Stereotypes, Underconfidence and Decision Making with an Application to Gender and Math*

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Abstract

We study the effects of the presence of a negative stereotype on the formation of self-confidence and on decision making, taking into account ego and self-esteem issues.
We show that any stereotype of lower ability leads to gaps in confidence, in participation in risky/ambitious options and in performance. Furthermore, we show how the stereotype survives and even gets reinforced. Considering gender and mathematics, we are able to explain girls lower self-confidence in mathematics, girls underrepresentation in STEM fields, as well as girls lower performance and choices of less ambitious options, especially at the right side of the ability distribution.

JEL: D03, D81, D84, J16, I24

1 Introduction

Boys outnumber girls in math intensive fields. For example, in the US, only 27% of bachelor degrees in math intensive fields go to women, and only 19% of Ph.D.s go to women. The gender gap widens after graduation. In academia for instance, in the top US universities, only 12% of tenure track and tenured positions in math intensive fields are occupied by women and among full professors, women are 8%[1].

This gender gap in participation rates persists controlling for ability which excludes ability-based explanations. As noted in Niederle and Vesterlund [2010][2], "among equally gifted students, males are many times more likely to select college majors that are considered to be high in math content". In France, girls are only 45% in the scientific section S of the baccalauréat[3], only 30% in scientific university degrees and only 23% in scientific preparatory classes to elite schools (scientific CPGE), even though girls obtain equivalent (or slightly better) grades at both moments of choice, i.e., when choosing their section of the baccalauréat, and when choosing their orientation after high school[4].

Another striking feature is the fact that girls are less self-confident than boys in mathematics, in the sense that they are less optimistic about their

[1]Each cited percentage is the average among all math intensive fields in Table 11 of Nelson and Brammer [2010].

[2]See also Weinberger [2005] who shows that white women enter math related fields at no more than half the rate of men with the same mathematics test scores. In Buser et al. [2014], students exhibit a "significant gender gap in math intensity of their chosen profiles controlling for objective academic performance".

[3]Analogous figures hold for the Netherlands, Denmark, Switzerland and Germany.

[4]All the figures are from the 2013 publication of "Filles et garçons sur le chemin de l'égalité, de l'école à l'enseignement supérieur" of the French Department of Education.

abilities in mathematics[5]. Buser et al. [2014], for instance, show that even though there is no significant difference in grades between boys and girls in their sample, girls claim that it is more difficult for them to obtain good grades and guess lower ranks for themselves. In French upper high schools, with equivalent results and same background in mathematics, 82% of boys and 53% of girls think that they are good enough to pursue scientific studies[6].

It has been argued in the literature that this lower self-confidence of girls was the reason for their lower self-selection in math intensive and selective fields. Following this argument, since girls need to be more able than boys to reach the same level of confidence, then only the most able girls should self-select. As underlined in, e.g., Wozniak et al. [2014] "a female would have to improve her percentile rank by 34% to be as likely to enter the tournament as a male". This should lead to a reduced gender gap in participation for high ability levels and to a better performance of girls who participate compared to boys.

However, there is no evidence of such a reduction of the gender gap in participation for high ability levels; on the contrary, the gender gap seems to be higher for high ability levels. Niederle and Vesterlund [2007], Niederle and Yestrumskas [2008] and Niederle et al. [2013], for instance, get in their experiments that men's participation (in the competition or in the challenging tasks) increases with ability while there is no significant relationship between ability and participation for women. More directly related to the issue of gender and maths, Ellison and Swanson [2010] analyze the participation and performance of boys and girls at American mathematics competitions and

[5]See, among others, Eccles [1984], Lundberg et al. [1994], Eccles [1998], Ülkü Steiner et al. [2000], Correll [2001], Goodman et al. [2002], OECD [2004], Buser et al. [2014].

[6]As a source see http://www.femmes-ingenieurs.org/index.php/promotions-des-fi/faqz.

underline that there is a substantial gender-related selection effect, able girls self-selecting less than able boys.

Moreover, there is a gender gap in performance, boys outperforming girls. In almost all OECD countries, average PISA scale scores in mathematics are (slightly) higher for males than for females. This performance gap widens as we move up the ability distribution. At the SAT, boys outperform girls in maths on average by around 30 points[7] and the proportion of girls scoring at a given level decreases with the scores, from around 60% for the lowest scores (below 500) to around 30% for the top scores (800), the proportion dipping below 50% around the 30th percentile [see also Hedges and Nowell, 1995, Xie and Shauman, 2003, Ellison and Swanson, 2010, for analogous findings].

This paper aims at providing a theoretical model that improves the understanding of the gender gaps (in confidence, in participation, in performance) and why they persist. We rely on a model of (optimal) beliefs formation and choices in the vein of models with ex-ante savoring and ex-post disappointment[8], taking into account not only consumption utility but also ego utility ex-ante, and self-esteem ex-post, and adding a stereotype component. We take as granted the existence of a stereotype of lower ability in maths concerning girls [see, e.g., Correll, 2001, Rudman et al., 2001, Kiefer and Sekaquaptewa, 2007, Reuben et al., 2014], inducing a biased/stereotyped attribution of success and failure in mathematics [see Parsons et al., 1976, Betz and Hackett, 1981, Dweck, 1986, Lupart et al., 2004, Dickhäuser and Meyer, 2006, Koch et al., 2008]: when performance is consistent with the stereotype, it is more easily interpreted in terms of ability, and boys will have their successes more easily at-

[7]This gap was of the same magnitude 40 years ago.

[8]See Gollier and Muermann [2010] and also Loewenstein and Linville [1986], Karlsson et al. [2004], Jouini et al. [2014], Macera [2014].

tributed to ability and on the contrary, girls will have their failures more easily attributed to lack of ability. We justify in detail this assumption in Section 2 through empirical evidence and through social psychology theories like in particular, expectations states theory and status characteristics theory [Berger et al., 1972, Foschi, 1996, Ridgeway, 2001]. We assume that boys and girls have the same distribution of ability. However, the presence of the stereotype, in the form of biased interpretation of success and failure in terms of ability, exposes boys and girls to different self-esteem risk. These different self-esteem risks for boys and girls generate different self-esteem protection strategies. The underconfidence of girls as well as their less ambitious/challenging, less risk tolerant or competitive choices are their best response to their self-esteem threat. By adopting modest confidence levels and by choosing less ambitious options, girls reduce the future psychological risk of disappointment and of self-esteem loss if reality does not keep up with expectations. Besides, we show that differences in self-esteem risk also generate differences in the nature of the relation between ability and self-selection in difficult tasks. The relation is more increasing for boys than for girls, in the sense that more able boys self-select more in difficult options whereas highly able girls self-select less than less able girls. This gender selection effect in challenging options leads to a higher participation gap for high ability levels and to a better ability conditional on participation, i.e., a better performance, of boys, especially for high ability levels. This self-selection bias is an important feature of our approach; even if the same (ex-ante) ability is assumed, the mere presence of the stereotype leads to girls' underperformance.

Note that we do not make the assumption that girls personally endorse the stereotype and in our model, the reason for girls' lower confidence is not that they adopt the collective belief of lower ability of girls. However, we make the assumption that girls take into account the existence of the stereotype of lower ability, and more precisely, its impact on how their future performance will be judged[9]: girls anticipate that they have a lower "right to fail" or higher self-esteem costs from failure, and protect their future self-esteem by adopting lower self-confidence levels.

To sum up, the static part of our model shows that the presence of a stereotype (in the form of biased interpretation of successes and failures in terms of ability) leads girls to adopt lower levels of confidence and to self-select less in math difficult options, especially for the most competent, which leads to the underperformance of girls in mathematics, especially at the right tail of the distribution. Our model explains the three gender gaps (in confidence, in participation, in performance) as well as their main features.

We then turn to dynamic considerations to analyze how stereotypes of lower/higher ability (or status hierarchies) can survive. As underlined above, if underconfidence were the main reason for the gender gap in mathematics, then only the most able girls should participate, leading to their overperformance; this, in turn, should lead more girls to participate and should lead to the reduction of the negative stereotype of lower ability. On the contrary, we show that in our model, the stereotypes of lower ability of girls and of higher ability of boys survive and even increase. More precisely, we consider a dynamics on the stereotypes of lower/higher ability such that the good (resp. bad) performance of one group relative to the other has a positive (resp. negative) impact on the stereotype concerning the given group. We show that this

[9] These assumptions are in line with what Ridgeway [2001] notes about status beliefs: "Because individuals expect others to judge them according to these beliefs, they must take status beliefs into account in their own behavior, whether or not they personally endorse them."

dynamics converges to a situation where the stereotype is stronger. An initial gap in the groups status or in the collective expectations of ability, one group being considered as less able than the other group, leads to a higher gap in the groups status. Our model then provides some new insight on a possible channel through which status hierarchies can survive and even get reinforced.

Our approach shares similarities with Mechtenberg [2009], one of the few theoretical approaches to the issue of gender and mathematics, that also leads to boys outperforming girls in maths and sciences and to the predominance of male students in maths at the university. However, while Mechtenberg's model relies on biased grading and on a cheap talk model of teachers and students, we rely on a stereotype model. Our model provides socio-cultural foundations for the gender gaps in mathematics and for their survival. It is fully in line with Correll [2001], who measures the extent to which cultural beliefs about gender and mathematics bias the formation of self-assessments of competence and contribute to the gender gap in careers in science, mathematics and engineering. It is also in the spirit of Leslie et al. [2015] who show empirically that girls underrepresentation in some given fields can be explained by the field specific talent hypothesis, girls being stereotyped as not possessing talent for some disciplines that include math.

Our approach is different from the recent work of Bordalo et al. [2014] on stereotypes. In their approach, stereotypes emerge and some groups are considered and consider themselves as less able, hence participate less in difficult options, because of an exaggeration of differences in objective ability distributions (due to imperfect recall and to a representativeness heuristic). In our approach, we assume no ex-ante difference in ability, we do not analyze how the stereotype emerges but its implications in terms of beliefs and decision making as well as how it survives.

The same approach can be applied to gender issues in general since there is a stereotype of lower ability of women in many fields and women enjoy a lower status than men in general [Foschi, 1996, Fiske et al., 2002]. We then obtain girls' lower self-confidence in general, as well as their less challenging choices, which can help explain the lower competitiveness of girls, their underrepresentation in most selective tracks and at the top of the labor market hierarchy, as well as the wage gap. Our approach can also explain women's reluctance to speak up or to contribute ideas in male-stereotyped fields [Coffman, 2014]. More generally, our approach can also be applied to any negatively stereotyped or low status group.

The paper is organized as follows. Section 2 presents the model. Section 3 deals with confidence levels and Section 4 with choices. Section 5 considers performance issues as well as stereotype dynamics. Section 6 summarizes and discusses the results about gender and maths and their generalization to other stereotypes. Section 7 presents numerical results that match all the above mentioned stylized facts and Section 8 concludes. All proofs are in Appendix A and Appendix B extends the model to effort.

2 The Model

Let us present our model with ego, self-esteem and stereotypes. We rely on models with ex-ante savoring and ex-post disappointment like Karlsson et al. [2004], Gollier and Muermann [2010], Jouini et al. [2014] adding a stereotype component. We consider an individual, who is confronted with a risky situation involving his ability. There are two dates, denoted by date 0 and date 1. The risky situation is represented by a random variable $\widetilde{x}$ with two possible outcomes at date 1: $\widetilde{x} = x_l$, representing failure and $\widetilde{x} = x_h$, representing success, with $0 \leq x_l \leq x_h \leq 1$. We can think of $\widetilde{x}$ as representing graduation risk for instance or any pass or fail exam. We let p denote the individual's objective probability of success; the individual's objective expectation of performance is then given by $E[\widetilde{x}] = px_h + (1-p)\,x_l \in [x_l, x_h]$. We assume that the individual's perception of the probability of success might differ from the objective one, and we let $y \in [x_l, x_h]$ denote the individual's subjective expectation of performance.

At date 0, the individual has ego utility [Kőszegi, 2006, Weinberg, 2009], in the form of an increasing function $v(y)$ of his subjective expectation of performance. Indeed, a high subjective expectation of performance increases the individual's satisfaction at date 0 by increasing his feeling of personal capacity. At date 1, the individual's utility depends upon the realized outcome $x \in \{x_l, x_h\}$, upon the support of the risky option $\{x_l, x_h\}$, upon the expectation of performance y and upon the intensity of the negative stereotype $\lambda \in [0,1]$ and is given by $u(x) + \varphi(x, y, \lambda)$. The first component $u(x)$ is standard 'consumption' or outcome utility. The second component $\varphi(x, y, \lambda)$ represents self-esteem utility. We let $W(y)$ denote the individual's intertemporal well-being

$$W(y) = v(y) + E[u(\widetilde{x})] + E[\varphi(\widetilde{x}, y, \lambda)]\,.$$

We make the following assumptions.

Assumption (A1): *u is increasing and strictly concave with $u_{yy} < 0$, $u(0) = 0$ and $v = ku$ for $k \in \mathbb{R}_+^*$.*

Assumption (A2): $\varphi(x_l, y, \lambda) \leq 0, \varphi(x_h, y, \lambda) \geq 0$.

Assumption (A3): $\varphi_y \leq 0, \varphi_{yy} \leq 0$.

Assumption (A4): $\varphi_y(x_h, y, \lambda) \geq \varphi_y(x_l, y, \lambda)$.

Assumption (A5): $\varphi_\lambda \leq 0$.

Assumption (A6): $\varphi_{y\lambda}(x_l, y, \lambda) \leq 0, \varphi_{y\lambda}(x_h, y, \lambda) \geq 0$.

The assumptions on u in $(A1)$ are standard. The assumption on $u(0)$ is a simplifying assumption. So is the assumption on v, that permits to analyze the impact of the weight on ego feelings. $(A2)$ amounts to assuming that success impacts self-esteem positively and failure impacts self-esteem negatively. $(A3)$ is an elation/disappointment condition (see Bell, 1985, Loomes and Sugden, 1986, Gul, 1991 for models of disappointment, and Kőszegi, 2006 for reference-dependent models): the ex-ante expectation y plays the role of a reference level for one's self-esteem and the higher the ex-ante expectations, the lower the self-esteem benefit. It is in line with the conception of self-esteem as the ratio of one's successes to one's expectations in James [1890][10], Diener et al. [1991], Mellers and McGraw [2001]. Moreover, we make the assumption $(A4)$ that disappointment effects are more important than elation effects; there is strong empirical support for this assumption [see, e.g., Mellers et al., 1997, 1999, Mellers and McGraw, 2001].

The last two assumptions are about the impact of stereotypes. We assume that stereotypes impact the way success and failure are interpreted in terms of ability. According to status characteristics theory and expectations states theory (Berger et al., 1972, Foschi, 1996[11], Ridgeway, 2001), an individual who is considered as less able (i.e., a negatively stereotyped individual or a low status individual) will have his failures attributed to lack of ability

[10]William James proposed that self-esteem could be expressed as the ratio of one's successes to one's expectations. According to Leary [1999], self-esteem is the relation between one's real self and one's ideal self. Other theorists have made similar observations.

[11]Foschi [1996] for instance argues that the standards used to determine if a given performance is indicative of ability are a function of the status of the individual.

and his successes attributed externally, to luck or effort. The reason is that for an individual who is expected to be less able, failure is consistent with expectations but success is not. The opposite holds for an individual who is considered as more able, who will have his successes attributed to ability and his failures attributed externally to lack of work or lack of luck. In line with these arguments, $(A5)$ and $(A6)$ assume that when the negative stereotype is higher, there is less self-esteem benefit and less marginal elation from success (since it is more externally attributed) and more self-esteem loss and more marginal disappointment from failure (since it is more attributed to lack of ability). Concerning gender and maths, there is evidence of the existence of a stereotype of lower ability in maths concerning girls [see, e.g., Correll, 2001, Rudman et al., 2001, Kiefer and Sekaquaptewa, 2007, Reuben et al., 2014]. There is also evidence of stereotyped attribution of success and failure [Parsons et al., 1976, Betz and Hackett, 1981, Eccles, 1983, Eccles and Jacobs, 1986, Dweck, 1986, Jacobs and Weisz, 1994, Tiedemann, 2000, Lupart et al., 2004, Dickhäuser and Meyer, 2006, Koch et al., 2008]. For instance, for Yee and Eccles [1988], mothers think that talent is a more important explanation for boys' math successes while effort is a more important explanation for girls' math successes.

We shall sometimes introduce the following assumption:

Assumption (B): a. $\varphi(x_l, y, 0) = 0$, **b.** $\varphi(x_h, y, 1) = 0$.

We know by (A5) that the self-esteem benefit from success decreases with the negative stereotype and that the loss from failure increases with the stereotype, and Assumption (B) adds that there is no self-esteem benefit from success when the negative stereotype is maximal, and no self-esteem loss from failure when the positive stereotype is maximal.

In our model, the individual faces a self-esteem risk $\varphi(\widetilde{x}, y, \lambda)$ at date 1, in addition to the standard consumption risk $u(\widetilde{x})$. Under our assumptions, self-esteem risk is higher (in the sense of first-order stochastic dominance[12]) for individuals with higher levels of λ: the negative stereotype increases self-esteem risk. Moreover, self-esteem risk increases with the individual's expectation of performance y.

As in Brunnermeier and Parker [2005], we assume that individuals optimally choose their subjective expectations. More precisely, we assume that the individual adopts the subjective expectation of performance y^* in $[x_l, x_h]$ that maximizes his intertemporal well-being[13] $W(y)$, i.e., solves

$$y^* \equiv \arg \max_{y \in [x_l, x_h]} W(y) = \arg \max_{y \in [x_l, x_h]} v(y) + E[\varphi(\widetilde{x}, y, \lambda)]. \tag{1}$$

The individual facing the risky situation $\widetilde{x}$ is then endowed with the well-being level $W(y^*)$. The individual's optimal expectation of performance or optimal self-confidence realizes the best trade-off between today's ego utility and tomorrow's self-esteem; it characterizes the level of ex-ante ego utility that the individual is ready to sacrifice to reduce future self-esteem risk, i.e., the

[12]We recall the definition of first-order stochastic dominance. A random variable X dominates a random variable Y in the sense of FSD if and only if the cumulative distribution functions satisfy $F_X \leq F_Y$.

[13]In our model, as in e.g., Akerlof and Dickens [1982], Brunnermeier and Parker [2005], Gollier and Muermann [2010], individuals use *objective* probabilities to evaluate future utility while experiencing ego utility related to *subjective* probabilities; indeed, optimal levels of confidence are those that maximize the individual's satisfaction on average across realizations of uncertainty, and uncertainty unfolds according to objective probability. This "schizophrenic" behavior is consistent with the existence of parallel cognitive and emotional processes, the cognitive process being represented by the objective probability and the emotional process by the subjective one. For instance, a student who predicts failing although having a high objective probability of success might *know* that he is not going to fail and yet *feel* that he is going to.

optimal self-insurance level against future self-esteem risk. As an illustration, consider the case of a student who takes an important pass or fail exam. The student's intertemporal well-being consists both of his satisfaction before uncertainty resolves, i.e., his ego utility, and of his satisfaction after uncertainty resolves, once he knows his results. Either he enjoys a high level of self-confidence and benefits from a good ego feeling ex-ante, but this comes at the risk of experiencing a loss of self-esteem ex-post, if reality is below expectations. Or he adopts a low level of self-confidence, which is associated with less ego utility ex-ante, but comes at the benefit of being less exposed to future loss of self-esteem.

Note that if there is no uncertainty involved, i.e., if $\widetilde{x} = x_l = x_h = A$, then according to (A2), we have $\varphi = 0$, i.e., there is no self-esteem risk, and according to (1), we have $y^* = A$, i.e., there is no ex-ante manipulation of self-confidence. Well-being is then given by $W(y^*) = (k+1)\,u(A)$ and in the next, we will denote it by W_A. Unless otherwise specified, we take $\{x_l, x_h\} = \{0, 1\}$ for a probability of success $p \in (0,1)$. In such a setting, p (resp. y^*) indifferently represents the objective (resp. subjective) expectation of performance or the objective (resp. subjective) probability of success or ability of the individual.

We shall often refer to the two following specifications of our model.

Example 1: $\varphi(1, y, \lambda) = (1-\lambda)\,K(1-y)$, $\varphi(0, y, \lambda) = -\lambda\eta y$ with $(K, \eta) \in \mathbb{R}_+^2$.

Example 2: $\varphi(1, y, \lambda) = (1-\lambda)\,K$, $\varphi(0, y, \lambda) = -\lambda\eta y$ with $(K, \eta) \in \mathbb{R}_+^2$.

Example 1 is Bell's model[14], adding weights for the level of the stereotype. It is easy to verify that Example 2 satisfies all Assumptions (A2)-(A6), and

[14] Bell [1985] considers two possible outcomes s_1 and s_2 with $s_1 < s_2$. Letting y denote

that Example 1 satisfies Assumptions (A2)-(A3) and (A5)-(A6). Example 1 further satisfies Assumption (A4) when $\lambda \geq \frac{K}{\eta+K}$. Moreover, both examples satisfy Assumption (B). With quadratic utility functions $u(x) = x - \frac{1}{2}\alpha x^2$ and $v = ku$, as considered in Section 7, these examples satisfy Assumption (A1).

3 Self-confidence

Due to Assumptions (A1) and (A3), the well-being function is concave, and the first order conditions characterize the optimal self-confidence level y^*, which is given by:

- $y^* = 0$ if $p \leq \frac{-\varphi_y(0,0,\lambda)-v_y(0)}{[\varphi_y(1,0,\lambda)-\varphi_y(0,0,\lambda)]}$,
- $y^* = 1$ if $p \geq \frac{-\varphi_y(0,1,\lambda)-v_y(1)}{[\varphi_y(1,1,\lambda)-\varphi_y(0,1,\lambda)]}$,
- $v_y(y^*) + p\varphi_y(1, y^*, \lambda) + (1-p)\varphi_y(0, y^*, \lambda) = 0$ otherwise.

Proposition 1 *1. Self-confidence increases with ability, i.e.,* $\frac{\partial y^*}{\partial p} \geq 0$.

2. Under Condition (C) below, self-confidence decreases with the intensity of the stereotype, i.e., $\frac{\partial y^*}{\partial \lambda} \leq 0$.

Condition (C): $v_y(1) \geq \frac{\varphi_{y\lambda}(1,y,\lambda)[-\varphi_y(0,1,\lambda)]+\left(-\varphi_{y\lambda}(0,y,\lambda)\right)[-\varphi_y(1,1,\lambda)]}{\varphi_{y\lambda}(1,y,\lambda)-\varphi_{y\lambda}(0,y,\lambda)}$.

Condition (C) is satisfied for high enough weights on ex-ante ego feelings relative to ex-post self-esteem feelings. It is always satisfied in the setting of Example 2 and it is given by $ku_y(1) \geq \frac{\eta K}{(\eta+K)}$ in the setting of Example 1.

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the expectation, disappointment felt in s_1 is given by $d(y - s_1)$ and elation felt in s_2 is given by $e(s_2 - y)$ for nonnegative constants d and e.

Although not immediate in our context, we get the desirable feature that self-confidence increases with ability. Concerning gender and maths, this prediction is in line with educational statistics and surveys as well as with experiments. For instance, in PISA, science self-efficacy, which is a good proxy for self-confidence about academic ability in science, is positively correlated with the PISA score [Filippin and Paccagnella, 2012]. Experiments about competitiveness show a strong link between subjective probabilities of success and past performance, which can be considered as a proxy for objective probabilities of success [Niederle et al., 2013].

Concerning the impact of the stereotype, we need an additional condition to ensure that a higher level of the negative stereotype (that is associated with higher self-esteem risk) leads to a lower self-confidence level for all levels of ability. Indeed, as shown in the proof of the proposition, self-confidence always decreases with the intensity of the stereotype for levels of ability below a given threshold, but for higher levels of ability, the effect of success is dominant, and since $\varphi_{y\lambda}(1, y, \lambda) \geq 0$, an increase in λ is associated to lower elation hence may lead to a higher y^*. Condition (C) rules out this mechanism, by ensuring that $y^* = 1$ for high ability levels and higher stereotype then leads to lower self-confidence for all levels of ability. Applied to gender and maths, we get (under Condition (C)) that girls are less confident than boys in their ability in mathematics. Two remarks can be made; first, in our model, girls are not underconfident because they believe in the stereotype but because they protect their future self-esteem. Second, the argument (mentioned in the introduction) according to which girls, or negatively stereotyped individuals, need to be more able to be as confident holds in our model.

We get as a consequence of Proposition 1 that an increase in the ability level, or an increase in the level of the stereotype have the expected impact on individual well-being.

Corollary 2 *Individual well-being increases with objective ability and decreases with the intensity of the stereotype, i.e.,* $\frac{\partial W(y^*)}{\partial p} \geq 0$ *and* $\frac{\partial W(y^*)}{\partial \lambda} \leq 0$.

A higher level of ability increases well-being because it permits higher levels of ego utility without fearing exposure to loss of self-esteem, as well as higher expected consumption utility. Higher stereotype levels are associated with lower well-being because a higher stereotype level is associated with more self-esteem risk and more precisely, with higher costs of failure and lower benefits from success. This result can explain the underrepresentation of girls in boys stereotyped fields like mathematics. Indeed, for the same level of ability, and the same consumption utility, if a girl can choose between a more or less stereotyped field, she will choose the field that maximizes her well-being, i.e., the less stereotyped one. Let us now consider choices between more or less challenging options within a given field.

4 Choice

We analyze the decision to participate in a challenging task, i.e., a risky situation involving one's ability (like the choice of a difficult track). We assume that the individual has the choice between the risky option $\widetilde{x}$ and a nonrisky option. The nonrisky option may depend upon the individual ability and we denote it by $A(p) \in (0,1)$. There is no self-esteem risk associated with the nonrisky option and we have $W_{A(p)} = (k+1)\, u(A(p))$. The individual chooses the option that maximizes his intertemporal well-being, i.e., participates in the risky option if and only if $W(y^*) \geq W_{A(p)}$, or equivalently if and only if

$$V \equiv [v(y^*) - v(A(p))] + [pu(1) + (1-p)u(0) - u(A(p))] + E[\varphi(\widetilde{x}, y, \lambda)] \geq 0, \tag{2}$$

where $V \equiv W(y^*) - W_{A(p)}$ denotes the value of participation. Letting as usual the certainty equivalent $CE_{\widetilde{x},u}$ denote the constant such that $u(CE_{\widetilde{x},u}) = E[u(\widetilde{x})]$, the second component $pu(1) + (1-p)u(0) - u(A(p))$ can equivalently be written $u(CE_{\widetilde{x},u}) - u(A(p))$. In the standard setting (corresponding to the case $v = \varphi = 0$ in our setting), the individual compares expected 'consumption utility levels' in both situations and chooses the risky option if and only if $CE_{\widetilde{x},u} \geq A(p)$. In our setting, two other components must be taken into account for decision-making. The first component, i.e., $v(y^*) - v(A(p))$, corresponds to the gain or loss in ego utility at date 0. It is in favor of the nonrisky option if the subjective expectation y^* is below the nonrisky outcome, i.e., if $y^* < A(p)$. The other component, i.e., $E[\varphi(\widetilde{x}, y, \lambda)]$, corresponds to the expected gain or loss of self-esteem at date 1. It is in favor of the nonrisky option if the average impact of the risky option on self-esteem is negative.

We assume that $u(A(p))$ is concave in p. If A is increasing in p, it amounts to assuming that $\frac{A''(p)}{A'(p)} \leq -\frac{u''[A(p)]}{u'[A(p)]}$. Natural examples are given by $A(p) = p$ (the objective expectation) or $A(p) = CE_{\widetilde{x},u}$. It is in particular always satisfied if A is concave in p.

As an illustration, consider the example of a high school graduate in France who hesitates between preparatory class to elite school (CPGE) and university. CPGE is (seen as) more prestigious, more difficult and risky, it leads to a difficult competitive exam, threatening one's self-esteem. University on the contrary is seen as less prestigious, graduation is not competitive and grades essentially reflect the students' ability. The random variable $\widetilde{x}$ represents success or failure at entering an elite school and p represents the individual's objective probability of success. Well-being for the option CPGE is given by $W(y^*)$ and well-being for the option university is given by $W_{A(p)}$. The choice involves the consumption values of both options (in terms of future career opportunities for instance) but also immediate psychological rewards (CPGE being more prestigious might be more ego satisfying) and future self-esteem consequences (failure in CPGE can be disappointing and painful but success can be self-esteem rewarding).

Proposition 3
1. *Participation in the risky option weakly decreases with the intensity of the stereotype* λ*.*
2. *Controlling for ability, and under Condition (C), participation in the risky option weakly increases with confidence* y^**.*
3. *There exists a threshold* $\widehat{p}(\lambda)$ *in* $[0,1]$ *such that the value of participation in the risky option is weakly decreasing with ability* p *on* $[0,\widehat{p}(\lambda)[$*, then weakly increasing on* $[\widehat{p}(\lambda),1]$*. The same is true for participation.*

As shown in 1., a higher level of the negative stereotype leads to a lower participation. More negatively stereotyped individuals face a higher self-esteem risk and not only choose more modest confidence levels (Proposition 1) but also make more timid choices. The higher the gap in the level of stereotype the larger the gap in participation. This explains the gender gap in participation rates in selective tracks or challenging options in mathematics.

Despite a lack of direct causality in our model between subjective probability of success and participation in the risky option, the positive correlation between y^* and participation is valid, as shown in 2. The reason in our model is that individuals who are more self-confident are those who face less self-esteem risk hence also those who dare to participate in more challenging options. Note however that lower self-confidence is not the only reason for underparticipation of more negatively stereotyped individuals, in the sense that in Examples 1 and 2 for instance (see Section 7), controlling for self-confidence, participation still decreases with the intensity of the stereotype λ. Applied to maths issues, our result in 2. is consistent with empirical and experimental evidence; for instance, Correll [2001] shows that controlling for objective ability, the higher students assess their mathematical ability, the greater the odds of enrolling in a high school calculus course and choosing a college major in science, math, or engineering.

But as shown in 3., participation in the challenging option is not always increasing with ability. For $\widehat{p}(\lambda)$ in $]0,1[$, an increase in p increases the incentive to participate only for high enough levels of ability. The reason is that for high ability levels, the increase in self-esteem rewards due to the increase of the probability of success are higher than the increase in ego rewards in the nonrisky option, which is not necessarily the case for low ability levels, that are associated with lower self-esteem rewards. This result implies that the individuals who self-select in challenging options are not necessarily the most able; more precisely, as shown in the proof of the proposition, the set of participation is of the form $p \in [0, p_0(\lambda)] \cup [p_1(\lambda), 1]$ for some $p_0(\lambda)$ and $p_1(\lambda)$ in $[0,1]$, which means that in addition to the most able, the least able may also participate because they have "less to lose".

The following result shows that high negative stereotypes not only reduce participation[15] as shown in Proposition 3 but also modify the nature of the relation between ability and participation, the relation being "less increasing"

[15] The participation set $S(\lambda) = \{p; V(p,\lambda) \geq 0\}$ weakly decreases with λ, in the sense of the inclusion, i.e., $S(\lambda_1) \subset S(\lambda_2)$ for $\lambda_1 \geq \lambda_2$.

in the following sense.

Corollary 4 *1. The derivative of the value of participation with respect to objective ability $V_p|_{V=0}$ decreases with λ under Condition (C).*

2a. If the negative stereotype is maximal, then, under Assumption (B.b), participation is weakly decreasing with ability.

2b. If the negative stereotype is low enough then participation is weakly increasing with ability if the self-esteem reward from success $\varphi(1, 0, \lambda)$ is above a given level.

As seen in 1., a higher negative stereotype reduces the incentive to participate when ability increases. The reason is that a higher stereotype reduces the self-esteem rewards associated with a higher probability of success. In the extreme, these self-esteem rewards are eliminated and so is the incentive to participate. Note that the derivative $V_p|_{V=0}$ characterizes the impact of an increase in ability on the pivotal individual, i.e., an individual who is indifferent between participating or not.

As shown in 2a., a maximal negative stereotype biases the self-selection in such a way that participation decreases with ability; contrarily to boys, more able girls then self-select less. A high enough gap in the level of the stereotypes leads to a participation gap that is higher for the highly able than for the less able. This self-selection bias is important for performance issues.

5 Performance and Stereotype Dynamics

5.1 Performance

We now turn to performance issues. We have seen that negatively stereotyped individuals are less confident (under Condition (C)), that they need to be more able to be as confident and that they participate less in challenging options. The argument according to which only more able girls participate is not valid, as the following proposition shows. We let Example 1Q denote Example 1 with quadratic utility functions and with $A(p) = CE_{\widetilde{x},u}$.

Proposition 5 *Consider two groups G_1 and G_2 with stereotype parameters $\lambda_1 > \lambda_2$ and with the same distribution of ability.*

1. *Under Assumption (B.b)., if λ_1 is high enough, then the average ability of the individuals of Group G_1 participating in the risky option is lower than that of group G_2.*

2. *In the setting of **Example 1Q,** group G_1 has a lower ability conditional on participation for all levels $\lambda_1 > \lambda_2$.*

Due to the self-selection bias (leading more able girls to participate less relative to boys), a high enough gap in the stereotype levels leads in the general case to a gap in performance (i.e., on ability conditional on participation). We emphasize that the only difference between G_1 and G_2 is a difference in the level of the stereotype. This result can explain the gender gap in performance in mathematics. Moreover, this result remains true if we restrict our attention to levels of ability above a given threshold, which means that our results not only permit to explain the overall lower performance of girls in maths but can account for their underrepresentation at the right tail of the performance distribution, without assuming any difference in ability.

Note that in addition to the lower ability conditional on participation due to the self-selection bias, our model also provides indirect effects possibly contributing to the lower performance of girls in mathematics. First, as seen in Proposition 1, girls are less confident in their ability in maths, which may lead to anxiety hindering performance, lower effort and to lower persistence. We develop this idea formally in Appendix B. Second, as seen in Corollary 2 and Proposition 3, girls avoid math-related fields and self-select less in math difficult courses, which may lead to lower competence, hence to lower performance.

5.2 Stereotype Dynamics

Consider now a dynamics on the intensity of the negative stereotype. We assume that the two groups G_1 and G_2 face repeated challenging options or tests concerning their ability and we let $\lambda_1(t)$ and $\lambda_2(t)$ denote the intensity of the stereotype of low ability at date t of Group G_1 and group G_2 respectively. We assume that the evolution of the levels $\lambda_i(t)$ of the stereotype depends on the relative performances of Groups G_1 and G_2 at the challenging tasks: the stereotype level of a given group evolves positively following relative success of the group, and negatively following relative failure. This dynamics reflects the fact that the performance of a group can be considered as a signal of its ability. More precisely, we assume that

$$\frac{d\lambda_i(t)}{\lambda_i(t)(1-\lambda_i(t))} = F(P_i(t) - P_j(t))dt \tag{3}$$

where F is negative on $\mathbb{R}^*_+$ and positive on $\mathbb{R}^*_-$, and where $P_i(t)$ denotes the performance of group G_i at date t (i.e., its ability conditional on participation). Note that $P_i(t)$ only depends upon the stereotype level at date t and can be denoted by $P(\lambda_i(t))$, hence the dynamics can equivalently be written as $\frac{d\lambda_i(t)}{\lambda_i(t)(1-\lambda_i(t))} = F(P(\lambda_i(t)) - P(\lambda_j(t)))dt$. By Assumption (A5), we have $\varphi_\lambda(x_h, y, \lambda) \leq 0$ and $\varphi_\lambda(x_l, y, \lambda) \leq 0$; since the stereotype parameter λ is defined up to an increasing transformation, we may assume without loss of generality that $\varphi_\lambda(x_h, y, \lambda) + \varphi_\lambda(x_l, y, \lambda) < 0$. Finally, we exclude the degenerate situation where all individuals participate for all levels of the stereotype.

Proposition 6 *Suppose that the stereotype dynamics is of the form* (3) *for some continuous function F negative on $\mathbb{R}_+^*$ and positive on $\mathbb{R}_-^*$ and suppose that G_1 and G_2 have the same distribution of ability $\bar{Q}$ satisfying $\bar{Q}[a,b] > 0$ for all $0 \leq a < b \leq 1$.*

1. *In the general setting, if $\lambda_1(0)$ is close enough to 1, then $\lambda_1(t)$ converges to* 1 *and $\lambda_2(t)$ converges to* 0.
2. *In the setting of* ***Example 1Q****, if $\lambda_1(0) > \lambda_2(0)$, then $\lambda_1(t)$ converges to* 1 *and $\lambda_2(t)$ converges to* 0*. Furthermore, at any date t, Group G_1 participates less and is less confident.*

A high enough gap in the stereotype levels leads to an increase of the gap, the more negatively stereotyped group becoming more negatively stereotyped and the more positively stereotyped group becoming more positively stereotyped.

Note that we would obtain the same results with a dynamics where the stereotype evolves as in Equation (3) but replacing the difference in performance by the difference in participation. Such dynamics would reflect the fact that the lower presence of girls in difficult tracks can be considered as a signal of their lower ability. We could also consider stereotype dynamics (3) with functions F depending upon the level of the stereotype[16]. Such specifications permit to take into account a biased updating process in which a difference in performance impacts more the stereotype dynamics when this difference is consistent with the current stereotype, like in the model with a confirmation bias of Rabin and Schrag [1999]. These extensions would contribute to the survival or to the worsening of the stereotype.

6 Gender and Maths and Other Stereotypes

6.1 Gender and Math

Applying our approach, with girls being endowed with higher levels of λ, we get that girls participate less than boys in difficult/challenging/competitive/risky tasks involving maths (Proposition 3), girls are less self-confident than boys in their abilities in maths (Proposition 1), and girls are less performant than boys in maths, if the level of negative stereotype is high enough (Proposition 5). Moreover, according to Propositions 3 and 5, the underparticipation and underperformance of girls are higher at the high end of the ability distribution if the gender gap in stereotype levels is high enough. We also get a lower participation of girls in domains where they are subject to the negative stereotype compared to other domains (Corollary 2). All these results hold assuming the same distribution of ability among boys and girls. Finally, as seen in the previous section, the negative stereotype survives and increases since it leads to

[16] Consider for instance the dynamics governed by $\frac{d\lambda_{\min}(t)}{\lambda_{\min}(t)(1-\lambda_{\min}(t))} = [P(\lambda_{\min}(t)) - P(\lambda_{\max}(t))] F(P(\lambda_{\min}(t)) - P(\lambda_{\max}(t)))dt$, and $\frac{d\lambda_{\max}(t)}{\lambda_{\max}(t)(1-\lambda_{\max}(t))} = -\frac{d\lambda_{\min}(t)}{\lambda_{\min}(t)(1-\lambda_{\min}(t))}$ with a convex function F and where $\lambda_{\max}(t) = \max(\lambda_1(t), \lambda_2(t))$ and $\lambda_{\min}(t) = \min(\lambda_1(t), \lambda_2(t))$.

worse self-selection hence to worse performance.

Our model predicts that a reduction in the negative stereotype of lower ability of girls in mathematics or any improvement of the status of girls (i.e., in our model, a reduction in the level of λ) should reduce the gender gaps. Consistent with this prediction, Guiso et al. [2008] find that the gender gap in performance disappears in countries with a more gender-equal culture. Pope and Sydnor [2010] analyze geographic variations in the gender gap across states in the US and point towards a strong role for different social forces. In the same line of ideas, Rich and Tsui [2002] and Wang [2010] have shown that the "only child policy" in China has led to a significant reduction of the gender gap in expectations and performance in mathematics.

Analogously, our model also predicts that a reduction in the stereotyped attributions in mathematics should lead to higher levels of confidence and to more participation of girls in math-related fields. This is consistent with Tiedemann [2000] and Schmader et al. [2004], who obtain a negative relation between parents' or teachers' stereotypic attributions and daughters' self-perceptions and decisions to engage in math-related activities. Children whose parents believe that they are able academically can allow themselves to be self-confident and to be ambitious because their failure will not be associated with a questioning of their ability and of their predictions.

Note that our model also predicts that girls in single-sex schools should be more confident and bold in math since in single sex schools, gender is no more a salient status characteristics or less so. This prediction is in line with the empirical findings of Booth and Nolen [2012b,a].

6.2 Other Stereotypes

6.2.1 Gender in General

Our approach can be applied not only to gender and maths but to gender issues in general since there is a stereotype of lower ability of women in many fields [Foschi, 1996, Fiske et al., 2002]. We get lower self-confidence of girls in general, lower participation in the ambitious tasks and finally possible lower performance, feeding the stereotype. There is evidence for girls' lower confidence and lower participation in difficult tasks [e.g., Niederle and Yestrumskas, 2008]. Note that the lower participation in the ambitious tasks can be interpreted as girls' lower competitiveness, for which there is experimental evidence [see Niederle and Vesterlund, 2011, for a survey]. We emphasize that in our approach, the reason for the lower competitiveness of girls is higher self-esteem risk[17], due to the negative stereotype of lower ability of girls. We can help explain the overrepresentation of boys in most selective tracks, as well as the wage gap. We can apply our approach to shed some light on girls' reluctance to contribute ideas and to speak up. Indeed, due to the status hierarchy, contributions of boys and girls are not judged according to the same standard, contributions of boys will be more easily considered as good and contributions of girls as bad. Speaking up or contributing ideas is then associated with higher self-esteem threat for girls, leading to their lower self-confidence and their higher reluctance to contribute ideas. In line with this reasoning, Coffman [2014] shows that the decisions to contribute ideas to the group depend upon the gender stereotype associated with the decision-making domain. In

[17]Note that girls' lower competitiveness is consistent with the fact the girls are more competitive when competition is team-based [see, e.g., Dargnies, 2012]. Indeed, competition in teams is less self-esteem threatening for girls.

the same line of ideas, our model provides insights on “why women, unlike boys, don’t apply for jobs unless they are 100% qualified” [Mohr, 2014]: the fact of being 100% qualified corresponds in our setting to $p = 1$, i.e., to the situation where there is no self-esteem risk involved, which is more beneficial to girls than to boys.

6.2.2 Low Status

More generally, our approach can be applied to any negatively stereotyped group or low status group, leading to underconfidence, to the avoidance of the stereotyped field, to less ambitious choices, to possible underperformance, and to the survival of the stereotype.

Remark that our model gives some new insight on a possible channel through which status translates into self-confidence and more risky/bolder choices[18]. In our approach, this effect does not result from persuasion or imitation but upon the following mechanism: status imposes others’ ability expectations and dictates the way failure and success will be interpreted, which translates into levels of self-confidence and choices through the individual’s need to protect his self-esteem. High status provides the individual’s self-esteem with an insulating layer enabling him to be more self-confident and more bold in his choices without fearing the impact of failure in terms of self-esteem loss. Lower status individuals have a lower right to fail, hence they will be more timid in their beliefs and choices, leading to the maintenance of status hierarchies. Applied to educational or occupational issues, this mechanism provides possible reasons why orientation choices and inequalities perpetuate across generations.

[18] The same remark applies to the Pygmalion effect, i.e., the phenomenon whereby the greater the expectations placed upon people, the better they perform.

7 Specifications and Numerical Results

We first provide explicit expressions for the self-confidence level and for the decision to participate, as well as their properties in the two specific settings of Examples 1Q and 2Q. We recall that Examples 1Q and 2Q denote Examples 1 and 2 with quadratic utility functions and with $A(p) = CE_{\widetilde{x},u}$.

Example 7 *In the setting of* ***Example 1Q:***

1. *The optimal self-confidence level is given by* $y^* = 0$ *for* $p \leq \frac{\lambda\eta - k}{\lambda\eta-(1-\lambda)K}$, *by* $y^* = 1$ *for* $p \geq \frac{\lambda\eta-k(1-\alpha)}{\lambda\eta-(1-\lambda)K}$ *and by* $y^* = \frac{k-p(1-\lambda)K-\lambda\eta(1-p)}{k\alpha}$ *otherwise.*

2. *The optimal self-confidence level* y^* *is weakly increasing in* p *if* $\lambda \geq \frac{K}{(\eta+K)}$, *and weakly decreasing in* λ *if* $k(1-\alpha) \geq \frac{\eta K}{(\eta+K)}$*; it is weakly increasing in* k, *weakly decreasing in* η, *and weakly decreasing in* K.

3. *Participation is characterized for* $p \leq \frac{\lambda\eta-k}{\lambda\eta-(1-\lambda)K}$ *by* $(1-\lambda)K \geq k\left(1-\frac{1}{2}\alpha\right)$, *for* $p \geq \frac{\lambda\eta-k(1-\alpha)}{\lambda\eta-(1-\lambda)K}$ *by* $k\left(1-\frac{1}{2}\alpha\right) \geq \lambda\eta$ *and for* $\frac{\lambda\eta-k}{\lambda\eta-(1-\lambda)K} \leq p \leq \frac{\lambda\eta-k(1-\alpha)}{\lambda\eta-(1-\lambda)K}$ *by* $\frac{1}{2}\left(k-p(1-\lambda)K - \lambda\eta(1-p)\right)^2 - k\alpha p\left[k\left(1-\frac{1}{2}\alpha\right) - K(1-\lambda)\right] \geq 0$.

4. *Participation is characterized by* $p \in [0, p_0(\lambda)]$ *for some* $p_0(\lambda) \in [0,1]$.

5. *If* $K \leq \left(1-\frac{1}{2}\alpha\right)$, *then controlling for confidence, participation still weakly decreases with the intensity of the stereotype* λ.

Example 8 *In the setting of* ***Example 2Q:***

1. *The optimal self-confidence level is given by* $y^* = 0$ *for* $p \leq \frac{\lambda\eta-k}{\lambda\eta}$, *by* $y^* = 1$ *for* $p \geq \frac{\lambda\eta-k(1-\alpha)}{\lambda\eta}$ *and by* $y^* = \frac{k-\lambda\eta(1-p)}{k\alpha}$ *otherwise.*

2. *The optimal self-confidence level* y^* *is weakly increasing in* p *and weakly decreasing in* λ*; it is weakly increasing in* k, *weakly decreasing in* η, *independent from* K.

3. *Participation is characterized for* $p \leq \frac{\lambda\eta-k}{\lambda\eta}$ *by* $K(1-\lambda) \geq k\left(1-\frac{1}{2}\alpha\right)$, *for* $p \geq \frac{\lambda\eta-k(1-\alpha)}{\lambda\eta}$ *by* $K(1-\lambda)p \geq \left(\lambda\eta - k\left(1-\frac{1}{2}\alpha\right)\right)(1-p)$ *and for* $\frac{\lambda\eta-k}{\lambda\eta} \leq p \leq \frac{\lambda\eta-k(1-\alpha)}{\lambda\eta}$ *by* $\frac{1}{2}\left(k-\lambda\eta\,(1-p)\right)^2 - k\alpha p\left[k\left(1-\frac{1}{2}\alpha\right) - K(1-\lambda)\right] \geq 0$.

4. *Participation is characterized by* $p \in [0, p_0(\lambda)] \cup [p_1(\lambda), 1]$ *for some* $p_0(\lambda)$ *and* $p_1(\lambda)$ *in* $[0,1]$.

These explicit results permit numerical simulations. We consider for boys and for girls a uniform distribution of objective ability on $[0,1]$.

For instance, in the setting of Example 1Q, with the specification[19] (α, k, η, K) = (0.8, 1.25, 1, 0.3), if we assume negative stereotype levels of $\lambda = 0.76$ for boys, and $\lambda = 0.90$ for girls, we get

- average overconfidence for both boys and girls with an average self-confidence level of 0.81 for boys and 0.76 for girls (instead of an average objective ability of 0.5),

- boys are more self-confident than girls, on average but also for all level of ability. The confidence level of boys increases with objective ability from 0.49 to 1 and the confidence level of girls increases from 0.35 to 1. For an objective probability of success of 0.5, boys' subjective probability of success is equal to 0.86 and girls' subjective probability of success is equal to 0.79,

- boys' participation rate is equal to 61% and girls' participation rate is equal to 18%. The proportion of girls among the participants is 22.8%,

[19] Condition (C) is satisfied, all (A1)-(A3) and (A5)-(A7) are satisfied without further restriction and (A4) is satisfied for $\lambda \geq 0.23$.

- the participation gap is higher among the more able,
- success rate for boys is 31% (i.e., more than three boys out of ten succeed among participating boys) and success rate for girls is 9% (i.e., less than one girl out of ten succeeds among participating girls). The proportion of girls among the "winners" is 13.6%.

All these results are summarized in Table 1.

Insert Table 1 here.

The table also contains the results in the setting of Example 2Q with the specification $(\alpha, k, \eta, K) = (0.8, 1, 0.8, 0.1)$, if we assume negative stereotype levels of $\lambda = 0.8$ for boys and $\lambda = 0.9$ for girls. In particular, we get

- average overconfidence for both boys and girls with an average self-confidence level of 0.81 for boys and 0.76 for girls (instead of an average objective ability of 0.5),
- boys are more self-confident than girls, on average, but also for all level of ability. The confidence level of boys varies from 0.45 to 1 and the confidence level of girls varies from 0.35 to 1. For an objective probability of success of 0.5, boys' subjective probability of success is equal to 0.85 and girls' subjective probability of success is equal to 0.8,
- boys' participation rate is equal to 81% and girls' participation rate is equal to 25%. The proportion of girls among the participants is 23%,
- the participation gap is higher among the more able: the proportion of girls among participants with ability below average is 27% and the proportion of girls among participants with ability above average is 19%,

- success rate for boys is 48% (i.e., approximately one boy out of two participating boys succeed) and success rate for girls is 35% (i.e., approximately one girl out of three among participating girls succeeds).

Figures 1 and 2 represent the value of participation and self confidence levels as a function of ability for boys and girls in both settings of Examples 1Q and 2Q. In particular, we observe that the value of participation first decreases then increases with ability, and that self confidence levels increase with ability.

Insert Figure 1 and Figure 2 here.

8 Conclusion

We have shown the impact of status hierarchies or of stereotypes on self-confidence and choices, hence on risk attitudes, competitiveness, participation in ambitious tasks and performance. We have seen how stereotypes can explain the gender gaps in confidence, participation in difficult options and performance in maths. Concerning gender and math, the impact of stereotypes is obviously detrimental to girls, due to their avoidance of math-related fields, to their avoidance of selective tracks and to their underperformance, but it is also detrimental to boys. If well performing girls hurt themselves by shying away from competition, poorly performing boys also hurt themselves by embracing it: boys make choices that do not necessarily fit their abilities. Finally, the impact of the gender stereotype in mathematics is *collectively* detrimental since we have shown that the selection procedures are biased and do not necessarily select the most competent.

We have seen that without intervention, stereotypes or status hierarchies survive and even get stronger.

As far as intervention policies are concerned, the conclusion of our model is that higher self-confidence, more participation in math related fields as well as more "risky" or ambitious choices of girls in mathematics (or of any negatively stereotyped or low status group) would come from the reduction of the stereotype, i.e., from a change in social consideration. More egalitarian collective expectations of ability would lead to more egalitarian self-esteem threats, hence to more egalitarian self-confidence levels and choices, a lower self-selection bias in risky/difficult/challenging options and more egalitarian performance. Programs showing that there is no scientific basis for the belief that men's math skills are superior to women's[20] or policies aimed at providing early and precise feedback about the cognitive skills of children could be beneficial.

More directly aimed at reducing the gender gaps in maths, efficient policies should not only aim at leading *more girls* to participate, but more *able* girls: such interventions should reduce the gender self-selection bias, leading to increased performance of girls and to a reduction of the stereotype of lower ability. Concerning affirmative actions, according to our model, simply introducing an explicitly more lenient standard, even though beneficial in terms of role models, is likely to maintain or increase the difference in collective expectations (since the ability conditional on participation would be lower) hence in confidence and choices. The same remark can be made about programs aimed at directly raising the aspirations of girls.

Overall our model suggests that girls should be provided with a safety

[20]The first Fields medal awarded to a woman, Maryam Mirzakhani in 2014, can play the same role.

cushion to their self-esteem in order to reduce the impact of stereotypes. The intervention that seems the most appropriate according to our model is the creation of mentor programs or of benevolent advice. Knowing that others expect one to do well at mathematics is precisely what boys enjoy and what girls lack, and what is at the origin of the differences in self-confidence and choices. As does a positive stereotype for boys, mentor programs/benevolent advice would provide negatively stereotyped groups with an insulating layer from the self-esteem threat that any achievement-related choice represents.

A Proofs

Proof of Proposition 1.

1. We have either $\frac{\partial y^*}{\partial p} = 0$ or $\frac{\partial y^*}{\partial p} = -\frac{\varphi_y(1,y^*,\lambda)-\varphi_y(0,y^*,\lambda)}{v_{yy}(y^*)+p\varphi_{yy}(1,y^*,\lambda)+(1-p)\varphi_{yy}(0,y^*,\lambda)}$, in which case $\frac{\partial y^*}{\partial p}$ has the sign of $\varphi_y(1, y^*, \lambda) - \varphi_y(0, y^*, \lambda)$, which is nonnegative according to $(A4)$.

2. We have $\frac{\partial y^*}{\partial \lambda} = -\frac{p\varphi_{y\lambda}(1,y^*,\lambda)+(1-p)\varphi_{y\lambda}(0,y^*,\lambda)}{v_{yy}(y^*)+p\varphi_{yy}(1,y^*,\lambda)+(1-p)\varphi_{yy}(0,y^*,\lambda)}$, hence $\frac{\partial y^*}{\partial \lambda}$ has the sign of $p\varphi_{y\lambda}(1, y^*, \lambda) + (1-p)\varphi_{y\lambda}(0, y^*, \lambda)$, which is nonpositive if and only if $\frac{p\left[\varphi_{y\lambda}(1,y^*,\lambda)-\varphi_{y\lambda}(0,y^*,\lambda)\right]}{-\varphi_{y\lambda}(0,y^*,\lambda)} \leq 1$. For $\frac{p\left[\varphi_{y\lambda}(1,y,\lambda)-\varphi_{y\lambda}(0,y,\lambda)\right]}{-\varphi_{y\lambda}(0,y,\lambda)} \geq 1$, and under Condition (C), we have $y^* = 1$ and $\frac{\partial y^*}{\partial \lambda} = 0$; indeed, under Condition (C), we have $[\varphi_y(1,1,\lambda) - \varphi_y(0,1,\lambda)] \geq [-\varphi_y(0,1,\lambda) - v_y(1)] \frac{\left[\varphi_{y\lambda}(1,y,\lambda)-\varphi_{y\lambda}(0,y,\lambda)\right]}{-\varphi_{y\lambda}(0,y,\lambda)}$ hence $p[\varphi_y(1,1,\lambda) - \varphi_y(0,1,\lambda)] \geq [-\varphi_y(0,1,\lambda) - v_y(1)] \frac{p\left[\varphi_{y\lambda}(1,y,\lambda)-\varphi_{y\lambda}(0,y,\lambda)\right]}{-\varphi_{y\lambda}(0,y,\lambda)}$ $\geq [-\varphi_y(0,1,\lambda) - v_y(1)]$ and we know by the first order conditions that this leads to $y^* = 1$. ■

Proof of Corollary 2.

We have $W(y^*) = v(y^*) + pu(1) + p\varphi(1, y^*, \lambda) + (1-p)\varphi(0, y^*, \lambda)$, hence $\frac{\partial W(y^*)}{\partial p} = v_y(y^*)\frac{\partial y^*}{\partial p} + u(1) + \varphi(1, y^*, \lambda) - \varphi(0, y^*, \lambda) + p\varphi_y(1, y^*, \lambda)\frac{\partial y^*}{\partial p} + (1-p)\varphi_y(0, y^*, \lambda)\frac{\partial y^*}{\partial p}$, with $v_y(y^*) + p\varphi_y(1, y^*, \lambda) + (1-p)\varphi_y(0, y^*, \lambda) = 0$ or $\frac{\partial y^*}{\partial p} = 0$, hence $\frac{\partial W(y^*)}{\partial p} = u(1) + \varphi(1, y^*, \lambda) - \varphi(0, y^*, \lambda) > 0$. Besides, we have $\frac{\partial W(y^*)}{\partial \lambda} = v_y(y^*)\frac{\partial y^*}{\partial \lambda} + p\varphi_y(1, y^*, \lambda)\frac{\partial y^*}{\partial \lambda} + (1-p)\varphi_y(0, y^*, \lambda)\frac{\partial y^*}{\partial \lambda} + p\varphi_\lambda(1, y^*, \lambda) + (1-p)\varphi_\lambda(0, y^*, \lambda)$ with $v_y(y^*) + p\varphi_y(1, y^*, \lambda) + (1-p)\varphi_y(0, y^*, \lambda) = 0$ or $\frac{\partial y^*}{\partial \lambda} = 0$, hence $\frac{\partial W(y^*)}{\partial \lambda} = p\varphi_\lambda(1, y^*, \lambda) + (1-p)\varphi_\lambda(0, y^*, \lambda) \leq 0$, due to Assumption (A5). ■

Proof of Proposition 3.

Participation is characterized by $V(p, \lambda) = pu(1) - (k+1)u(A(p)) + v(y^*) + p\varphi(1, y^*, \lambda) + (1-p)\varphi(0, y^*, \lambda) \geq 0$.

1. We have $V_{\lambda}(p,\lambda)=v_y(y^*)\frac{\partial y^*}{\partial\lambda}+p\varphi_y(1,y^*,\lambda)\frac{\partial y^*}{\partial\lambda}+(1-p)\varphi_y(0,y^*,\lambda)\frac{\partial y^*}{\partial\lambda}$ $+p\varphi_{\lambda}(1,y^*,\lambda)+(1-p)\varphi_{\lambda}(0,y^*,\lambda)$ with either $v_y(y^*)+p\varphi_y(1,y^*,\lambda)+(1-p)$ $\varphi_y(0,y^*,\lambda)=0$ or $\frac{\partial y^*}{\partial\lambda}=0$, hence $V_{\lambda}(p,\lambda)=p\varphi_{\lambda}(1,y^*,\lambda)+(1-p)\varphi_{\lambda}(0,y^*,\lambda)\leq 0$ due to Assumption (A5).

2. Controlling for ability, participation only depends upon λ. The result then follows from 1. and from Proposition 1.

3. We have $V_p(p,\lambda)=u(1)-(k+1)\frac{\partial u(A(p))}{\partial p}+v_y(y^*)\frac{\partial y^*}{\partial p}+\varphi(1,y^*,\lambda)-\varphi(0,y^*,\lambda)+p\varphi_y(1,y^*,\lambda)\frac{\partial y^*}{\partial p}+(1-p)\varphi_y(0,y^*,\lambda)\frac{\partial y^*}{\partial p}$, with either $v_y(y^*)+p\varphi_y(1,y^*,\lambda)+(1-p)\varphi_y(0,y^*,\lambda)=0$ or $\frac{\partial y^*}{\partial p}=0$, hence $V_p(p,\lambda)=u(1)-(k+1)\frac{\partial u(A(p))}{\partial p}+\varphi(1,y^*,\lambda)-\varphi(0,y^*,\lambda)$. We have $V_{pp}(p,\lambda)=-(k+1)\frac{\partial^2 u(A(p))}{\partial^2 p}$ $+[\varphi_y(1,y^*,\lambda)-\varphi_y(0,y^*,\lambda)]\frac{\partial y^*}{\partial p}$. We know that $\frac{\partial^2 u(A(p))}{\partial^2 p}<0$ (by assumption), that $\varphi_y(1,y^*,\lambda)-\varphi_y(0,y^*,\lambda)\geq 0$ by Assumption (A4), and that $\frac{\partial y^*}{\partial p}\geq 0$ by Proposition 1. The function $V(\cdot,\lambda)$ is then convex hence its section $\{p;V(p,\lambda)\leq 0\}$ is convex and participation is characterized by $p\in[0,p_0(\lambda)]\cup[p_1(\lambda),1]$. The function V_p is increasing in p. If $V_p(0,\lambda)$ is positive then V_p is positive and participation is weakly increasing in p. If $V_p(1,\lambda)$ is negative then V_p is negative, and participation is weakly decreasing in p. If $V_p(0,\lambda)$ is nonpositive and $V_p(1,\lambda)$ nonnegative, then the function V_p is first nonpositive, and nonnegative above a given threshold $\widehat{p}_{\lambda}$ characterized by $V_p(\widehat{p}_{\lambda})=0$. The value of participation and participation are weakly decreasing on $[0,\widehat{p}_{\lambda}[$ then weakly increasing on $]\widehat{p}_{\lambda},1]$. ■

Proof of Corollary 4.

1. As in the proof of Proposition 3, and adopting the same notations, we have $V_p(p,\lambda)=u(1)-(k+1)\frac{\partial u(A(p))}{\partial p}+\varphi(1,y^*,\lambda)-\varphi(0,y^*,\lambda)$.

When $V=0$, we have $pu(1)-(k+1)u(A(p))+v(y^*)+p\varphi(1,y^*,\lambda)=-(1-p)\varphi(0,y^*,\lambda)$, hence $(1-p)V_p(p,\lambda)|_{V=0}=u(1)-(1-p)(k+1)\frac{\partial u(A(p))}{\partial p}-$

$(k+1)\,u\,(A\,(p)) + v\,(y^*) + \varphi\,(1, y^*, \lambda)$, whose derivative with respect to λ is equal to $B = v_y\,(y^*)\,\frac{\partial y^*}{\partial \lambda} + \varphi_y\,(1, y^*, \lambda)\,\frac{\partial y^*}{\partial \lambda} + \varphi_\lambda\,(1, y^*, \lambda)$. Now, $B = [-p\varphi_y\,(1, y^*, \lambda)$ $- (1-p)\,\varphi_y\,(0, y^*, \lambda)]\,\frac{\partial y^*}{\partial \lambda} + \varphi_y\,(1, y^*, \lambda)\,\frac{\partial y^*}{\partial \lambda} + \varphi_\lambda\,(1, y^*, \lambda) = (1-p)\,[\varphi_y\,(1, y^*, \lambda)$ $-\varphi_y\,(0, y^*, \lambda)]\,\frac{\partial y^*}{\partial \lambda} + \varphi_\lambda\,(1, y^*, \lambda)$. We have $\varphi_y\,(1, y^*, \lambda) - \varphi_y\,(0, y^*, \lambda) \geq 0$ by Assumption (A4), $\varphi_\lambda\,(1, y^*, \lambda) \leq 0$ by Assumption (A5), and $\frac{\partial y^*}{\partial \lambda} \leq 0$ under Condition (C) by Proposition 1, hence $B \leq 0$.

2. As in the proof of Proposition 3, and adopting the same notations, we know that participation is weakly increasing if $V_p\,(0, \lambda)$ is positive, and in particular, if $\varphi\,(1, 0, \lambda)$ is high enough. We know by the proof of Proposition 3 that the function V is convex in p. It suffices to show that $V_p\,(p, \lambda)\,|_{V=0} \leq 0$ in order to get that participation is weakly decreasing with ability. As seen in the proof of 2., we have $(1-p)\,V_p\,(p, \lambda)\,|_{V=0} = u\,(1) - (1-p)\,(k+1)\,\frac{\partial u(A(p))}{\partial p} -$ $(k+1)\,u\,(A\,(p)) + v\,(y^*) + \varphi\,(1, y^*, \lambda)$. Since $u(A(p))$ is concave by assumption, we have $\frac{\partial u(A(p))}{\partial p} \geq \frac{u(1) - u(A(p))}{1-p}$ hence $(1-p)\,V_p\,(p, \lambda)\,|_{V=0} \leq -v\,(1) +$ $\varphi\,(1, y^*, \lambda) + v\,(y^*)$. For $\lambda = 1$, we have $\varphi\,(1, y^*, 1) = 0$ and $(1-p)\,V_p\,(p, \lambda)\,|_{V=0}$ ≤ 0 for all $y^* \leq 1$. ■

Proof of Proposition 5.

1. For $\lambda_1 = 1$, we have under Assumption (B.b) $\varphi\,(1, y, 1) = 0$, hence according to Corollary 4, participation of group G_1 is then given by $[0, p_0\,(1)]$. By Proposition 3, we know that participation of group G_2 is given by $p \in [0, p_0\,(\lambda_2)] \cup [p_1\,(\lambda_2)\,, 1]$, and since participation weakly decreases with λ (Proposition 3), with $p_0\,(\lambda_2) \geq p_0\,(1)$. We compare the expected values conditional on participation $E^{G_1}\,[\widetilde{p}] = \frac{E\left[\widetilde{p} 1_{[0, p_0(1)]}(\widetilde{p})\right]}{E\left[1_{[0, p_0(1)]}(\widetilde{p})\right]}$ and $E^{G_2}\,[\widetilde{p}] = \frac{E\left[\widetilde{p} 1_{[0, p_0(\lambda_2)] \cup [p_1(\lambda_2), 1]}(\widetilde{p})\right]}{E\left[1_{[0, p_0(\lambda_2)] \cup [p_1(\lambda_2), 1]}(\widetilde{p})\right]}$. Since $1_{[0, p_0(1)]}\,(\widetilde{p}) = 1_{[0, p_0(1)]}\,(\widetilde{p})\,1_{[0, p_0(\lambda_2)] \cup [p_1(\lambda_2), 1]}\,(\widetilde{p})$ and $1_{[0, p_0(1)]}\,(\widetilde{p})$ is a weakly decreasing function of $\widetilde{p}$, we then have $E^{G_1}\,[\widetilde{p}] \leq E^{G_2}\,[\widetilde{p}]$ for $\lambda_1 = 1$. Indeed, we have $E^{G_2}\left[\widetilde{p} 1_{[0, p_0(1)]}\,(\widetilde{p})\right] \leq E^{G_2}\,[\widetilde{p}]\;E^{G_2}\left[1_{[0, p_0(1)]}\,(\widetilde{p})\right]$ hence $\frac{E^{G_2}\left[\widetilde{p} 1_{[0, p_0(1)]}(\widetilde{p})\right]}{E^{G_2}\left[1_{[0, p_0(1)]}(\widetilde{p})\right]} \leq$

$E^{G_2}[\widetilde{p}]$ and $\frac{E^{G_2}\left[\widetilde{p}1_{[0,p_0(1)]}(\widetilde{p})\right]}{E^{G_2}\left[1_{[0,p_0(1)]}(\widetilde{p})\right]} = E^{G_1}[\widetilde{p}]$. Note that if $P(\widetilde{x} \in [p_1(\lambda_2), 1]) > 0$, then the previous inequality is strict and by continuity, also holds for λ_1 high enough. If $P(\widetilde{x} \in [p_1(\lambda_2), 1]) = 0$, then $E^{G_2}[\widetilde{p}] = \frac{E\left[\widetilde{p}1_{[0,p_0(\lambda_2)]}(\widetilde{p})\right]}{E\left[1_{[0,p_0(\lambda_2)]}(\widetilde{p})\right]}$ and $E^{G_1}[\widetilde{p}] \leq E^{G_2}[\widetilde{p}]$ for all $\lambda_1 \in [\lambda_2, 1]$, we have

2. In the setting of Example 1, as shown in Example 8, participation for G_1 is given by $p \in [0, p_0(\lambda_1)]$ and participation for G_2 is given by $[0, p_0(\lambda_2)]$ with $p_0(\lambda_2) \geq p_0(\lambda_1)$. We have $E^{G_i}[\widetilde{p}] = \frac{E\left[\widetilde{p}1_{[0,p_0(\lambda_i)]}(\widetilde{p})\right]}{E\left[1_{[0,p_0(\lambda_i)]}(\widetilde{p})\right]}$. Since $1_{[0,p_0(\lambda_1)]}(\widetilde{p}) = 1_{[0,p_0(\lambda_1)]}(\widetilde{p})\, 1_{[0,p_0(\lambda_2)]}(\widetilde{p})$ and $1_{[0,p_0(\lambda_1)]}(\widetilde{p})$ is a weakly decreasing function of $\widetilde{p}$, we then have $E^{G_1}[\widetilde{p}] \leq E^{G_2}[\widetilde{p}]$. ■

Proof of Proposition 6.

By the proof of Proposition 3, we know that $V_\lambda(p, \lambda) = p\varphi_\lambda(1, y^*, \lambda) + (1-p)\varphi_\lambda(0, y^*, \lambda)$. Under the additional assumption $\varphi_\lambda(x_h, y, \lambda) + \varphi_\lambda(x_l, y, \lambda) < 0$, we then get $V_\lambda(p, \lambda) < 0$. Since we excluded the degenerate situation where all individuals participate for all λ, we have $p_0(1) \neq 1$. According to the proof of 5, and since $p_0(1) \neq 1$, we get that $p_0(\lambda)$ decreases with λ. For a given $\lambda_2 < 1$, and with the notations of the proof of Proposition 5, we have $p_0(1) < p_0(\lambda_2)$; now, since $\bar{Q}[p_0(\lambda_2), p_0(1)] > 0$, the inequality $E^{G_1}[\widetilde{p}] \leq E^{G_2}[\widetilde{p}]$ is strict and $P_1 < P_{\lambda_2}$, where P_λ denotes the ability conditional on participation of a group with stereotype level λ. If $\lambda_{G_1}(0)$ is close enough to 1 for a given $\lambda_{G_2}(0)$, we have $\sup_{\lambda \in [\lambda_{G_1}(0), 1]} P_\lambda < \inf_{\lambda \in [0, \lambda_{G_2}(0)]} P_\lambda$. We then have $d\lambda_{G_1}(0) > 0$ and $d\lambda_{G_2}(0) < 0$. It is obvious that the differential of performance cannot change sign and we get that λ_{G_1} is increasing and λ_{G_2} decreasing and $\lambda_{G_1}(t) \to 1$, $\lambda_{G_2}(t) \to 0$. ■

Proof of Example 7.

1. Immediate. 2. Condition (C) is given by $k(1-\alpha) \geq \frac{\eta K}{(\eta + K)}$. The rest is immediate. 3. For $p \leq \frac{\lambda\eta - k}{\lambda\eta - (1-\lambda)K}$, $y^* = 0$, and participation is character-

ized by $K(1-\lambda) \geq k\left(1-\frac{1}{2}\alpha\right)$. For $p \geq \frac{\lambda\eta - k(1-\alpha)}{\lambda\eta-(1-\lambda)K}$, $y^* = 1$, and participation is characterized by $k\left(1-\frac{1}{2}\alpha\right) \geq \lambda\eta$. Otherwise, participation is characterized by $k\left(y-\frac{1}{2}\alpha y^2\right) - kp\left(1-\frac{1}{2}\alpha\right) + p(1-\lambda)K(1-y) - (1-p)\lambda\eta y \geq 0$. Direct computations lead to $\frac{1}{2}k\alpha y^2 - p\left[k\left(1-\frac{1}{2}\alpha\right) - K(1-\lambda)\right] \geq 0$ or $\frac{1}{2}(k - p(1-\lambda)K - \lambda\eta(1-p))^2 - k\alpha p\left[k\left(1-\frac{1}{2}\alpha\right) - K(1-\lambda)\right] \geq 0$.

4 a. If $\lambda\eta > k\left(1-\frac{1}{2}\alpha\right)$, then, as seen in 3., participation is never chosen for $p \geq \frac{\lambda\eta - k(1-\alpha)}{\lambda\eta-(1-\lambda)K}$, hence according to Proposition 3, given by $p \leq p_0(\lambda)$.

b. If $\lambda\eta \leq k\left(1-\frac{1}{2}\alpha\right)$, then let us show that $S = [0,1]$. We have $k - p(1-\lambda)K - \lambda\eta(1-p) \geq k - p(1-\lambda)K - k\left(1-\frac{1}{2}\alpha\right)(1-p) = p\left[k\left(1-\frac{1}{2}\alpha\right)\right.$ $\left.-(1-\lambda)K\right] + \frac{1}{2}k\alpha$, hence $\frac{1}{2}(k - p(1-\lambda)K - \lambda\eta(1-p))^2 \geq \frac{1}{2}\left(p\left[k\left(1-\frac{1}{2}\alpha\right)\right.\right.$ $\left.\left.-(1-\lambda)K\right] + \frac{1}{2}k\alpha\right)^2 \geq 2p\left[k\left(1-\frac{1}{2}\alpha\right) - (1-\lambda)K\right]\left(\frac{1}{2}k\alpha\right) = k\alpha p\left[k\left(1-\frac{1}{2}\alpha\right)\right.$ $\left.-(1-\lambda)K\right]$.

5. Consider two levels of the stereotype λ_1 and λ_2 associated with the same level of confidence y^*. For $y^* = 0$, we have $V = -pv(1) + p(1-\lambda_i)K = p[-v(1) + (1-\lambda_i)K]$ hence $V < 0$ if $v(1) > K$. For $y^* = 1$, we have $V = (1-p)[v(1) - \lambda\eta]$, hence V has the sign of $v(1) - \lambda\eta$ and participation weakly decreases with λ. Otherwise, consider two individuals denoted by 1 and 2 with $y_1^* = y_2^* = y$ for $(\lambda_1, p_1) \geq (\lambda_2, p_2)$ and letting $V^{(i)} = v(y) - p_i v(1) + p_i\varphi(1, y, \lambda_i) + (1-p_i)\varphi(0, y, \lambda_i)$, let us show that $V^{(1)} \leq V^{(2)}$. We have $V^{(1)} - V^{(2)} = (p_2 - p_1)v(1) + [p_1(1-\lambda_1) - p_2(1-\lambda_2)]K(1-y) + [(1-p_2)\lambda_2 - (1-p_1)\lambda_1]\eta y$. Since $y_1^* = y_2^*$, we have $\lambda_2\eta(1-p_2) - \lambda_1\eta(1-p_1) = p_1(1-\lambda_1)K - p_2(1-\lambda_2)K$, hence $V^{(1)} - V^{(2)} = (p_2 - p_1)v(1) + [p_1(1-\lambda_1)$ $-p_2(1-\lambda_2)]K = (p_2 - p_1)[v(1) - K] + [-p_1\lambda_1 + p_2\lambda_2]K \leq 0$. ■

Proof of Example 8.

Immediate, proceeding as in the proof of Example 7. ■

B Effort

The aim of this section of the appendix is to sketch how our model can be consistent with the idea that lower self-confidence leads by itself to lower performance. To obtain this prediction we assume as in Akerlof and Kranton [2002] that the probability of success not only depends on the objective ability of the individual but also on his level of effort. Let the objective probability of success be given by $p = ae$, with a being the ability of the individual and e his effort. The discussion in the main paper can be seen as a specific case in which effort is fixed at $e = 1$ and hence $p = a$. The cost of effort is given by $g(e)$ and the cost is increasing and convex in effort, i.e., $g' > 0$ and $g'' > 0$. The decision maker chooses the optimal level of effort, $e \in [0, 1/a]$, and his level of self-confidence to maximize his intertemporal utility

$$W(y,e) = v(y) - g(e) + eau(x_h) + (1-ea)u(x_l) + ea\varphi\left(x_h, y, \lambda\right) + (1-ea)\varphi\left(x_l, y, \lambda\right).$$

Since we maximize with respect to both y and e, it is equivalent to maximize first with respect to y then with respect to e and conversely. If (y^*, e^*) is the optimum, it satisfies then

$$-g'(e^*) + a\left[u(x_h) - u(x_l) + \varphi\left(x_h, y^*, \lambda\right) - \varphi\left(x_l, y^*, \lambda\right)\right] \begin{cases} \leq 0 & \text{if } e^* = 0, \\ = 0 & \text{if } e^* \in \left[0, \frac{1}{a}\right], \\ \geq 0 & \text{if } e^* = \frac{1}{a}. \end{cases}$$

The next proposition shows that individuals with higher self-confidence levels exert more effort which is consistent with our conjecture that lower self-confidence levels may lead by themselves to lower performance.

Proposition 9 *The optimal level of effort is non-decreasing in self-confidence.*

Proof. For interior solutions of effort, we get with the implicit function theorem

$$\frac{\partial e^*}{\partial y^*} = \frac{\varphi_y(x_h, y^*, \lambda) - \varphi_y(x_l, y^*, \lambda)}{g''(e^*)} > 0,$$

and for corner solutions $\frac{\partial e^*}{\partial y^*} = 0$. ■

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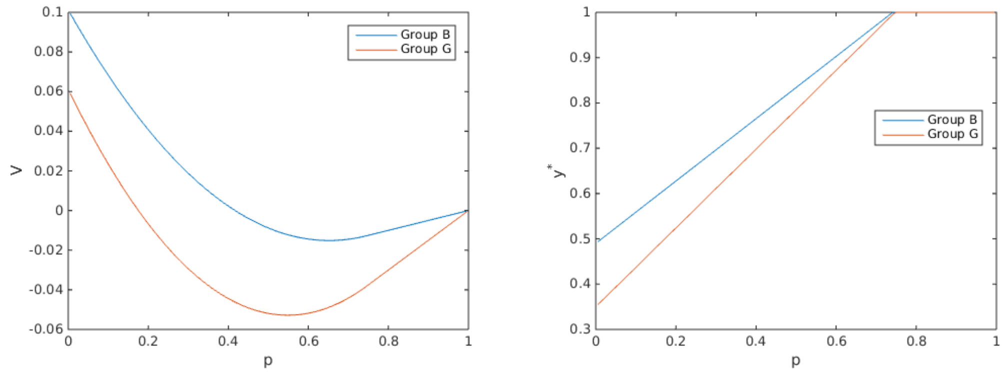

Figure 1: The left-hand side figure shows the value of participation as a function of ability and the right-hand side figure shows the self-confidence level as a function of ability. Both figures are for Example 1Q and the specification $(\alpha,\, k,\, \eta,\, K) = (0.8,\, 1.25,\, 1,\, 0.3)$ and $\lambda = 0.76$ for Group B (boys) and $\lambda = 0.9$ for Group G (girls).

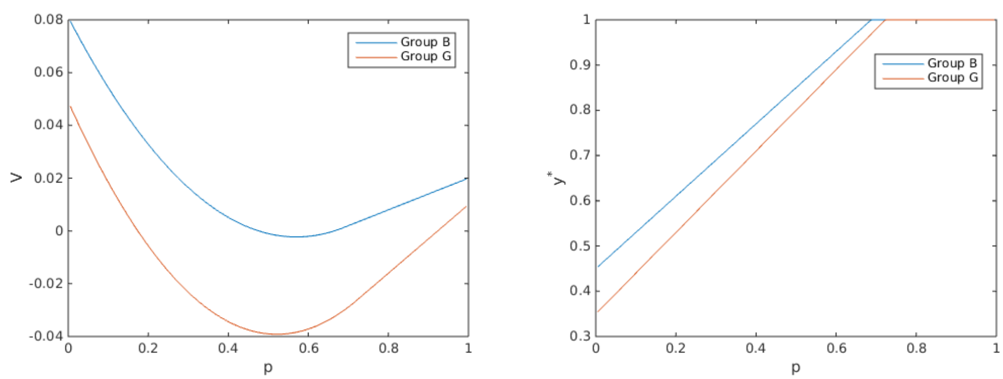

Figure 2: The left-hand side figure shows the value of participation as a function of ability and the right-hand side figure shows the self-confidence level as a function of ability. Both figures are for Example 2Q and the specification $(\alpha,\, k,\, \eta,\, K) = (0.8,\, 1,\, 0.8,\, 0.1)$ and $\lambda = 0.8$ for Group B (boys) and $\lambda = 0.9$ for Group G (girls).

Table 1: The table contains numerical results for Example 1Q with the specification $(\alpha,\, k,\, \eta,\, K) = (0.8,\, 1.25,\, 1,\, 0.3)$ and for Example 2Q with the specification $(\alpha,\, k,\, \eta,\, K) = (0.8,\, 1,\, 0.8,\, 0.1)$. The probability of success follows a uniform distribution on $[0, 1]$.

	Example	Stereotype	Self-confidence y^*		Participation	Ability cond
		λ	Interval	Average	for $A = CE$	on participation
Group B	1	0.76	[0.49, 1]	0.81	61%	0.31
Group G	1	0.90	[0.35, 1]	0.76	18%	0.09
Group B	2	0.80	[0.45, 1]	0.81	81%	0.48
Group G	2	0.90	[0.35, 1]	0.77	25%	0.35