogenous number of children $\mu \in [0;1+n]$ in each families are successfully migrating per period. In his home country, an agent born in period $t-1$, who decided to not migrate, draws utility from consumption in period t when middle-aged, c_t, and in period $t+1$ when old, d_{t+1}. He nevertheless draws disutility from work when middle-aged, l_t, due to tough working conditions[4]. This agent stayed in home country, supplies therefore elastically labor in period t and the received wage is dedicated to consumption, saving and education of children who can emigrate in another country with more favorable economic conditions. We assume that the last period is a retirement period, and the consumption in this last period of life is allowed by the return of saving and the amount of received remittances.

Hence, agent in the home country wants to maximize his life-cycle utility function under each period budget constraints, which gives the following program:

$$\begin{aligned}
\underset{c_t,l_t,e_t,s_t,d_{t+1}}{\text{Max}} \quad & \epsilon \ln c_t + \eta \ln (1-l_t) + \delta \ln d_{t+1} \\
\text{s.t.} \quad & w_t l_t = c_t + s_t + \mu e_t \\
& \mu B_{t+1} + s_t R_{t+1} = d_{t+1} \\
& \epsilon + \eta + \delta = 1
\end{aligned}$$

where the parameter $\epsilon \in [0;1]$ represents weight of first period consumption in total utility, $\eta \in [0;1]$ represents subjective hardness of work and $\delta \in [0;1]$ is the weight of second period consumption, defined as the discount factor. We also assume that $\delta \in [0;1]$, and most of the time lower than the first period weight consumption under the well-known preference for present axiom. Total available productive time is normalized to one and $1-l_t$ represents the leisure time. In period t the worker's income is defined by $w_t l_t$ (a wage rate w_t multiplied by worked hours l_t). The amount dedicated to education is defined by μe_t, and s_t represents saving. In period $t+1$, the amount of received remittances is defined by μB_{t+1}, with B_{t+1} amount of inflows sent by each emigrated child. The return of saving is defined as $s_t R_{t+1}$ (which is the interest factor R_{t+1} multiplied by the saving s_t). These amount of workers' remittances and saving makes consumption.

In the foreign country[5], an immigrated agent (born one period later his parents in home country) draws utility from consumption when middle-aged and old, but also from the the amount of remittances (altruistic motivation). This last point is the most important in our

[4]The utility function is therefore mathematically increasing with leisure

[5]Variables representing foreign country are described by a star

modelization of sending decision. Indeed, utility can depend on sent amounts for different reasons. The literature (see Chami, Fullenkamp and Jahjah [16], Docquier and Rapoport [18] and Grigorian and Melkonyan [21]), actually, proposes two main determinants which are altruism and self interest exchange. The first is the most obvious and relevant. Migrant knows that additional income decreases poverty and makes sure his family stayed in developing country against economic shocks (diversification of income). The altruism determinant can be explained too, in some cases, by the desire of repaying the education financed by parents. The second, developed by some economists is that migrant sends money to family, expecting a future resource for him though house parents' outlays. In that case, one deals about "merit good", migrant can diversify his future expected resources. In the empirical literature, the most determinant is supposed to be the altruism factor. That is why we consider in this model, a pure altruistic motive to remit.

As we have said before, an immigrated agent supplies inelastically labor when middle aged. We assume that children educated by their parents in period t begin to work at period $t+1$. The acquired income during the labor period allows to consume, save and remit. In the last period, return of saving allows to consume. The child's program in the foreign country can be written by the following way:

$$\begin{array}{cl} \underset{c^*_{t+1}, B^*_{t+1}, s^*_{t+1}, d^*_{t+2}}{\text{Max}} & \sigma \ln c^*_{t+1} + \beta \ln d^*_{t+2} + \gamma \ln B^*_{t+1} \\ \text{s.t.} & w^*_{t+1}(e_t) = c^*_{t+1} + B^*_{t+1} + s^*_{t+1} \\ & s^*_{t+1} R^*_{t+2} = d^*_{t+2} \\ & \sigma + \beta + \gamma = 1 \end{array}$$

where $\sigma \in [0;1]$ represents the weigh of consumption in first period of life, $\beta \in [0;1]$ is the weight of second period consumption, as in the parents' program. The parameter $\gamma \in [0;1]$ represents the altruism of the migrant toward family. The labor income is defined by $w^*_{t+1}(e_t)$ and only depends on child's education

Assumption 3. *The emigrated agent's wage $w^*_{t+1}(e_t)$, is increasing with respect to education but with decreasing return. We assume the following equation: $w^*_{t+1}(e_t) = \alpha e_t^\lambda$ with $0 < \lambda < 1$ to depict decreasing returns of education.*

We solve these two programs by backward induction as in Grigorian and Melkonyan [21]. In other words, parents integrate child's decisions in their owns. We first must ascertain child's remittances and integrate them in the parents' program

By solving the Lagrangian associated to the child program, we obtain the following expressions for the amount of saving and money sent by the migrant to his family in the home country:

$$B^*_{t+1} = \gamma w^*_{t+1}(e_t) = \gamma \alpha e_t^\lambda \qquad s^*_{t+1} = \beta w^*_{t+1}(e_t) = \beta \alpha e_t^\lambda \tag{3}$$

The first thing to notice is that, obviously, amounts of remittances and saving are increasing with the wage. Sent amount to family is ceteris paribus increasing with the altruism parameter γ (and implicitly decreasing with respect to the consumption weight σ and β, due to the last constraint of the program $\sigma + \beta + \gamma = 1$). Migrant's saving is obviously increasing with β and implicitly decreasing with σ and γ. These are intuitive results, when the altruism parameter increases, remittances are more important, saving and consumption decline. In the same way, an rise of σ decreases saving and sent financial flows, and a rise of β increases saving and drops first period consumption and amount of remittances for a constant wage.

The backward induction induces that we can insert these results in the parents' program who take into consideration the remittances they will receive from their child to decide the amount of money they will invest in education. By inserting the amount of remittances defined in equation (3), the program becomes:

$$\begin{aligned} \underset{c_t,l_t,e_t,s_t,d_{t+1}}{\text{Max}} \quad & \epsilon \ln c_t + \eta \ln (1 - l_t) + \delta \ln d_{t+1} \\ \text{s.t.} \quad & w_t l_t = c_t + s_t + \mu e_t \\ & \mu\gamma\alpha e_t^{\lambda} + s_t R_{t+1} = d_{t+1} \\ & \epsilon + \eta + \delta = 1 \end{aligned}$$

The first order conditions of the Lagrangian associated to this maximization program give the following expressions for education function, $e_t(R_{t+1})$, saving, $s_t(w_t, R_{t+1})$, and labor supply function, $l_t(w_t, R_{t+1})$.

$$e_t = \left(\frac{\gamma\alpha\lambda}{R_{t+1}}\right)^{\frac{1}{1-\lambda}} \tag{4}$$

$$s_t = \delta w_t - \frac{\mu(\lambda\delta + \eta + \epsilon)}{\lambda}\left(\frac{\gamma\alpha\lambda}{R_{t+1}}\right)^{\frac{1}{1-\lambda}} \tag{5}$$

$$l_t = 1 - \eta - \frac{\mu\eta(1-\lambda)}{\lambda w_t}\left(\frac{\gamma\alpha\lambda}{R_{t+1}}\right)^{\frac{1}{1-\lambda}} \tag{6}$$

As with child's decision, it can be interesting to see impacts of parameters on variables. Nevertheless, we can remind that effects are only on isolated variables, not at the equilibrium. The aim here is only to see how each variable is impacted all things being equal. The amount of education, is, ceteris paribus increasing with altruism parameter γ, foreign wage parameters α and λ, but decreasing with interest rate R_{t+1}. Agent provides education to children by equalizing the marginal return of education with the marginal return of saving. In this framework, education becomes a substitute of saving to allow consumption in last period of life. When interest rate increases, saving becomes more profitable, so households spend less on education as it is a substitute of saving. On the other hand, saving is positively related with domestic wage w_t. Indeed, when the wage raises, agents save more, but they consume also more, so they proportionally save less than the increase of wage. The marginal propensity to save is included between 0 and 1, reflecting consumption of normal goods. The expression of saving is also positively correlated with the discount factor δ and the interest rate all things being equal, and negatively correlated with the subjective hardness of work η, the weight for first period consumption ϵ and education. Finally, labor supply is increasing with wage and interest rate and decreasing with the harsh working conditions and education ceteris paribus.

We can now determine the macroeconomic equilibrium of the long run knowing both agents and firm decision.

3 The inter-temporal equilibrium

Previous saving and labor functions describe temporary competitive equilibrium at each period. Our analysis will be based in the long run. With the assumption of complete depreciation of capital, the starting point to determine inter-temporal equilibrium is that the capital stock in a period is equal to the saving accumulated by all workers during the previous period.

$$K_{t+1} = N_t^w s_t$$

Period	t_{-1}	t	t_{+1}
Births	N_{t-1}	$(1+n)(1-p)N_{t-1}$	$(1+n)^2(1-p)^2N_{t-1}$
		$\searrow$ $\Uparrow$	$\searrow$ $\Uparrow$
Workers		$(1-p)N_{t-1}$	$(1+n)(1-p)^2N_{t-1}$
			$\searrow$
Retired			$(1-p)N_{t-1}$

Table 1: The Evolution of Population

with N_t^w is the number of workers in period t. In order to express capital per capita, we need to determine the evolution of population and therefore labor force by taking into account of migration in addition to births. We have assume that N_{t-1} denoted the number of births in period $t-1$. By assuming an exogenous migration rate of children $p = \frac{\mu}{1+n}$, the number of workers in period t is defined by $N_t^w = (1-p)N_{t-1}$. The growth of births is $1+n$ implying that there are $(1+n)(1-p)N_{t-1}$ births in period t and therefore $(1+n)(1-p)^2N_{t-1}$ workers in period $t+1$. Hence, the evolution of workers between two period satisfies:

$$\frac{N_{t+1}^w}{N_t^w} = (1+n)(1-p) = 1+n-\mu$$

This evolution of population is summarized in table 1. We easily get the following equation describing the long run equilibrium on capital market with the initial condition k_0 given.

$$k_{t+1}\left[(1+n-\mu)\,l_{t+1}\right] = s_t \tag{7}$$

Using equation (1) to (7) allows to obtain, after some algebra, the dynamical equation of the macroeconomic equilibrium:

$$\begin{aligned}&(1+n-\mu)(1-\eta)\,k_{t+1} - \delta(1-s)\,Ak_t^s + \mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\\ &\times\left(\frac{(\lambda\delta+1-\delta)}{\lambda}k_{t+1}^{\frac{1-s}{1-\lambda}} - \frac{(1+n-\mu)\;\eta(1-\lambda)}{\lambda(1-s)A}k_{t+1}^{1-s}k_{t+2}^{\frac{1-s}{1-\lambda}}\right) = 0\end{aligned} \tag{8}$$

The previous equation evaluated in the long run (at the steady state such that $k_{t+2} = k_{t+1} = k_t = k$) gives:

$$\begin{aligned}&(1+n-\mu)(1-\eta)\,k - \delta(1-s)\,Ak^s + \mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\\ &\times\left(\frac{\lambda\delta+1-\delta}{\lambda}k^{\frac{1-s}{1-\lambda}} - \frac{(1+n-\mu)\,\eta(1-\lambda)}{\lambda(1-s)A}k^{\frac{(1-s)(2-\lambda)}{1-\lambda}}\right) = 0\end{aligned} \tag{9}$$

Our analysis will be based on comparison between the same economy receiving or not remittances in order to evaluate impact on capital equilibrium in long run. We starts by analysis of our benchmark, the situation where there is no inflows.

3.1 The benchmark: Equilibrium without remittances

We can provide an analytic and graphical explanation of equilibrium using equation (9) in order to highlight potential steady-state(s). To analyze first the equilibrium without remittances, we evaluate equation (9) in this benchmark when the parameter of altruism γ is equal to zero. In this case, as parents do not receive remittances, they do not educate children who leave the home country. This equation becomes:

$$(1+n-\mu)(1-\eta)k - \delta(1-s)Ak^s = 0 \tag{10}$$
$$\Leftrightarrow f(k) = 0$$

Proposition 3.1. *In the benchmark case without remittances (when $\gamma = 0$), it exists under assumptions 1 and 2, a trivial steady state with no capital accumulation defined by $k = 0 \equiv \overline{k}_0^{wr}$ and only one steady state with a positive stock of capital, which is: $k = \left(\frac{\delta(1-s)A}{(1+n-\mu)(1-\eta)}\right)^{\frac{1}{1-s}} \equiv \overline{k}^{wr}$*

Proof. The proof is obvious, the equation (10), is satisfied if $k = 0$ which implies that it exists a trivial steady state and $\overline{k}^{wr}$ is the unique positive solution of equation 10. See appendix for a graphical argument. □

The steady state value of capital accumulation per head is as usual increasing with the discount factor (δ) and hardness of work (η), and decreasing with the elasticity of revenue with respect to capital (s) and demographic growth ($1 + n - \mu$). Therefore, a rise in number of migrants increases capital accumulation per capita, by lowering the number of workers in the country.

An analysis of local dynamics is required for the two particular values of steady state, the corner $\overline{k}_0^{wr}$ and the positive steady state $\overline{k}^{wr}$, to makes sure that the initial condition k_0 converges to the unique admissible steady state.

Proposition 3.2. *Under assumptions 1 and 2, only the non-trivial steady state is stable with monotonous convergence.*

Proof. The dynamical equation (8) becomes in the no remittances case:

$$(1+n-\mu)(1-\eta)k_{t+1} - \delta(1-s)Ak_t^s = f(k_{t+1}, k_t) = 0$$
$$\Leftrightarrow k_{t+1} = \frac{\delta(1-s)A}{(1+n-\mu)(1-\eta)}k_t^s = k_{t+1}(k_t)$$

The analysis is the easiest (dimension 1). The derivative of $k_{t+1}(k_t)$ evaluated at the positive steady state value is equal to the parameter s which is positive and lower than one implying a stable equilibrium. Nevertheless, one verifies that the derivative evaluated at the trivial steady state tends to $+\infty$ implying a non stable equilibrium. Therefore, there is a unique equilibrium path with a monotonous convergence starting from k_0 to $\overline{k}^{wr}$. □

Therefore too, there is two possible steady states in the no remittances case. The first being with no capital accumulation and is unstable. The second with positive accumulation is stable with monotonous convergence. It is now necessary to analyze situation on this unique last equilibrium capital stock when $\gamma > 0$ implying that the economy is receiving remittances.

3.2 Equilibrium with remittances

By using equation (9) and (10), the equilibrium at the steady state with remittances is now defined as follow:

$$h(k) = f(k) + g(k) = 0 \tag{11}$$

with:

$$g(k) = \mu \left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}} \left(\frac{\lambda\delta + 1 - \delta}{\lambda} k^{\frac{1-s}{1-\lambda}} - \frac{(1+n-\mu)\,\eta\,(1-\lambda)}{\lambda\,(1-s)\,A} k^{\frac{(1-s)(2-\lambda)}{1-\lambda}}\right)$$

Due to the different powers of the variable representing capital accumulation (k), we cannot obtain expression of steady state with remittances. Nevertheless, we can provide graphical arguments by studying firstly the curve $g(k)$, to determine the equilibrium defined by $h(k) = f(k) + g(k) = 0$ in order to explain impacts of remittances on capital accumulation.

Proposition 3.3. *Under assumptions 1, 2 and 3, it also exists two steady states in this developing economy which is receiving workers' remittances, whose one trivial defined by $\overline{k}_0^r$ and the second defined by $\overline{k}^r$.*

Proof. See appendix □

An analysis of local stability must be provided in order to can explain then impacts of remittances on capital accumulation. In this configuration of developing countries receiving remittances, labor supply depends on capital stock, and not only on utility function parameter, implying a change of dimension. The equation of dynamical equilibrium (8) is:

$$\begin{aligned}
&(1+n-\mu)(1-\eta)\,k_{t+1} - \delta(1-s)\,Ak_t^s + \mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}} \\
&\times\left(\frac{(\lambda\delta+1-\delta)}{\lambda}k_{t+1}^{\frac{1-s}{1-\lambda}} - \frac{(1+n-\mu)\,\eta\,(1-\lambda)}{\lambda\,(1-s)\,A}k_{t+1}^{1-s}k_{t+2}^{\frac{1-s}{1-\lambda}}\right) \\
&= h(k_{t+2}, k_{t+1}, k_t) = 0
\end{aligned}$$

To study the local properties of the equilibrium, the usual method is to linearize the dynamical equation in the neighborhood of the steady state as Nourry [27]. This gives:

$$\begin{bmatrix} dk_{t+2} \\ dk_{t+1} \end{bmatrix} = \begin{bmatrix} \dfrac{-\frac{\partial h(k_{t+2},k_{t+1},k_t)}{\partial k_{t+1}}\left(\overline{k}^r\right)}{\frac{\partial h(k_{t+2},k_{t+1},k_t)}{\partial k_{t+2}}\left(\overline{k}^r\right)} & \dfrac{-\frac{\partial h(k_{t+2},k_{t+1},k_t)}{\partial k_t}\left(\overline{k}^r\right)}{\frac{\partial f(k_{t+2},k_{t+1},k_t)}{\partial k_{t+2}}\left(\overline{k}^r\right)} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} dk_{t+1} \\ dk_t \end{bmatrix} \tag{12}$$

Computing the roots of the Jacobian matrix is equivalent to computing the roots of the corresponding characteristic polynomial given by:

$$P(\Lambda) = \Lambda^2 - \Lambda\left(\frac{-\frac{\partial h(k_{t+2},k_{t+1},k_t)}{\partial k_{t+1}}}{\frac{\partial h(k_{t+2},k_{t+1},k_t)}{\partial k_{t+2}}}\left(\overline{k}^r\right)\right) + \frac{\frac{\partial h(k_{t+2},k_{t+1},k_t)}{\partial k_t}}{\frac{\partial f(k_{t+2},k_{t+1},k_t)}{\partial k_{t+2}}}\left(\overline{k}^r\right)$$

Proposition 3.4. *Under assumptions 1, 2 and 3, only the non-trivial steady-state is stable in this recipient economy.*

Proof. See appendix □

The main consequence of the last proposition is that remittances have not a destabilizing effect, they do not bring macroeconomic fluctuations in this framework. The target is now to evaluate impacts of remittances on the unique steady state named $\overline{k}^r$.

Proposition 3.5. *Under assumptions 1, 2 and 3, capital per head is lower in the recipient economy.*

Proof. See appendix □

Proposition 3.6. *Under assumptions 1, 2 and 3, the parameters γ and α decline the stock of capital per head.*

Proof. See Appendix □

The impact of studied inflows on capital accumulation is therefore negative. By extrapolation, we deduce that other parameters which decrease amount of remittances, as β and σ in the child's utility function, have positive impacts on capital accumulation. It is necessary to add that remittances decrease relatively more capital accumulation K than labor L, so $\overline{k}$ decreases. However, it seems necessary to pay attention to parameter λ which depicts the return of education. By using the implicit function theorem, the sign of $\frac{\mathrm{d}\overline{k}^r}{\mathrm{d}\lambda}$ is ambiguous and may depend on the utility function parameters, but also the value of capital per head. Even if remittances have negative impacts on capital accumulation, a variation of the return of education could attenuate the negative effect or amplify this effect according to conditions on parameters.

Thus, in this framework workers' remittances act like a new financial asset different from usual saving. Agent finances education to the retirement period. Therefore, remittances have negative effects on saving in the long run being a substitute for saving.

4 Efficiency of equilibrium in recipient countries

We will first determine the golden rule of capital accumulation in the economy, before to provide an optimal economic policy which bring economies to their golden rule.

4.1 The Golden Rule

The aim of the part is to maximize the welfare of agents in order to derive an optimal equilibrium. In order to do that, we need to maximize the stationary utility function under the resource constraint of the economy which is defined in period t as:

$$F(K_t, L_t) + \mu N^w_{t-1} B^*_t = N^w_t c_t + N^w_{t-1} d_t + \mu N^w_t e_t + K_{t+1}$$

This constraint is such that all resources in the economy allow to each expense. Indeed, under the one defined above, the production in period t and amount of remittances received by all the retired persons in this period allow to consumption of workers and retired agents, but also to investment in education provided by the workers and investment in physical capital. The program which maximizes welfare in the long run is the following:

$$\begin{aligned}
\max_{c,l,e,s,d,k} \quad & \epsilon \ln c + \eta \ln(1-l) + \delta \ln d \\
\text{s.t.} \quad & l\left(Ak^{s} - (1+n-\mu)k\right) + \frac{\mu\alpha\gamma e^{\lambda}}{1+n-\mu} = c + \mu e + \frac{d}{1+n-\mu} \\
& \epsilon + \eta + \delta = 1
\end{aligned}$$

The first order conditions associated to the Lagrangian give the optimal amount of production factors, the capital and labor supply[6].

$$\widehat{k} = \left(\frac{sA}{1+n-\mu}\right)^{\frac{1}{1-s}} \tag{13}$$

$$\widehat{l} = 1 - \eta - \frac{\eta\mu(1-\lambda)\left(\frac{\gamma\alpha\lambda}{1+n-\mu}\right)^{\frac{1}{1-\lambda}}}{\lambda A(1-s)\left(\frac{sA}{1+n-\mu}\right)^{\frac{s}{1-s}}} \tag{14}$$

Nevertheless, the variables are optimal if the amount of education provided by parent is also optimal. This value is defined by the following equation:

$$\widehat{e} = \left(\frac{\gamma\alpha\lambda}{1+n-\mu}\right)^{\frac{1}{1-\lambda}} \tag{15}$$

Remark 4.1. *$\overline{k}^r > \widehat{k}$ implies $\overline{l}^r < \widehat{l}$ and $\overline{e}^r > \widehat{e}$.*

If the capital accumulation is too large with respect to the golden rule, it directly follows that the labor supply is too low and education too high.

The first thing to notice is that efficient value of capital accumulation does not depend on utility function parameters and remittances. Thus, capital accumulation could be too high and labor supply to low (over-accumulation) or capital could be too low and labor supply to high (under-accumulation) according to the value of parameters. Secondly, if $k = \widehat{k}$ all other variables (and principally education and labor supply) are efficient. In other terms an optimal value of k is enough to guaranty the greater efficiency as possible. This result can be similar to Michel and Pestieau [25]. The main consequence is that an optimal economic policy should only target to obtain the optimal value of capital per head without distorting equilibrium labor supply and education. The second important point is that the optimal labor supply is also decreasing with altruism and foreign wage parameter. The intuition behind this result is that as remittances can be perceived as an increase of revenue, it is also optimal, in order to increase welfare, to work less. In other terms, the golden rule determines amount of production factors which maximize the welfare of agents. Therefore, to maximize utility, there is a trade-off between maximization of consumption and minimization of labor. Furthermore, according to Acosta [1], a decrease of labor supply can not be perceived as a negative effect adopting a development point of view, agent can concentrate more in parenting for instance with an increase of revenue.

[6] We indicate efficient values of variable by a hat

Remark 4.2. *Without remittances, the labor supply is always optimal and economy is over-accumulated if* $\eta > \frac{s(1+\delta)-\delta}{s} \equiv \widehat{\eta}$

If the hardness of work is high enough, capital per worker is too high relative to labor supply implying over-accumulation. In this situation, aggregated welfare increases if capital per head declines, implying a non-optimal equilibrium. We easily see that $\frac{\partial \widehat{\eta}}{\partial \delta} < 0$ and $\frac{\partial \widehat{\eta}}{\partial s} > 0$. The more important the discount factor is, the lower $\widehat{\eta}$ is, and the more the probability for the economy to be inefficient (in over-accumulation) for a particular value of η is. Conversely, the more important s is, the higher $\widehat{\eta}$ is, and the more the probability for the economy to be in under-accumulation for a particular value of η is. The explanation is obvious, if δ is high, all things being equal, saving is large and $\frac{K}{L}$ can be too high compared to the golden rule. Is s is high, return of saving is low and so saving will decrease implying that $\frac{K}{L}$ can be too low compared to the golden rule. We can remark that if $s < \frac{\delta}{1+\delta}$ the economy is inefficient ($\eta > 0 > \widehat{\eta}$).

In recipient economies, the labor supply is not necessary optimal as in supposed developing economies without remittances. The efficiency of equilibrium in recipient economy now depend on amount of received inflows. We determine conditions to have an optimal equilibrium such that $\overline{k}^r = \widehat{k}$, $\overline{e} = \widehat{e}$ and $\overline{l}^r = \widehat{l}$.

Proposition 4.1. *Under assumptions 1, 2 and 3, it exists an optimal amount of remittances which brings the economy to the golden rule only if* $\eta > \widehat{\eta}$. *This amount is such that*

$$\gamma = \left(\frac{1+n-\mu}{\alpha\lambda}\right)\left(\frac{\left(\frac{sA}{1+n-\mu}\right)^{\frac{s}{1-s}} A\lambda\left(\delta\left(1-s\right)-\left(1-\eta\right)s\right)}{\mu\left(\lambda\delta+1-\delta-\frac{\eta s(1-\lambda)}{1-s}\right)}\right)^{1-\lambda} \equiv \widehat{\gamma}$$

and implies $\widehat{B}^* = \mu\widehat{\gamma}\alpha\widehat{e}^{\lambda}$

1. *If* $\gamma < \widehat{\gamma}$ *then* $\overline{k}^r > \widehat{k}$, $\overline{l}^r < \widehat{l}$ *and* $\overline{e}^r > \widehat{e}$. *The economy is over-accumulated.*
2. *If* $\gamma = \widehat{\gamma}$ *then* $\overline{k}^r = \widehat{k}$, $\overline{l}^r = \widehat{l}$ *and* $\overline{e}^r = \widehat{e}$. *The economy is optimal.*
3. *If* $\gamma > \widehat{\gamma}$ *then* $\overline{k}^r < \widehat{k}$, $\overline{l}^r > \widehat{l}$ *and* $\overline{e}^r < \widehat{e}$. *The economy is under-accumulated.*

Proof. The proof is obvious. First of all, $\overline{k}^r > \widehat{k} \Leftrightarrow h(\widehat{k}) < 0 \Leftrightarrow \gamma < \widehat{\gamma}$. Secondly, $\widehat{\gamma} > 0 \Leftrightarrow \eta > \widehat{\eta}$ □

The value $\widehat{\gamma}$ determines the optimal amount of remittances. The condition on η defining positivity of $\widehat{\gamma}$ comes from the fact that if an economy was under-accumulated without remittances, it is evident given proposition 3.5 that it would be under-accumulated with remittances. Therefore, an optimal amount should be negative which is absurd. The last part of the proposition is obvious through the previous condition on η. In other terms, if $\gamma = \widehat{\gamma}$ remittances bring the economy to the golden rule and are totally efficient. If amount of these inflows is higher, capital accumulation decreases and so the economy is under-accumulated. In this situation k and e are too small compared to the golden rule and l is overly large. Conversely, if amount of remittances is lower, capital accumulation and education are large and labor supply too law. In this case, economy is over-accumulated.

In this way, remittances should decrease (such that $\gamma = \widehat{\gamma}$) to bring under-accumulated economies to their golden rule if $\eta > \widehat{\eta}$ and $\gamma > \widehat{\gamma}$. However, they should increase to bring over-accumulated economies to their golden rule if $\eta > \widehat{\eta}$ and $\gamma > \widehat{\gamma}$. Nevertheless, in the case where $\eta < \widehat{\eta}$, the more efficient situation is such that $\gamma = 0$. Indeed, in this case remittances bring k, l and e further to the golden rule ($\overline{k}^r < \overline{k}^{wr} < \widehat{k}$, $\overline{e}^r < \widehat{e}^r$ while $\overline{e}^{wr} = \widehat{e}^{wr} = 0$ and finally, $\overline{l}^r > \widehat{l}^r$ while $\overline{l}^{wr} = \widehat{l}^{wr}$).

Remark 4.3. *If $\gamma < \widehat{\gamma}$ then labor supply need to increase but saving need to decrease only if $\overline{k}^r\overline{l}^r > \widehat{kl}$. Conversely, if $\gamma > \widehat{\gamma}$ then labor supply need to decrease but saving need to increase only if $\overline{k}^r\overline{l}^r < \widehat{kl}$.*

The direct implication of this remark is that saving must not necessary raise in under-accumulated economy as the value of labor supply influences capital accumulation.

The aim of the next part is to determine an economic policy in order to bring economies to their golden rule. We will suggest the most simple policy to restore efficiency in developing countries.

4.2 An Optimal Policy

We will determine an economic policy to restore efficiency without directly affecting amount of remittances. First of all, these flows decline poverty and increase education. Then it seems empirically difficult to propose taxes on remittances. Only few countries have experienced these types of policies but under indirect taxes. For instance, in Cuba, remittances sent from the United-States were only paid to recipients in Cuban Convertible or with a tax of 20 percent for conversion of US dollars to national currency. In other countries like Ethiopia, Pakistan, and Venezuela for instance, authorities implemented also an implicit tax on remittances with a conversion of foreign inflows to local currency at an non-competitive exchange rates (over-valuation of the official exchange rate). In India, government proposed a tax on services being a fixed commission payed to Indian agents delivering currency. These few examples are not a direct tax on remittances which seems difficult to implement. Nevertheless, few propositions to tax remittances in source countries have been proposed as in the United Arab Emirates where oil revenues are declining. The last reason to not tax remittances is described by the following lemma, which depicts that with an appropriate economic policy, the stationary utility at the golden is always greater with remittances. Hence, an optimal policy should not a tax on remittances, but a modification of saving and consumption to take the advantage of remittances to increase stationary utility under the resource constraint.

Lemma 1. *The stationary utility at the golden rule in always higher in presence of remittances. Furthermore, the more γ is important, the more the stationary utility is at the golden rule.*

Proof. See appendix □

This lemma is important. Indeed, even if remittances can have bad impact on efficiency in some case, authorities can always improve the utility of agents in recipient countries in a greater level than without remittances. In other term, with a public policy which brings the economy to the golden rule, remittances improve the efficiency by increasing welfare of citizens through the saving-education trade-off.

The target is now to determine an economic policy to restore efficiency in the economy. As we have said before, if k is optimal such that $k = \widehat{k}$, then other variables are also optimal. The policy could be to modify savings to obtain an optimal capital stock without distorting labor supply and education.

Lemma 2. *An appropriate taxation with lump-sump taxes or subsidies for working agents (τ_w) and retired agents (τ_r) is sufficient to bring the economy to the golden rule.*

These amounts are such that the following decentralized program allow to obtain the optimal situation through a satisfied government budget constraint:

$$\begin{aligned} \underset{c_t,l_t,e_t,s_t,d_{t+1}}{\text{Max}} \quad & \epsilon \ln c_t + \eta \ln (1-l_t) + \delta \ln d_{t+1} \\ \text{s.t.} \quad & w_t l_t = c_t + s_t + \mu e_t + \tau_w \\ & \mu\gamma\alpha e_t^{\lambda} + s_t R_{t+1} = d_{t+1} + \tau_r \\ & \epsilon + \eta + \delta = 1 \end{aligned}$$

The solution of this program gives the decentralized saving (s_t^d), the decentralized education (e_t^d) and the decentralized labor supply (l_t^d). The budget constraint of the government is given by the following equation:

$$\tau_w + \frac{\tau_r}{1+n-\mu} = 0$$

Proof. See Appendix □

The value τ_w is the tax or subsidy addressed to workers and τ_r is the tax or subsidy addressed to retired persons. Hence, these two amounts are positive for taxes and negative for subsidies and are such that the budget constraint is balanced. In others words, amount dedicated to workers must be equal to the opposite of amounts dedicated to retired agents. By solving the system associated to the optimal decentralized steady state, we obtain optimal taxes and transfers. Then it is easy to compute the decentralized saving, education and labor supply which are now equal to the optimal values.

Proposition 4.2. *Under assumptions 1, 2 and 3, it exists an optimal lump-sump taxation and redistribution for the workers (τ_w) and retired agents (τ_r) such that:*

$$\begin{aligned} \tau_w^r =& \delta (1-s) A \left(\frac{sA}{1+n-\mu}\right)^{\frac{s}{1-s}} - (1+n-\mu)(1-\eta)\left(\frac{sA}{1+n-\mu}\right)^{\frac{1}{1-s}} \\ & - \mu \left(\frac{\gamma\alpha\lambda}{1+n-\mu}\right)^{\frac{1}{1-\lambda}} \left(\frac{\lambda\delta+1-\delta}{\lambda} - \frac{\eta s(1-\lambda)}{\lambda(1-s)}\right) \end{aligned}$$

and

$$\begin{aligned} \tau_r^r =& (1+n-\mu)^2 (1-\eta)\left(\frac{sA}{1+n-\mu}\right)^{\frac{1}{1-s}} - \delta(1-s) A (1+n-\mu)\left(\frac{sA}{1+n-\mu}\right)^{\frac{s}{1-s}} \\ & + \mu (1+n-\mu) \left(\frac{\gamma\alpha\lambda}{1+n-\mu}\right)^{\frac{1}{1-\lambda}} \left(\frac{\lambda\delta+1-\delta}{\lambda} - \frac{\eta s(1-\lambda)}{\lambda(1-s)}\right) \end{aligned}$$

Hence,

1. *If $\gamma < \widehat{\gamma}$ then $\tau_w^r > 0$ and $\tau_r^r < 0$ implying a tax for workers and a subsidy for retired agents.*
2. *If $\gamma = \widehat{\gamma}$ then $\tau_w^r = 0$ and $\tau_r^r = 0$ implying no need of policy.*
3. *If $\gamma > \widehat{\gamma}$ then $\tau_w^r < 0$ and $\tau_r^r > 0$ implying a subsidy for workers and a tax for retired agents.*

Proof. See appendix □

The intuition behind this result is clear. In under-accumulated economies, an optimal policy could be a subsidy for workers and a tax for old agents in order to raise capital accumulation and decrease labor supply. However, in over-accumulated economies, the policy is inverted in order to decrease capital accumulation and increase labor supply. For instance, if the economy is under-accumulated due to weak saving, then a subsidy for workers in second period of life tends to raise saving and decline labor supply as workers have an higher income. Moreover tax in last period of life also tends to increase saving. This phenomenon holds through income effects, a tax is perceived as a lower revenue in last period, so agent needs to save more in order to consume in this last period. Mechanisms are the same for an over-accumulated economy. A tax addressed to workers increases labor supply and declines possibility to save. A subsidy for retired agents lowers the needed of saving in order to consume in the retirement period.

Remark 4.4. *In an economy without remittances, then this optimal policy would be:*

$$\tau_w^{wr} = \delta\,(1-s)\,A\left(\frac{sA}{1+n-\mu}\right)^{\frac{s}{1-s}} - (1+n-\mu)\,(1-\eta)\left(\frac{sA}{1+n-\mu}\right)^{\frac{1}{1-s}}$$

and

$$\tau_r^{wr} = (1+n-\mu)^2\,(1-\eta)\left(\frac{sA}{1+n-\mu}\right)^{\frac{1}{1-s}} - \delta\,(1-s)\,A\,(1+n-\mu)\left(\frac{sA}{1+n-\mu}\right)^{\frac{s}{1-s}}$$

1. *If $\eta < \widehat{\eta}$ then $\tau_w^{wr} < 0$ and $\tau_r^{wr} > 0$ implying a subsidy for workers and a tax for retired agents.*
2. *If $\eta = \widehat{\eta}$ then $\tau_w^{wr} = 0$ and $\tau_r^{wr} = 0$ implying no policy.*
3. *If $\eta > \widehat{\eta}$ then $\tau_w^{wr} > 0$ and $\tau_r^{wr} < 0$ implying a tax for workers and a subsidy for retired agents.*

Economic mechanisms are the same as in the case with remittances. If saving is too low, then a subsidy for workers and a tax for retired increase capital accumulation.

It is now easy to compare the two policies (i.e in the recipient economy and in the hypothetical same economy without remittances) in order to see the impacts of remittances on this optimal policy. In order to to this, it is useful to remark that with remittances, $\tau_w^r = -h(\widehat{k})$ and $\tau_r^r = h(\widehat{k})\,(1+n-\mu)$. On the other hand, without remittances, this optimal policy would be such that $\tau_w^{wr} = -f(\widehat{k})$ and $\tau_r^{wr} = f(\widehat{k})\,(1+n-\mu)$. We will compare magnitude of policies according to parameters.

- If $\eta < \widehat{\eta}$, then $\widehat{\gamma} < 0 < \gamma$. In this case, $h(\widehat{k}) > f(\widehat{k}) > 0$ which implies $\tau_w^r < \tau_w^{wr} < 0$ and $0 < \tau_r^{wr} < \tau_r^r$. The scale of the public policy to bring economy to the golden rule (subsidies for workers and taxes for retired agents) is higher with remittances but brings higher stationary utility.
- If $\eta = \widehat{\eta}$, then $0 = \widehat{\gamma} < \gamma$. In this case, $h(\widehat{k}) > f(\widehat{k}) = 0$ which implies $\tau_w^r < \tau_w^{wr} = 0$ and $0 = \tau_r^{wr} < \tau_r^r$. With remittances, there is necessity of public policy (subsidies for workers and taxes for retired agents). Without remittances, it would not be necessary to have public policy but stationary utility would be lower.
- If $\eta > \widehat{\eta}$:

– $\widehat{\gamma} < \gamma$. In this case, $f(\widehat{k}) < h(\widehat{k}) < 0$ which implies $0 < \tau_w^r < \tau_w^{wr}$ and $\tau_r^{wr} < \tau_r^r < 0$. The magnitude of the public policy to bring economy to the golden rule (taxes for workers and subsidies for retired agents) is lower with remittances and brings more important utility.
– $\widehat{\gamma} = \gamma$. In this case, $f(\widehat{k}) < h(\widehat{k}) = 0$ which implies $0 = \tau_w^r < \tau_w^{wr}$ and $\tau_r^{wr} < \tau_r^r = 0$. There is no need of public policy to restore efficiency with remittances. Without remittances it would be necessary to have taxation (taxes for workers and subsidies for retired agents) and stationary utility would be lower.
– $\widehat{\gamma} > \gamma$. In this case, $f(\widehat{k}) < 0 < h(\widehat{k})$ which implies $\tau_w^r < 0 < \tau_w^{wr}$ and $\tau_r^{wr} < 0 < \tau_r^r <$. The 2 optimal policies have opposed impacts, but stationary utility is higher in presence of remittances.

Therefore, the required public policy varies according to amount of remittances. They can increase or decrease necessity and amplitude of the policy but also totally modify the required policy. We now quantify phenomenons with a calibration for some developing countries.

5 Calibration

The aim of this part is to move closer to the reality in order to see the different effects on efficiency in different recipient countries. We will evaluate impacts of workers' remittances on variables (saving rate, working time, capital accumulation) but also on efficiency. This empirical analysis will also allow to quantify theoretical results in term of policy recommendation.

The most important thing about parameters, to calibrate our model, is that we consider "long" periods (i.e. 25 years). It is for this reason that we have considered a total capital depreciation. First off all, we need to calibrate each parameter of the theoretical model using empirical data. There is two types of parameters. The first category is directly estimated using data as capital share or return of education for instance, but with a conversion over 25 years for some of them as the evolution of population. The second category is calibrated by computing economic variables in the model as the saving rate, the labor time or remittances over GDP for instance, to quantify weigh of consumption, importance of hardness of work or even altruism parameter in the utility functions. Of course, we also need to have value of k to compute these variables and calibrate parameters, even if k is also determined by these parameters for the calibration. We begin with the first category.

We calibrate parameters n and μ which respectively depict the growth of births (as agent has $1 + n$ children in the model), and those who leave the countries. We use data from the World Bank which gives growth rate of evolution of population but also migration over five-years periods (2010-2015). It is firstly necessary to quantify the evolution of population at each period in our model, given by $(1 + n)(1 - p)$ or also $1 + n - \mu$. We directly obtain this term by converting the rate of evolution of population over 25 years. Nevertheless, we need to calibrate the parameter μ influencing amount of inflows. For simplicity, we assume a constant migration over the five-year period. We compute the average migration rate for the middle of the period (year 2012) as the migration between 2010 and 2015 divided by 5 times the total population in the middle of the period (i.e. 2012). We then can convert this rate for 25 years which allows to obtain $-p$ in our model. Having the value of $(1 + n)(1 - p)$ and $1 - p$, then it is easy to compute $1 + n$. Finally, we obtain the parameter μ by knowing that $\mu = p(1 + n)$. Parameter s is considered as the capital share in total income. According to the estimations of labor share given by Bernanke and Gürkaynak [11], and used in Caseli's database [12] we easily obtain the parameter s. We calibrate the parameter λ in agreement with Gente *et al.* [20] for the method and with Psacharopoulos and Patrinos [31] for the data. First of all, we consider the same parameter for all studied countries by assuming a return of education of migrants in a developed countries. We use average data concerning high income countries. Secondly, we exploit Mincerian equation estimations which give a wage gain of one supplementary year of education (semi elasticity of wage with respect to education). In our framework, by normalizing the cost of one year of education, parameter λ represents elasticity of wage with respect to education implying that we must multiply the semi-elasticity coefficient by the average years of education[7]. By considering average years of education of 9.4 in high income countries and average return of 7.4%, from Psacharopoulos and Patrinos [31], we use $\lambda = 0.6956$ for all studied countries. Otherwise, parameter A is normalized to 1.

To calibrate the second category of parameters, η, δ, ϵ, α and γ, we use data on worked hours, saving rates and remittances over GDP. In our framework, parameter η represents the importance of harshness of work in the utility function. The more η is important, the more the utility function in decreasing with labor. Prescott [30] considered that the total productive available time is 100 hours per weeks. In our model, this time is normalized to one. Using ILO data on worked hours per employed person[8], we can easily compute the labor supply in our model needed to calibrate η. In order to estimate δ and then ϵ, we compute the saving rate in our model as the saving divided by the income $\frac{s_t}{w_t l_t}$. For the δ calibration we use World Bank data on saving over GDP. We approximate the saving rate by assuming that total wages in the economy are given by $(1 - s)\,\text{GDP}$. Hence, we divided the saving over GDP by $1 - s$ in order to obtain the saving over the wage. Parameter ϵ is estimated using the relation $\epsilon + \eta + \delta = 1$. Finally, parameters α and γ are calibrated using World Bank data on remittances over GDP. In our model, remittances over GDP are computed as $\frac{\mu\alpha\gamma e_{t-1}^{\lambda}}{Ak_t^s l_t(1+n-\mu)}$. We only calibrate $\gamma \times \alpha$.

Therefore, parameters, n, μ, s and λ are directly given. For the other parameters, we numerically solve a system with 5 equations and 5 variables including the value of capital accumulation k.

We calibrate our model for 11 countries in different regions of the World: Algeria, Bolivia, Colombia, Egypt, El Salvador, Morocco, Mexico, Peru, The Philippines, Sri-Lanka and finally Tunisia. The results of our parameter calibration are given in Table 2. We will discuss about this calibration with another table summarizing results for all countries and then we will explain detailed results with some examples. Nevertheless, we differentiate about under-accumulated and over-accumulated countries, which is the main difference across countries that we are interested in.

5.1 Remittances and Golden Rule across World

The table 3 summarizes the consequences of our calibration on the main economic variables in each recipient country and the estimations of values of these variables in each same country if remittances were removed. Obviously, values without remittances are estimations by assuming

[7] $\frac{\partial w_{t+1}}{\partial e_t}\frac{1}{w_{t+1}} = \frac{\lambda}{e_t}$

[8] We prefer to use the "mean weekly hours actually worked per employed person" than the "mean weekly hours actually worked per employee" because the first definition includes employed person who temporarily do not work, reflecting more, for us, the labor supply. Due to missing information for Tunisia, we use legal working time.

Country	n	μ	s	ϵ	δ	η	$\alpha \times \gamma$	λ
Algeria	0.6318	0.0309	0.39	0.08931	0.33284	0.57785	0.38184	0.6956
Bolivia	0.5327	0.0456	0.33	0.24828	0.18730	0.56442	1.30445	0.6956
Colombia	0.3024	0.02	0.35	0.30050	0.12918	0.57032	1.18960	0.6956
Egypt	0.7448	0.0218	0.23	0.36093	0.10109	0.53798	2.97054	0.6956
El Salvador	0.3147	0.2370	0.42	0.32899	0.08845	0.58256	1.60037	0.6956
Morocco	0.4839	0.0683	0.42	0.24075	0.21310	0.54615	1.41050	0.6956
Mexico	0.4463	0.0307	0.45	0.24625	0.17507	0.57868	1.27081	0.6956
Peru	0.4372	0.0561	0.44	0.23403	0.17093	0.59504	0.90436	0.6956
Philippines	0.5424	0.0552	0.41	0.06049	0.36408	0.57543	1.29510	0.6956
Sri-lanka	0.4452	0.1628	0.22	0.24615	0.17478	0.57906	0.85181	0.6956
Tunisia	0.3022	0.0198	0.38	0.35163	0.13078	0.51759	2.23914	0.6956

Table 2: Calibrated parameters for each recipient county

an hypothetical total regulation of remittances[9] (not empirically sustainable). Nevertheless, this assumption allows to approximately quantify effect of remittances according to our framework. Columns "l" gives the labor supply[10] in the two configurations. Following to our theoretical results, labor supply would be greater in each country without remittances. The columns "$\frac{s}{wl}$" reveal the saving rate in each configuration. Here again, the saving rate would be greater without remittances. Finally, columns "$\frac{e}{wl}$" and "$\frac{B}{GDP}$" respectively inform about amount of child education with respect to income when agent works and amount of remittances over GDP. Then, the table 4 depicts the estimations of variation of the main economic variables under our hypothesis of a total regulation of inflows. We respectively observes variation of absolute saving, labor supply, capital accumulation, wage rate, interest factor, income of worker and then saving rate. Following to our theoretical results, only the estimation of the variation of interest factor is negative due to negative effects of remittances (an therefore positive effect of regulation) on capital accumulation.

Finally, the table 5 summarizes situation of recipient countries according to the golden rule. The first thing to notice, is that under our calibration, the majority of them are under-accumulated countries. Indeed, as it is shown in this table, 8 countries over 11 are under-accumulated. This table summarizes the main result of our calibration. The column 2 allows to see if capital accumulation is too high or too law with respect to the golden rule in each recipient country. Columns 3 and 4 give the same information but related to education and labor supply. Theoretical results of remittances are confirmed, if $\overline{k} > \widehat{k}$ then $\overline{e} > \widehat{e}$ and $\overline{l} < \widehat{l}$. On the other and, if $\overline{k} < \widehat{k}$ then $\overline{e} < \widehat{e}$ and $\overline{l} > \widehat{l}$. The column 5 literally summarizes previous columns. As previously, the two last columns, represent situation of each country if they did not receive remittances. Obviously, only the equilibrium capital stock over the optimal capital stock is represented because labor supply would always be optimal according to the assumption of no inflows, and education would be null, what would be optimal. As the column 5, the last literally summarizes previous column. Clearly, each under-accumulated economy without remittances is also under-accumulated with remittances. Likewise, each over-accumulated economy with remittances would also be over-accumulated without remittances. Nevertheless, there is not in our sample an under-accumulated country which would be over-accumulated without remittances.

[9] $\gamma = 0$.

[10] Labor supply formulated in percentage also gives the number of worked hours. For Instance, in Algeria, $l = 0.422$ means that worked hours per week are in average 42.2 for each worker.

Country	With remittances				Without remittances	
	l	$\frac{s}{wl}$	$\frac{\mu e}{wl}$	$\frac{B}{GDP}$	l	$\frac{s}{wl}$
Algeria	42.20%	78.69%	0.14%	0.10%	42.21%	78.84%
Bolivia	43.20%	38.81%	3.35%	4.10%	43.56%	43.00%
Colombia	42.90%	29.23%	0.64%	1.10%	42.97%	30.06%
Egypt	45.80%	16.88%	3.73%	7.30%	46.20%	21.88%
El Salvador	41.30%	15.52%	4.22%	16.40%	41.74%	21.19%
Morocco	44.90%	41.38%	4.52%	6.60%	45.39%	46.95%
Mexico	42.00%	40.00%	1.24%	2.00%	42.13%	41.55%
Peru	40.40%	41.07%	0.91%	1.40%	40.50%	42.21%
Philippines	41.20%	72.88%	12.12%	9.80%	42.46%	85.75%
Sri-lanka	41.20%	30.77%	8.56%	8.80%	42.09%	41.52%
Tunisia	48.00%	24.19%	2.21%	5.00%	48.24%	27.11%

Table 3: Economic variables with and without remittances

The direct implication is that for all under-accumulated economies considered here, it does not exist an optimal amount of remittances. Indeed, for each of them, $\eta < \widehat{\eta}$. Nevertheless, for each over-accumulated economy, the optimal amount of remittances is greater than the current.

In order to understand more the impacts, we will explain the detailed results for some countries. We focus first on the under-accumulated economies and explain impacts of remittances, to focus then on over-accumulated economies.

5.1.1 Under-accumulated countries

We will explain our empirical results for Bolivia and Mexico and then for Tunisia in North Africa.

Bolivia The first Latin-American example considered here is Bolivia. In this country, the growth of population (including migration) is estimated to 1.6% per year and the migration rate to 0.12% what allow to obtain the growth of birth. We have calibrated the parameter η to obtain a labor supply equivalent to 43.2 hours per week as depicted by the ILO. Then, parameter δ and ϵ are calibrated to obtain a saving rate (saving over wage) equal to 38.81% according to our computation using labor share (saving over GDP is equivalent to 26% according to the World Bank). Finally, parameter α and γ are estimated here to obtain an amount of remittances over GDP to 4.1% according to World Bank data. As previously said, under this calibration, Bolivia is an under-accumulated economy. In other words, capital stock per head and education need to raise to improve efficiency and therefore to increase aggregated welfare. However, labor needs to decline to bring the economy to the golden rule. Indeed, according to our computations given in table 5, we see, $\frac{\overline{k}}{\widehat{k}} = 0.7007$, $\frac{\overline{e}}{\widehat{e}} = 0.4571$ and $\frac{\overline{l}}{\widehat{l}} = 1.0079$.
In other terms, the equilibrium capital stock represents 70% of the optimal capital stock. It is easy to understand that this economy is under-accumulated. By considering an hypothetical total regulation, capital accumulation would grow by 16.54%, with a saving rate estimated to 43% instead of 38.81%, representing a rise of absolute amount of saving evaluated to 17.5%. However, as remittances lower the capital accumulation, the interest factor would decrease by

Country	Variation without remittances						
	s	l	k	w	R	wl	$\frac{s}{wl}$
Algeria	+0.36%	+0.04%	+0.32%	+0.12%	−0.19%	+0.16%	+0.19%
Bolivia	+17.50%	+0.83%	+16.54%	+5.18%	−9.75%	+6.05%	+10.80%
Colombia	+4.59%	+0.16%	+4.43%	+1.53%	−2.78%	+1.69%	+2.86%
Egypt	+41.30%	+0.88%	+40.07%	+8.06%	−22.85%	+9.01%	+29.63%
El Salvador	+72.90%	+1.07%	+71.06%	+25.29%	−26.76%	+26.64%	+36.53%
Morocco	+25.68%	+1.08%	+24.34%	+9.58%	−11.87%	+10.77%	+13.47%
Mexico	+7.51%	+0.31%	+7.17%	+3.17%	−3.74%	+3.49%	+3.88%
Peru	+5.25%	+0.24%	+5.01%	+2.17%	−2.70%	+2.41%	+2.77%
Philippines	+35.76%	+3.05%	+31.74%	+11.97%	−15.01%	+15.38%	+17.66%
Sri-lanka	+50.03%	+2.17%	+46.84%	+8.83%	−25.90%	+11.18%	+34.94%
Tunisia	+20.78%	+0.50%	+20.18%	+7.24%	−10.77%	+7.77%	+12.07%

Table 4: Evolution of economic variables without remittances

Country	Efficiency with remittances				Efficiency without remittances	
	$\frac{\overline{k}}{\widehat{k}}$	$\frac{\overline{e}}{\widehat{e}}$	$\frac{\overline{l}}{\widehat{l}}$	Category	$\frac{\overline{k}}{\widehat{k}}$	Category
Algeria	1.4055	1.9782	0.9998	Over-accumulated	1.4100	Over-accumulated
Bolivia	0.7007	0.4571	1.0079	Under-accumulated	0.8166	Under-accumulated
Colombia	0.3907	0.1344	1.0070	Under-accumulated	0.4080	Under-accumulated
Egypt	0.4765	0.1534	1.0411	Under-accumulated	0.6675	Under-accumulated
El Salvador	0.0703	0.0063	2.1949	Under-accumulated	0.1202	Under-accumulated
Morocco	0.3810	0.1591	1.0357	Under-accumulated	0.4738	Under-accumulated
Mexico	0.2722	0.0953	1.0154	Under-accumulated	0.2917	Under-accumulated
Peru	0.3140	0.1187	1.0097	Under-accumulated	0.3297	Under-accumulated
Philippines	1.0840	1.1693	0.9965	Over-accumulated	1.4282	Over-accumulated
Sri-lanka	1.1180	1.3310	0.9950	Over-accumulated	1.6418	Over-accumulated
Tunisia	0.2232	0.0472	1.0583	Under-accumulated	0.2683	Under-accumulated

Table 5: Efficiency of the recipient countries

9.75% without remittances. Otherwise, labor supply would raise by 0.83% to attain 43.56 worked hours per week. By decreasing capital per head, remittances have also impact on the wage rate, by lowering it. For this reason, an hypothetical total regulation of remittances would rise the wage rate by 5.18% and the income by 6.05%.

Mexico The second American example is Mexico. Same method is used as in Bolivian calibration, we compute parameters to follow empirical studies. In Mexico, the growth of population is estimated to 1.4% per year and migration rate to 0.086%. We compute a saving rate around 40%, needed to calibrate δ and ϵ. We then determine α and γ to obtain a relative amount of remittances over GDP to 2%. Without remittances, capital accumulation would increase by 7.17%, with a raise of absolute saving estimated to 7.51%. Indeed, under this calibration, saving rate would increase from 40% to 41.55% (+3.88%). Labor supply would moderately grow by 0.31%. Mexican workers would work in average 42.13 hours per week instead of 42. As Bolivia, Mexico is in our framework with remittances in under-accumulation. Capital stock per head also needs to increase to improve efficiency and so to improve aggregated welfare. Indeed, the equilibrium capital stock only represents 27% of the optimal capital stock.

Tunisia The last example considering an under-accumulated economy is Tunisia. In this north-African country, the population growth is 1% per year according to the World Bank and migration rate to 0.061% per year. In this country, computed saving rate is lower than in the 2 previous (24.19%) and remittances are more important, representing approximately 5% of GDP. Nevertheless, amount of estimated education related to income is equivalent to 2.21%, ranged between Mexico (1.24%) and Bolivia (3.35%). By considering no remittance inflows, absolute saving would increase by 20.78%, implying a rise of saving rate estimated to 12.07%. Finally, this assumption would imply an augmentation of workers' income estimated to 7.77% due to an increase of labor supply (+0.50%) and wage rate (+7.24%). As previously said, it does not exist an optimal amount of remittances in this country. The reason is that Tunisia is an under-accumulated economy in our framework but would also be under-accumulated without remittances.

We will now explain three other examples, 3 over-accumulated economies which are Algeria, Philippines and then Sri-Lanka.

5.1.2 Over-accumulated countries

The first studied example of over-accumulated economy is located again in North-Africa. The second is located in Pacific Ocean and the third in Indian Ocean.

Algeria As previous examples, tables 3, 4 and 5 summarize all our results. In Algeria, evolution of population is estimated to increase by 1.9% per year and migration rate is evaluated to 0.077%. We founded a high saving rate around 78% (saving over GDP is estimated to 48% by the World-Bank) to calibrate δ and ϵ. The important thing about remittances, is that according to the Wold-Bank again, they only represent 0.1% of the GDP, a very law amount related to other countries in our sample. For this reason, parameters α and γ are lower than in other examples. The parameter η is calibrated to represent an average worked hours of 42.2 per week. Due to low level of remittances relative to the GDP, the variation of variables with a removal of remittances would be very low. For instance, capital accumulation would only increase by 0.32% with a rise of saving estimated to 0.36% and an growth of labor supply estimated to 0.04%. However, this country is an over-accumulated economy with remittances and obviously would also be over-accumulated without remittances. The equilibrium capital stock is 1.4 times higher than the optimal. Following our computation about $\widehat{\gamma}$ it exists an optimal amount of remittances which restore the optimality of capital accumulation. Indeed, this optimal amount is such that $\alpha \times \gamma = 1.79952$ what implies an increase of this two parameters of 371.28%.

Philippines Always with the same method, parameters are calibrated in order to obtain a growth of workers computed to 1.6% per year by the World Bank and migration rate 0.146%. Estimated parameters η, δ and ϵ model in average 41.2 worked hours per week according to ILO and an amount of saving over GDP of 43% according to the World Bank. First of all, impacts of a total regulation would have huge effects on capital accumulation and labor supply due to high migration rate and amount of remittances over GDP (9.8%). For instance labor supply would increase by 3.05% and saving rate by 17.66%, representing a rise of absolute saving estimated to 35.76%. As previously, income would grow (15.38%), with an increase of wage rate (11.97%). Interest factor would be reduced (-15.01%). The main difference about efficiency, with respect to Bolivia, Mexico and Tunisia, is that, here, this economy is in over-accumulation receiving remittances. In others words, capital per head is too high and needs to decrease to heighten aggregated welfare. As it is proved in last theoretical section, it exists therefore an optimal solution of entering remittances. Nevertheless as economy appears to be in over-accumulation with current remittances ($\frac{\bar{k}}{\hat{k}} = 1.0840$), the optimal amount is greater than the current. This is indeed the case, we compute an optimal equilibrium amount of remittances such that $\alpha \times \gamma = 1.44036$ representing an increase of this two parameters of 11.22%. This optimal evolution is lower than in Algeria due to the fact that in Algeria, the relative difference between equilibrium capital stock and relative capital stock is higher ($\frac{\bar{k}}{\hat{k}} = 1.4055$), and amount of relative remittances with respect to GDP is lower implying a low value of $\alpha \times \gamma$ and therefore a greater relative augmentation. The gap between equilibrium capital per head and golden rule decreases with remittances until an optimal amount.

Sri-Lanka The last studied country in our sample is the Sri-Lanka. In this country, migration rate is more than 3 times higher than in the Philippines. Indeed, our computed migration rate is equivalent to 0.477%. Growth of population is estimated to 1% per year. Saving rate is quantified according to our computations to 30.77% and worked hours per employed person are equivalent as in Philippines according to ILO. The main difference of this economy is that the capital share is very low with respect to other recipient countries. Finally, remittances over GDP represent 8.8%. A situation without remittances would imply a huge rise of saving and capital accumulation (+50.03% and +46.84%) implying a huge lowering of interest factor (-25.90%). As previously for other over-accumulated economies, it exists an optimal amount of remittances because $\eta > \widehat{\eta}$. This optimal amount is such that $\alpha \times \gamma = 0.9829$. In other terms, this 2 related parameter must to increase by 15.39% to get an optimal equilibrium.

According to empirical literature, impacts of remittances are country-specific, and principally on growth. Following this phenomenon, we have showed that impact of remittances would also be country-specific on efficiency. The aim of the next part is to quantify the optimal policy previously defined.

5.2 An optimal policy for recipient countries

In this last part, we quantify our previous results for the optimal policy. It consists in lump-sump taxes and transfers. We do not directly influence amount of remittances, because it seems to be empirically difficult to tax remittances for instance in under-accumulated countries. For

Country	With remittances		Without remittances		Variation of τ_w
	$\frac{\tau_w}{wl^d}$	$\frac{\tau_r}{s^d R+B}$	$\frac{\tau_w}{wl^d}$	$\frac{\tau_r}{s^d R+B}$	
Algeria	14.82%	−23.14%	14.91%	−23.32%	+0.62%
Bolivia	−14.47%	24.65%	−6.25%	12.70%	−56.08%
Colombia	−28.28%	48.10%	−23.78%	44.17%	−15.19%
Egypt	−36.62%	60.48%	−7.99%	26.75%	−77.08%
El Salvador	−694.04%	91.38%	−51.22%	70.74%	−83.63%
Morocco	−49.66%	49.34%	−25.46%	35.16%	−46.32%
Mexico	−49.48%	53.57%	−40.27%	49.21%	−17.11%
Peru	−42.18%	49.48%	−36.36%	46.28%	−12.75%
Philippines	4.92%	−5.80%	16.26%	−23.40%	+239.25%
Sri-lanka	5.08%	−13.50%	13.32%	−47.22%	+166.55%
Tunisia	−71.25%	70.06%	−34.18%	55.77%	−48.97%

Table 6: An optimal taxation with lump-sump taxes and transfers and comparison of cases with and without remittances
Note: In the column 6, $\frac{\tau_w^{wr}-\tau_w^r}{\tau_w^r} = \frac{\tau_r^{wr}-\tau_r^r}{\tau_r^r}$

the same reason a subsidy on remittances needed in over-accumulated country would imply a taxation to have a balanced government budget constraint. According to our theoretical results, this optimal policy would be to tax workers and to give subsidies to retired agents in over-accumulated economies and to give subsidies to workers and tax retired agents in under-accumulated economies.

The table 6 shows this optimal policy for each studied country in our sample. For a better interpretation we explain amount dedicated to workers, relative to income (column 2 and 4) and the amount dedicated to retired persons relative to total amount of saving and remittances (column 3 and 5) in the decentralized equilibrium. As previously, this table gives amount of taxes and subsidies for each recipient country, but in columns 4 and 5, hypothetical amounts are given, corresponding to optimal amounts if there were no remittances. Finally, the last column allows a comparison between the needed policy in each configuration. A negative evolution implies a greater magnitude of policy in the configuration with remittances. However, a positive evolution implies a lower scale of the taxation in the situation with remittances. The main result for the studied sample is that in under-accumulated economies, the magnitude of the subsidies for workers and taxes for retired agents is greater with remittances. In this case, remittances bring the capital accumulation further to the golden rule. The optimal policy is more important but brings more utility. Nevertheless, in over-accumulated countries, the scale of taxes for workers and subsidies for retired is lower in the configuration with remittances. This inflows bring capital accumulation closer to the golden rule.

We will briefly detail the policy in the previous examples, Bolivia, Mexico, Tunisia, and then Algeria, Philippines and Sri-Lanka.

5.2.1 An optimal policy in under-accumulated countries

As previously said, remittances represent in Bolivia 4.1% of the GDP. Under our calibration, this economy is under-accumulated. According to our framework, a moderated lump-sum taxation allows to bring this economy to the golden rule. This optimal policy would consist on a subsidy for workers and a tax for old agents. Indeed, at the optimal decentralized equilibrium, the labor supply is lower than in the competitive equilibrium. A subsidy for worker, implies a decrease of labor supply and an increase of saving. Furthermore a tax for retired implies the needed for the agent to save more to consume because the tax implies lower revenue in the last period of life. The amount of lump-sump subsidy for worker is equivalent to 14.47% of income in the optimal decentralized equilibrium. Nevertheless, the amount of the tax for retired agents is equivalent to 24.65% of the return of saving and the amount of remittances. However, if remittances were cut, the optimal subsidy would represent 6.25% of income and the tax 12.70% of the amount of saving and interests. By comparing the absolute amount of taxes and subsidies, we remark that with remittances, they are more than 2 times higher that amount without remittances. Indeed, the tax and the subsidy would decrease by 56.08% if remittances were cut. As they bring capital accumulation further to the golden rule, the scale of policy would be lower without remittances. A last point of these Bolivian calibration, is that if remittances were cut, the utility at the golden rule, as said in the theoretical part would be lower. This drop occurs for all countries in the sample (with different magnitude). The Mexican capital accumulation is further to the golden rule compared to Bolivian capital accumulation. For this reason an optimal taxation would be relatively more important. Indeed, in Mexico, an optimal subsidy for workers would represent almost 50% of income and the tax for retired more than 50%. As in Bolivia, this amount would be lower if there were no remittances in Mexico. Nevertheless as remittances over GDP are less important than in Bolivia, the difference of policy between the two situations is less important. The scale would be reduced by 17.11% without remittances. Due to a low capital accumulation in Tunisia, the tax and subsidy would be very important representing approximately 70% of revenue for workers and retired. Nevertheless, this relative amount would be lower in a situation without remittances. Indeed, optimal absolute amount would be approximately 2 times lower without remittances (-48.97%).

5.2.2 An optimal policy in over-accumulated countries

In over-accumulated economies, the optimal policy is reverted. In other terms, it is sufficient to tax workers and give a subsidy to retired persons in order to bring each over-accumulated economy to the golden rule. In Algeria, this policy would consist on a tax representing 14.82% of the worker's income, and a subsidy equivalent to 23.14% of the revenue of retired persons. By this instrument, labor supply would grow (to pay the tax) and saving would fall (subsidy allows for consumption in last period). Without remittances, these amounts would respectively be 14.91% and 23.32%. Contrary to the under-accumulated recipient countries, the optimal policy would now be more important without remittances. Indeed, in over-accumulated economies, remittances bring capital accumulation closer to the golden rule and reduce the intensity of taxation. Nevertheless, due to low amount of remittances in Algeria, the absolute amounts are almost equal between the two situations. In the Philippines, this optimal tax for workers would represent 4.92% of income and subsidy 5.80%. Without remittances, we would have a tax equivalent to 16.26% of income subsidy 23.40% of the amount of saving and interests. In this country the scale of policy is lower with remittances, absolute lump sump taxes and transfers would increase by 239.25% if remittances were removed. Finally, in Sri-lanka, amount are respectively 5.08% and 13.50% with remittances and 13.32% and 47.22% without remittances, with an higher taxation in this last configuration.

Therefore, this part shows that it is sufficient to have a lump-sump taxation in order to bring economies to their golden rule. This policy is inverted according to the fact that economies are under-accumulated or over-accumulated.

6 Concluding remarks

Since the second part of the twentieth century, migratory flows have greatly increased. In 2013, the OECD estimated more than 230 millions of migrants in the World with a more important growth during the last decade. These human flows went along with financial flows from emigrated areas to immigrated regions but also with inverted flows named workers' remittances. Workers' remittances are considered as a transfer of money from a migrant to his family living in the home country. Frequency of these flows is more important than migratory flows because conversely to these last flows, workers' remittances are repeated in time. During the last 40 years, amounts have substantially increased to speak about an exponential growth. Flows of remittances are more important than development aids. By the way, some institutions classify them as a "development resource" like the saving and private investment. The World Bank evaluated amount of these studied flows between 1% and 25% of GDP for the majority of developing recipient countries. The significance of this phenomenon implies effects in recipient economies. Indeed, remittances affect the behavior of economic agents. The more important effect is the increase of consumption. According to some empirical studies, more than 80% of remittances are dedicated to consumption spendings in the recipient country. However, they also affect saving and labor supply. As remittances are supplementary income, agents work less. The challenge for economists is to evaluate the macroeconomic impacts of these private transfers of money. Empirically, there is no consensus yet on this question. Results are country-specific and usually, depend on the consequences of remittances on investment. For instance, economists predict bad impacts if investment does not increase and better impacts if remittances can allow investment.

The aim of this paper was to show the effects of remittances in the long run on capital accumulation but also on efficiency. The main assumption is that parents educate their child in order to obtain inflows from him when he has migrated in another country with more favorable economic conditions. These remittances depend therefore on investment in education but also on child's altruism. Two overlapping economies were studied in order to explain both sending child decision in the foreign country and his impact in the parents' economy. With this general framework and a "backward induction" resolution, we explained that effects of remittances on capital accumulation tend to be negative if agents only consider them as an increase of revenues by realizing a portfolio choice between saving and child's education. Indeed, the negative consequence of these inflows is a drop of saving and labor supply and finally on capital accumulation with a greater effect on saving than on capital accumulation. Nevertheless, they don't have effects on stability and therefore on macroeconomic fluctuations. It is also shown that it exists only under some conditions an optimal amount of remittances which brings the economy to the optimal steady state in term of capital accumulation, education and labor supply. Authorities can set up a fiscal policy in order to restore efficiency. This simple policy consists on a tax for workers and a subsidy for retired persons in over-accumulated countries and a subsidy for workers and a tax for retired persons in under-accumulated economies. Finally, remittances have always positive impacts on efficiency if an optimal policy is implemented, through an increase of welfare.

Appendix

Proof of Proposition 3.1

A graphical argument can illustrate the two propositions of existence when $\gamma = 0$ (no remittances case) by representing the function $f(k)$ in figure 1

Lemma 3. *The function $f(k)$, which determines equilibrium, starts from 0, is decreasing and then increasing after the point k_1^{wr}, tends to ∞ and is convex, what satisfies proposition 3.1, with* $k_1^{wr} = \left(\frac{s\delta(1-s)A}{(1+n-\mu)(1-\eta)}\right)^{\frac{1}{1-s}}$

Proof. One verifies:

$$f(0) = 0 \qquad \lim_{k\to+\infty} f(k) = \infty$$

The function stars from zero and tends to ∞. To analyze the growth, one computes the first order derivative with respect to k.

$$\frac{\partial f(k)}{\partial k} = (1+n-\mu)(1-\eta) - s\delta(1-s)Ak^{s-1}$$

$$\frac{\partial f(k)}{\partial k} > 0 \Leftrightarrow k > \left(\frac{s\delta(1-s)A}{(1+n-\mu)(1-\eta)}\right)^{\frac{1}{1-s}} \equiv k_1^{wr} < \overline{k}^{wr}$$

$$\lim_{k\to 0^+} \frac{\partial f(k)}{\partial k} = -\infty \qquad \lim_{k\to+\infty} \frac{\partial f(k)}{\partial k} = (1+n-\mu)(1-\eta)$$

To see the convexity or concavity of $f(k)$, one computes the second order derivative with respect to k

$$\frac{\partial^2 f(k)}{\partial k^2} = s\delta(1-s)^2 Ak^{s-2} > 0$$

Thus, the function $f(k)$ starts from zero, is convex, decreasing and then increasing. It exists two solutions to the equation $f(k) = 0$ as it is represented in figure 1. $\square$

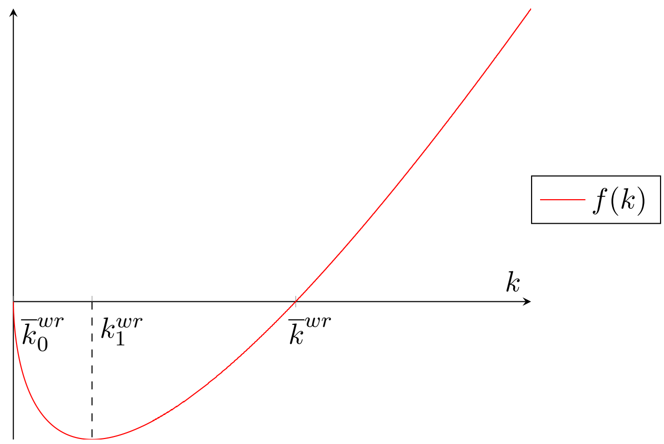


Figure 1: Representation of equilibrium in the no remittances case ($\gamma = 0$)

Proof of Proposition 3.3

For the trivial steady state, reasoning is the same as in proposition 3.1, equation (9) is satisfied if $\overline{k} = 0$. To express the second steady state, we describe the function $g(k)$ in order to determine then solutions of $h(k) = 0 \Leftrightarrow f(k) + g(k) = 0$

Lemma 4. *Under assumptions 1, 2 and 3, and if $\lambda > s$ the function $g(k)$ starts from 0, is increasing and convex. It becomes concave after the point k_1^r, and then decreasing after the point k_2^r and tends to $-\infty$. It is positive on $]0; k_3^r[$ and negative on $]k_3^r; \infty[$. If the condition on λ is not satisfied, this function is concave, starts from 0, is increasing and then decreasing after the point k_2^r and tends to $-\infty$. It is positive on $]0; k_3^r[$ and negative on $]k_3^r; \infty[$, with:*
$k_1^r = \left(\frac{(\lambda\delta+1-\delta)(\lambda-s)(1-s)A}{(1+n-\mu)\eta(1-\lambda)(2-\lambda)(1-2s+\lambda s)}\right)^{\frac{1}{1-s}}$, $k_2^r = \left(\frac{(\lambda\delta+1-\delta)(1-s)A}{(1+n-\mu)\eta(1-\lambda)(2-\lambda)}\right)^{\frac{1}{1-s}}$,
$k_3^r = \left(\frac{(\lambda\delta+1-\delta)(1-s)A}{(1+n-\mu)\eta(1-\lambda)}\right)^{\frac{1}{1-s}}$

Proof. One verifies:

$$g(0) = 0 \qquad\qquad \lim_{k\to+\infty} g(k) = -\infty$$

To analyze the growth and convexity or concavity of $g(k)$ one evaluates the first and second order derivatives with respect to k.

$$\frac{\partial g(k)}{\partial k} = \mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\left(\frac{(\lambda\delta+1-\delta)(1-s)}{\lambda(1-\lambda)}k^{\frac{\lambda-s}{1-\lambda}} - \frac{(1+n-\mu)\eta(2-\lambda)}{\lambda A}k^{\frac{1-2s+\lambda s}{1-\lambda}}\right)$$

$$\frac{\partial g(k)}{\partial k} > 0 \Leftrightarrow k < \left(\frac{(\lambda\delta+1-\delta)(1-s)A}{(1+n-\mu)\eta(1-\lambda)(2-\lambda)}\right)^{\frac{1}{1-s}} \equiv k_2^r$$

$$\lim_{k\to0^+}\frac{\partial g(k)}{\partial k} = \begin{cases} 0 & \text{if } \lambda > s \\ \infty & \text{if } \lambda < s\end{cases} \qquad\qquad \lim_{k\to+\infty}\frac{\partial g(k)}{\partial k} = -\infty$$

$$\frac{\partial^2 g(k)}{\partial k^2} = \mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\phi(k)$$

with:

$$\phi(k) = \frac{(\lambda\delta+1-\delta)(1-s)(\lambda-s)}{\lambda(1-\lambda)^2}k^{\frac{2\lambda-s-1}{1-\lambda}} - \frac{(1+n-\mu)\eta(2-\lambda)(1-2s+\lambda s)}{\lambda A(1-\lambda)}k^{\frac{1-2s+\lambda s}{1-\lambda}}$$

$$\frac{\partial^2 g(k)}{\partial k^2} > 0 \Leftrightarrow \phi(k) > 0$$

$$\Leftrightarrow k < \left(\frac{(\lambda\delta+1-\delta)(\lambda-s)(1-s)A}{(1+n-\mu)\eta(1-\lambda)(2-\lambda)(1-2s+\lambda s)}\right)^{\frac{1}{1-s}} = k_1^r$$

Concavity or convexity of the function $g(k)$ depends on the following condition:

- If $\lambda > s$ then k_1^r is positive and the function is convex and then concave over $[0; \infty[$.
- If $\lambda < s$ then k_1^r is negative and the function is concave over $[0; \infty[$.

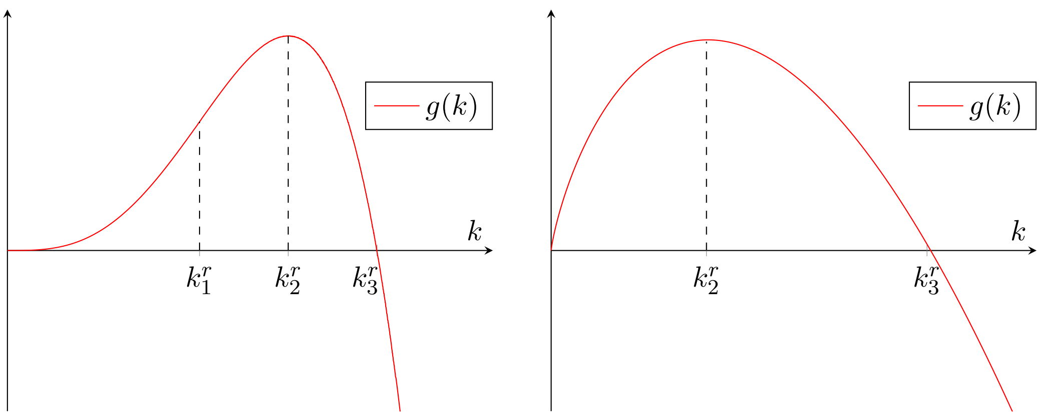


Figure 2: Representations of $g(k)$

Then, one has:

$$
\begin{aligned}
g(k) > 0 &\Leftrightarrow (\lambda\delta + 1 - \delta)\, k^{\frac{1-s}{1-\lambda}} - \frac{(1+n-\mu)\,\eta\,(1-\lambda)}{(1-s)\,A} k^{\frac{(1-s)(2-\lambda)}{1-\lambda}} \\
&\Leftrightarrow k < \left(\frac{(\lambda\delta + 1 - \delta)\,(1-s)\,A}{(1+n-\mu)\,\eta\,(1-\lambda)}\right)^{\frac{1}{1-s}} = k_3^r
\end{aligned}
$$

Finally, one verifies:

$$k_1^r < k_2^r \Leftrightarrow \lambda - s > 1 + \lambda s - 2s \Leftrightarrow (1-s)\,(1-\lambda) > 0$$

$$k_2^r < k_3^r \Leftrightarrow 1 < 2 - \lambda \Leftrightarrow \lambda < 1$$

$$k_3^r > k^{wr} \Leftrightarrow \lambda\delta + \epsilon > 0$$

If $\lambda < s$ the function $g(k)$ starts from zero, is concave, increasing, then decreasing and finally tends to $-\infty$. If $\lambda > s$ it starts from zero (with a zero slope), is increasing and convex then becomes concave and then decreasing and finally tends to $-\infty$. This function is represented by figure 2. □

It is now necessary to determine the equilibrium in situation where $\gamma > 0$. One can define using equations (11):

$$
\begin{aligned}
f(k) + g(k) &= (1+n-\mu)\,(1-\eta)\,k - \delta\,(1-s)\,Ak^s + \left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}} \times \\
&\quad \left(\frac{\mu\,(\lambda\delta + 1 - \delta)}{\lambda} k^{\frac{1-s}{1-\lambda}} - \frac{(1+n-\mu)\,\mu\eta\,(1-\lambda)}{\lambda\,(1-s)\,A} k^{\frac{(1-s)(2-\lambda)}{1-\lambda}}\right) \\
&= h(k) = 0
\end{aligned}
$$

The aim is now to determine the solution(s) of $h(k) = 0$ knowing the properties of functions $f(k)$ and $g(k)$.

Lemma 5. *The function $h(k)$ starts from zero, is decreasing and convex, becomes increasing, concave and then, decreasing. It cuts the axis of abscissa twice.*

Proof. One verifies:

$$\begin{aligned} h(0) &= 0 \\ \lim_{k\to+\infty} h(k) &= \lim_{k\to+\infty} -\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\left(\frac{(1+n-\mu)\,\eta\,(1-\lambda)}{\lambda\,(1-s)\,A}k^{\frac{(1-s)(2-\lambda)}{1-\lambda}}\right) = -\infty \end{aligned}$$

One can easily compute:

$$\begin{aligned} \frac{\partial h(k)}{\partial k} =& (1+n-\mu)\,(1-\eta) - s\delta\,(1-s)\,Ak^{s-1} + \mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}} \\ &\times\left(\frac{(\lambda\delta+1-\delta)\,(1-s)}{\lambda\,(1-\lambda)}k^{\frac{\lambda-s}{1-\lambda}} - \frac{(1+n-\mu)\,\eta\,(2-\lambda)}{\lambda A}k^{\frac{1+\lambda s-2s}{1-\lambda}}\right) \end{aligned}$$

The sign of this derivative is not directly identifiable. Nevertheless, one proves:

$$\lim_{k\to 0^+}\frac{\partial h(k)}{\partial k} = -\infty \qquad \lim_{k\to+\infty}\frac{\partial h(k)}{\partial k} = -\infty$$

$h(k)$ starts from zero with a negative slope and tends to $-\infty$. If one computes $h(k) > 0$ for one particular k, one can conclude that the curve cuts at least twice the abscissa axis.

For the point $k = \left(\frac{\delta(1-s)A}{(1+n-\mu)(1-\eta)}\right)^{\frac{1}{1-s}} \equiv \overline{k}^{wr}$, one compute : $h(\overline{k}^{wr}) = f(\overline{k}^{wr}) + g(\overline{k}^{wr})$
It is easy to check that $\overline{k}^{wr} < k_3^r$, and so $f(\overline{k}^{wr}) = 0$, and $g(\overline{k}^{wr}) > 0$. One concludes: $h(\overline{k}^{wr}) > 0$

If there is no more than one change of concavity, it is easy to verify that the equation $h(k) = 0$ has no more than two positive solutions.

$$\frac{\partial^2 h(k)}{\partial k^2} = s\,(s-1)^2\,\delta Ak^{s-2} + \mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\phi(k)$$

$$\frac{\partial^2 h(k)}{\partial k^2} > 0 \Leftrightarrow \psi(k) < \frac{s\,(s-1)^2\,\delta A}{\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}}$$

With:

$$\psi(k) = \frac{(1+n-\mu)\,\eta\,(2-\lambda)\,(1-2s+\lambda s)}{\lambda A\,(1-\lambda)}k^{\frac{2\lambda s-\lambda-3s+2}{1-\lambda}} - \frac{(\lambda\delta+1-\delta)\,(1-s)\,(\lambda-s)}{\lambda\,(1-\lambda)^2}k^{\frac{1-2s+\lambda s}{1-\lambda}}$$

This inequation can be solved with a graphical argument by studying the function $\psi(k)$. Knowing that $\frac{2\lambda s-\lambda-3s+2}{1-\lambda} > \frac{1-2s+\lambda s}{1-\lambda} > 0$ it is easy to check that:

$$\psi(0) = 0 \qquad \lim_{k\to+\infty}\psi(k) = \infty$$

$$\begin{aligned} \frac{\partial\psi(k)}{\partial k} =& \frac{(2\lambda s-\lambda-3s+2)\,(1+n-\mu)\,\eta\,(2-\lambda)\,(1-2s+\lambda s)}{\lambda A\,(1-\lambda)^2}k^{\frac{2\lambda s-\lambda-3s+2}{1-\lambda}-1} \\ &- \frac{(1-2s+\lambda s)\,(\lambda\delta+1-\delta)\,(1-s)\,(\lambda-s)}{\lambda\,(1-\lambda)^3}k^{\frac{1-2s\lambda s}{1-\lambda}-1} \end{aligned}$$

$$\frac{\partial\psi(k)}{\partial k} > 0 \Leftrightarrow \begin{cases} k > 0 & \text{if } \lambda < s \\ k > \left(\frac{(1-2s+\lambda s)(\lambda\delta+1-\delta)(1-s)(\lambda-s)A}{(2\lambda s-\lambda-3s+2)(1+n-\mu)\eta(2-\lambda)(1-2s+\lambda s)}\right)^{\frac{1}{1-s}} & \text{if } \lambda > s \end{cases}$$

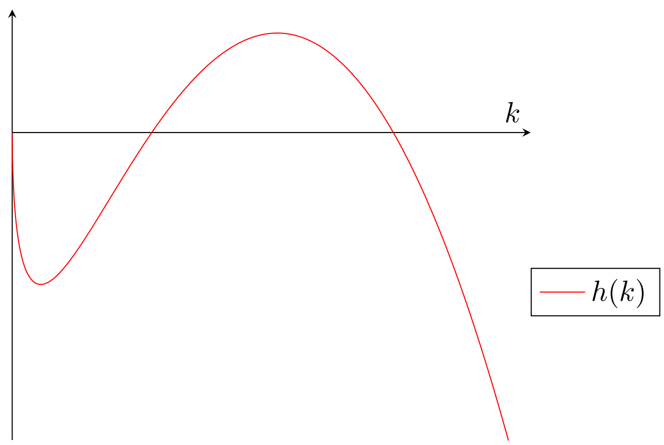


Figure 3: Representation of $h(k)$

It is easy to understand that in the two cases, $\psi(k)$ is firstly lower than $\frac{s(1-s)^2\delta A}{\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}}$, and then greater, so the function $h(k)$ is convex and then concave. The implication is that it exists exactly two positive solutions to the equation $h(k) = 0$. Indeed, we are able to prove that $h(k)$ starts from zero with a negative slope, is decreasing, convex, then becomes increasing and concave. It cuts the axis of abscissa before becoming decreasing, and so cuts again the axis of abscissa and tends to $-\infty$. □

The function $h(k)$ is represented by figure 3. It exists so two possible steady states. In order to overcome strong income effect due to remittances which can imply negative stock of capital and negative labor supply ($\overline{k}$ is also positive if $K < 0$ and $L < 0$) it is necessary to check that saving is positive to define a solution of equation (11) as a steady state. Indeed, remittances could steer agent to borrow in first period to "buy" leisure (which is not possible because $l < 1$) and repay the next period. The stock of capital is positive if the saving function, $s(k)$ defined by equation (5), is positive.

Firstly, one has :

$$s(0) = 0 \qquad\qquad \lim_{k\to+\infty} s(k) = -\infty$$

$$s(k) > 0 \Leftrightarrow k < \left(\frac{\delta\,(1-s)\,\lambda A}{\mu\,(\lambda\delta+1-\delta)\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}}\right)^{\frac{1-\lambda}{1+\lambda s-2s}} \equiv k_{max}$$

It is obvious that k cannot be greater than k_{max}, in order to have positive saving and capital accumulation. By substituting k_{max} into $h(k)$ defined in equation (11), one obtains:

$$h(k_{max}) = (1+n-\mu)\left((1-\eta)\left(\frac{\delta\,(1-s)\,\lambda A}{\mu\,(\lambda\delta+1-\delta)\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}}\right)^{\frac{1-\lambda}{1+\lambda s-2s}}\right.$$
$$\left. -\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\left(\frac{\mu\eta\,(1-\lambda)}{\lambda\,(1-s)\,A}\right)\left(\frac{\delta\,(1-s)\,\lambda A}{\mu\,(\lambda\delta+1-\delta)\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}}\right)^{\frac{(1-s)(2-\lambda)}{1-2s+\lambda s}}\right)$$

One verifies: $h(k_{max}) > 0 \Leftrightarrow \lambda\delta + 1 - \eta - \delta > 0 \Leftrightarrow \lambda\delta + \epsilon > 0$
One checks that if $k = k_{max}$, thus $s(k_{max}) = 0$, and the curve $h(k)$ representing the equilibrium

is positive. One concludes that $K > 0$ in the first solution of equation (11), and $K < 0$, $L < 0$ in the second solution. It exists therefore one unique steady state in this economy with positive capital accumulation which is the first intersection of the curve $h(k)$ to the abscissa axis when the derivative with respect to k is positive.

Proof of Proposition 3.4

The trace of the Jacobian matrix is the following:

$$\frac{-\frac{\partial h(k_{t+2},k_{t+1},k_t)}{\partial k_{t+1}}}{\frac{\partial h(k_{t+2},k_{t+1},k_t)}{\partial k_{t+2}}}\left(\overline{k^r}\right) =$$
$$\frac{(1+n-\mu)(1-\eta)+\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\left(\frac{(1-s)(\lambda\delta+1-\delta)}{\lambda(1-\lambda)}k^{\frac{\lambda-s}{1-\lambda}}-\frac{(1+n-\mu)\eta(1-\lambda)}{\lambda A}k^{\frac{1-2s+\lambda s}{1-\lambda}}\right)}{\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\left(\frac{(1+n-\mu)\eta}{\lambda A}k^{\frac{1-2s+\lambda s}{1-\lambda}}\right)}$$

And the determinant is:

$$\frac{\frac{\partial h(k_{t+2},k_{t+1},k_t)}{\partial k_t}}{\frac{\partial f(k_{t+2},k_{t+1},k_t)}{\partial k_{t+2}}}\left(\overline{k^r}\right) = \frac{s\delta(1-s)Ak^{s-1}}{\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\left(\frac{(1+n-\mu)\eta}{\lambda A}k^{\frac{1-2s+\lambda s}{1-\lambda}}\right)}$$

One studies first $P(0)$ and $P(-1)$ for the positive steady state :

$$P(0) = \frac{s\delta(1-s)Ak^{\frac{3s+\lambda-2+2\lambda s}{1-\lambda}}}{\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\left(\frac{(1+n-\mu)\eta}{\lambda A}\right)} > 0\ \forall\, k > 0$$

$$\begin{aligned}P(-1) =& 1+\frac{s\delta(1-s)Ak^{s-1}+(1+n-\mu)(1-\eta)}{\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\left(\frac{(1+n-\mu)\eta}{\lambda A}k^{\frac{1-2s+\lambda s}{1-\lambda}}\right)}\\ &+\frac{\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\left(\frac{(1-s)(\lambda\delta+1-\delta)}{\lambda(1-\lambda)}k^{\frac{\lambda-s}{1-\lambda}}-\frac{(1+n-\mu)\eta(1-\lambda)}{\lambda A}k^{\frac{1-2s+\lambda s}{1-\lambda}}\right)}{\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\left(\frac{(1+n-\mu)\eta}{\lambda A}k^{\frac{1-2s+\lambda s}{1-\lambda}}\right)}\end{aligned}$$

One remarks that $P(0)$ is the determinant of the Jacobian matrix and is positive for all positive equilibrium. After some algebra one obtains that $P(-1) > 0$ if the following inequation is satisfied.

$$\begin{aligned}P(-1) > 0 \Leftrightarrow & s\delta(1-s)Ak^{s-1}+(1+n-\mu)(1-\eta)+\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\\ &\times\left(\frac{(1-s)(\lambda\delta+1-\delta)}{\lambda(1-\lambda)}k^{\frac{\lambda-s}{1-\lambda}}+\frac{(1+n-\mu)\eta}{A}k^{\frac{1-2s+\lambda s}{1-\lambda}}\right) > 0\end{aligned}$$

Finally, one computes $P(1)$:

$$\begin{aligned}P(1) =& 1+\frac{s\delta(1-s)Ak^{s-1}-(1+n-\mu)(1-\eta)}{\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\left(\frac{(1+n-\mu)\eta}{\lambda A}k^{\frac{1-2s+\lambda s}{1-\lambda}}\right)}\\ &-\frac{\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\left(\frac{(1-s)(\lambda\delta+1-\delta)}{\lambda(1-\lambda)}k^{\frac{\lambda-s}{1-\lambda}}-\frac{(1+n-\mu)\eta(1-\lambda)}{\lambda A}k^{\frac{1-2s+\lambda s}{1-\lambda}}\right)}{\mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}\left(\frac{(1+n-\mu)\eta}{\lambda A}k^{\frac{1-2s+\lambda s}{1-\lambda}}\right)}\end{aligned}$$

$$P(1) > 0 \Leftrightarrow s\delta\,(1-s)\,Ak^{s-1} - (1+n-\mu)\,(1-\eta) + \mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}$$
$$\times\left(\frac{(1+n-\mu)\,(2-l)\,\eta}{\lambda A}k^{\frac{1-2s+\lambda s}{1-\lambda}} - \frac{(1-s)\,(\lambda\delta+1-\delta)}{\lambda\,(1-\lambda)}k^{\frac{\lambda-s}{1-\lambda}}\right) > 0$$

The sign of $P(1)$ is not directly identifiable. Nevertheless, by taking the expression of the equilibrium curve derivative, one has:

$$\frac{\partial h(k)}{\partial k} = (1+n-\mu)\,(1-\eta) - s\delta\,(1-s)\,Ak^{s-1} + \mu\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}}$$
$$\times\left(\frac{(\lambda\delta+1-\delta)\,(1-s)}{\lambda\,(1-\lambda)}k^{\frac{\lambda-s}{1-\lambda}} - \frac{(1+n-\mu)\,\eta\,(2-\lambda)}{\lambda A}k^{\frac{1+\lambda s-2s}{1-\lambda}}\right)$$

The sign of $P(1)$ is equivalent to the sign of $-\frac{\partial h(k)}{\partial k}$, and according to figure 3, the sign of the derivative at the unique steady state is positive implying that $P(1)$ is negative at the steady state with $\overline{k}^r \in \left]0; \overline{k}^{wr}\right]$.

As $P(0) > 0$ and $P(1) < 0$, the first root of the Jacobian Matrix is inside the unit circle and the other outside, implying a unique saddle path. Amount of receiving remittances has not impact on stability of the unique positive equilibrium.

Analysis is the same for the trivial steady state.

$$\lim_{k\to 0^+} \text{Det} = +\infty > 1 \qquad \lim_{k\to 0^+} \text{Tr} = +\infty$$

One computes:

$$\lim_{k\to 0^+} P(1) = \lim_{k\to 0^+} \text{Det} = +\infty$$
$$\lim_{k\to 0^+} P(-1) = \lim_{k\to 0^+} \text{Det} = +\infty$$

Consequently, the trivial steady state is a source because $P(0) > 1$, $P(-1) > 0$, $P(1) > 0$, roots are outside the unit circle implying an unstable trivial steady state.

Proof of Proposition 3.5

One easily see that $h(\overline{k}^{wr}) > 0$. This fact implies under lemma 5 and figure 3 that $h(\overline{k}^r) = 0 \Leftrightarrow \overline{k}^r < \overline{k}^{wr}$. The implication is that the capital per head in this framework is lower in economies receiving remittances.

Proof of Proposition 3.6

In order to evaluate impact, we can see how $\overline{k}^r$ varies with respect to the parameter γ. As we have not mathematical expression of this unique steady state, it is possible to evaluate the sign of impact of remittances parameter by using the implicit function theorem related to the equilibrium equation (11) defined here as $h(\overline{k}^r, \gamma)$:

$$\frac{\mathrm{d}\overline{k}^r}{\mathrm{d}\gamma} = \frac{-\frac{\partial h(\overline{k}^r,\gamma)}{\partial\gamma}}{\frac{\partial h(\overline{k}^r,\gamma)}{\partial\overline{k}^r}} < 0$$

With:

$$\frac{\partial h(\overline{k}^r,\gamma)}{\partial\gamma} = \left(\frac{\mu\alpha\lambda}{sA\,(1-\lambda)}\right)\left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{\lambda}{1-\lambda}}$$
$$\times\left(\frac{\lambda\delta+1-\delta}{\lambda}k^{\frac{1-s}{1-\lambda}} - \frac{(1+n-\mu)\,\eta\,(1-\lambda)}{\lambda\,(1-s)\,A}k^{\frac{(1-s)(2-\lambda)}{1-\lambda}}\right)$$

$$\frac{\partial h(\overline{k}^r,\gamma)}{\partial \overline{k}^r} = (1+n-\mu)(1-\eta) - s\delta(1-s)A\overline{k}^{r^{s-1}} + \left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}} \times \mu\left(\frac{(\lambda\delta+1-\delta)(1-s)}{\lambda(1-\lambda)}\overline{k}^{r^{\frac{l-s}{1-\lambda}}} - \frac{(1+n-\mu)\eta(2-\lambda)}{\lambda A}\overline{k}^{r^{\frac{1+\lambda s-2s}{1-\lambda}}}\right)$$

It is easy to see that $\frac{\partial h(\overline{k}^r,\gamma)}{\partial \overline{k}^r} > 0 \Leftrightarrow \overline{k}^r < k_3^r$. By knowing that $\overline{k}^r < \overline{k}^{wr} < k_3^r$, this derivative is positive. Using the lemma 5 and figure 3 allows to prove that $\frac{\partial h(\overline{k}^r,\gamma)}{\partial \overline{k}^r} > 0$ implying that a rise of γ decreases the value of $h(\overline{k}^r)$. The proof is the same for parameter α.

Proof of Lemma 1

In an economy without remittances, the stationary utility at the golden rule is given by:

$$\widehat{U} = \epsilon \ln\left[\epsilon A(1-s)\left(\frac{sA}{1+n-\mu}\right)^{\frac{s}{1-s}}\right] + \eta\ln\eta + \delta\ln\left[\delta(1+n-\mu)A(1-s)\left(\frac{sA}{1+n-\mu}\right)^{\frac{s}{1-s}}\right]$$

In a recipient economy, this stationary utility at the golden rule becomes:

$$\begin{aligned}\widehat{U} =& \epsilon\ln\left[\epsilon\left(A(1-s)\left(\frac{sA}{1+n-\mu}\right)^{\frac{s}{1-s}} + \frac{\mu(1-\lambda)\left(\frac{\gamma\alpha\lambda}{1+n-\mu}\right)^{\frac{1}{1-\lambda}}}{\lambda}\right)\right] \\ &+ \eta\ln\left[\eta\left(1+\frac{\mu(1-\lambda)\left(\frac{\gamma\alpha\lambda}{1+n-\mu}\right)^{\frac{1}{1-\lambda}}}{\lambda A(1-s)\left(\frac{sA}{1+n-\mu}\right)^{\frac{s}{1-s}}}\right)\right] \\ &+ \delta\ln\left[\delta(1+n-\mu)\left(A(1-s)\left(\frac{sA}{1+n-\mu}\right)^{\frac{s}{1-s}} + \frac{\mu(1-\lambda)\left(\frac{\gamma\alpha\lambda}{1+n-\mu}\right)^{\frac{1}{1-\lambda}}}{\lambda}\right)\right]\end{aligned}$$

It is easy to see that the utility is higher in the second case.

Proof of Lemma 2

The government budget constraint is such that the sum of the amount dedicated to workers and the amount dedicated to retired is equal to zero implying that public deficit is not allowed. Mathematically, this gives for the period t: $N_t^w\tau_w + N_{t-1}^w\tau_r = 0$ By evaluating this equation in the long run, this gives: $\tau_w + \frac{\tau_r}{1+n-\mu} = 0$

Concerning the agent, the maximization of the decentralized program gives the expressions for decentralized education, saving and labor supply:

$$e_t^d = \left(\frac{\gamma\alpha\lambda}{R_{t+1}}\right)^{\frac{1}{1-\lambda}} \tag{16}$$

$$s_t^d = \delta(w_t-\tau_w) + \frac{(1-\delta)\tau_r}{R_{t+1}} - \frac{\mu(\lambda\delta+\eta+\epsilon)}{\lambda}\left(\frac{\gamma\alpha\lambda}{R_{t+1}}\right)^{\frac{1}{1-\lambda}} \tag{17}$$

$$l_t^d = 1-\eta+\frac{\eta\tau_w}{w_t} + \frac{\eta\tau_r}{w_tR_{t+1}} - \frac{\mu\eta(1-\lambda)}{\lambda w_t}\left(\frac{\gamma\alpha\lambda}{R_{t+1}}\right)^{\frac{1}{1-\lambda}} \tag{18}$$

As previously, the decentralized equilibrium (k^d) is the solution of the following equation.

$$k_{t+1}^d \left[(1+n-\mu)\, l_{t+1}^d \right] = s_t^d$$

By replacing expression of decentralized variable, the decentralized equilibrium evaluated in the long run is solution of:

$$\begin{aligned}
&(1+n-\mu)(1-\eta)\, k^d - \delta(1-s)\, A \left(k^d\right)^s + \mu \left(\frac{\gamma\alpha\lambda}{sA}\right)^{\frac{1}{1-\lambda}} \\
&\times \left(\frac{\lambda\delta + 1 - \delta}{\lambda} \left(k^d\right)^{\frac{1-s}{1-\lambda}} - \frac{(1+n-\mu)\,\eta\,(1-\lambda)}{\lambda(1-s)\,A} \left(k^d\right)^{\frac{(1-s)(2-\lambda)}{1-\lambda}} \right) \\
&+ \tau_w \left(\delta + \frac{(1+n-\mu)\,\eta}{(1-s)\,A} \left(k^d\right)^{1-s} \right) + \tau_r \left(\frac{(1+n-\mu)\,\eta}{(1-s)\, sA^2} \left(k^d\right)^{2-2s} - \frac{1-\delta}{sA} \left(k^d\right)^{1-s} \right) = 0 \qquad (19)
\end{aligned}$$

The optimal policy for the government consists to determine τ_w and τ_r such that all variables are optimal and the government budget constraint is satisfied. In other terms, the decentralized equilibrium is optimal is the following system is satisfied:

$$\begin{cases}
k^d = \widehat{k} & (20) \\
e^d = \widehat{e} & (21) \\
l^d = \widehat{l} & (22) \\
\tau_w = -\dfrac{\tau_r}{1+n-\mu} & (23)
\end{cases}$$

It is easy to check that if equation (20) is satisfied, then equation (21) is satisfied and if equation (20) and (23) are satisfied, then equation (22) is also satisfied. Inded, equation (22) is satisfied when equation (20) is satisfied if $\frac{\eta\tau_w}{w_t} + \frac{\eta\tau_r}{w_t R_{t+1}} \Leftrightarrow \tau_w + \frac{\tau_r}{1+n-\mu} = 0$.

Hence, the decentralized equilibrium is optimal if $k^d = \widehat{k}$ and the government budget constraint is satisfied. The appropriate system is then the following:

$$\begin{cases}
(1+n-\mu)(1-\eta)\left(\dfrac{sA}{1+n-\mu}\right)^{\frac{1}{1-s}} - \delta(1-s)\,A\left(\dfrac{sA}{1+n-\mu}\right)^{\frac{s}{1-s}} + \tau_w\left(\delta + \dfrac{\eta s}{1-s}\right) & \\
+\dfrac{\tau_r}{1+n-\mu}\left(\dfrac{\eta s}{1-s} - 1 - \delta\right) + \mu\left(\dfrac{\gamma\alpha\lambda}{1+n-\mu}\right)^{\frac{1}{1-\lambda}}\left(\dfrac{\lambda\delta\eta+\epsilon}{\lambda} - \dfrac{\eta s(1-\lambda)}{\lambda(1-s)}\right) = 0 & (24) \\
\tau_w = -\dfrac{\tau_r}{1+n-\mu} & (25)
\end{cases}$$

This is a simple system with 2 equations and 2 variables, implying than only lump-sum taxes and transfers can bring the economy to the golden rule.

Proof of Proposition 4.2

By inserting equation (25) in equation (24), we directly obtain after simplification:

$$h(\widehat{k}) + \tau_w = 0 \Leftrightarrow \tau_w = -h(\widehat{k})$$

By using equation (25), we obtain $\tau_r = h(\widehat{k})\,(1+n-\mu)$

We know that if $\gamma < \widehat{\gamma}$ then $h(\widehat{k}) > 0$ implying that $\tau_w = -h(\widehat{k}) < 0$.

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