# Quasi-hyperbolic discounting and the taxation of capital income

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January 2016

# Introduction

- The paper considers the implications of quasi-hyperbolic preferences for capital taxation
- It builds on two observations:
  - That quasi-hyperbolic preferences lead to "under-saving" so justify intervention
  - That the Chamley-Judd result demonstrates captial should not be taxed
- The quasi-hyperbolic preferences are embedded within a Ramsey growth model and an overlapping generations model

# Introduction

- The Mirrlees Review placed considerable emphasis on the tax treatment of capital income
- The motive for this were a set of *equivalence results*: a consumption tax is equivalent to an income tax with exemption for interest income
- These results are based on budgets, not preferences
- This reduces the relevant policy choices to:
  - A comprehensive income tax
  - A tax on labour income with tax exemption for capital income
  - A tax on labour income with a tax at a different rate on capital income
- Chamley-Judd supports the second option in Ramsey growth models

# Introduction

- The first section introduces the different choice problems for quasi-hyperbolic preferences
- The choice problems are then explored for log utility and a fixed wealth level
- The Ramsey growth model with quasi-hyperbolic preferences is then analyzed
  - Existing conclusions are not changed
- Taxation is then considered in an overlapping generations economy
  - A capital tax can have a role

# Preferences and choices

- We deal with the class of preferences described by

$$U = u(c_0) + \beta\delta u(c_1) + \beta\delta^2 u(c_2) + \ldots + \beta\delta^T u(c_T)$$

- With these can define three different types of consumer:
- Commited: $\{c_0, \ldots, c_T\}$ chosen at 0
- Naive: $\{c_0, \ldots, c_T\}$ chosen at 0, $\{c_1, \ldots, c_T\}$ chosen at 1, . . .
- Sophisticated: $\{c_0, \ldots, c_T\}$ chosen at 0 taking into account actions of future selves
- Each type of consumer generates a different growth path

# Log utility

- For the naive consumer with a $T$ period lifetime

$$s_{T-1} = \frac{\beta\delta}{1+\beta\delta} W_{T-1}, ..., s_{T-2} = \frac{\beta\left[\delta+\delta^2\right]}{1+\beta\left[\delta+\delta^2\right]} W_{T-2},$$

$$s_0 = \frac{\beta\sum_{i=1}^{T}\delta^i}{1+\beta\sum_{j=1}^{T}\delta^j} W_0$$

- As $T \to \infty$

$$s_0 \to \frac{\beta\frac{\delta}{1-\delta}}{1+\beta\frac{\delta}{1-\delta}} W_0, \quad s_t \to \frac{\beta\frac{\delta}{1-\delta}}{1+\beta\frac{\delta}{1-\delta}} W_t$$

# Log utility

- For an exponential consumer with discount factor $\tilde{\delta}$

$$s_{T-1} = \frac{\tilde{\delta}}{1+\tilde{\delta}} W_{T-1}, \; s_{T-2} = \frac{\tilde{\delta}+\tilde{\delta}^2}{1+\tilde{\delta}+\tilde{\delta}^2} W_{T-2}, \; s_0 = \frac{\sum_{i=1}^{T} \tilde{\delta}^i}{1+\sum_{j=1}^{T} \tilde{\delta}^j} W_0$$

- So as $T \to \infty$

$$s_0 \to \frac{\frac{\tilde{\delta}}{1-\tilde{\delta}}}{1+\frac{\tilde{\delta}}{1-\tilde{\delta}}} W_0, \quad s_t \to \frac{\frac{\tilde{\delta}}{1-\tilde{\delta}}}{1+\frac{\tilde{\delta}}{1-\tilde{\delta}}} W_t$$

- Naive and exponential are identical if

$$\tilde{\delta} = \frac{\beta}{\frac{1}{\delta} - 1 + \beta}$$

# Ramsey growth model

- Consider a Ramsey growth model with quasi-hyperbolic preferences
- The preferences affect the growth path when the economy is finite
- To illustrate this assume a three-period economy with CRRA utility

$$U = \frac{c_0^{1-\gamma} - 1}{1-\gamma} + \beta\delta \frac{c_1^{1-\gamma} - 1}{1-\gamma} + \beta\delta^2 \frac{c_2^{1-\gamma} - 1}{1-\gamma}$$

- And the standard production function

$$y_t = Ak_t^{\alpha}, \quad 0 < \alpha < 1.$$

- The outcomes for Committed and Sophisticated consumers are shown in the table

| $\beta = 0.9$ | | | | | $\beta = 0.5$ | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Committed | | Sophisticated | | | Committed | | Sophisticated | |
| $\gamma$ | $k_1$ | $k_2$ | $k_1^*$ | $k_2^*$ | $\gamma$ | $k_1$ | $k_2$ | $k_1^*$ | $k_2^*$ |
| 0.5 | 1.631 | 0.589 | 1.631 | 0.529 | 0.5 | 1.631 | 0.589 | 1.631 | 0.529 |
| 1.0 | 3.068 | 0.763 | 3.068 | 0.705 | 1.0 | 3.068 | 0.763 | 3.068 | 0.705 |
| 1.5 | 4.253 | 0.931 | 4.253 | 0.876 | 1.5 | 4.253 | 0.931 | 4.253 | 0.876 |
| 2.0 | 5.082 | 1.066 | 5.082 | 1.015 | 2.0 | 5.082 | 1.066 | 5.082 | 1.015 |
| 2.5 | 5.650 | 1.168 | 5.651 | 1.123 | 2.5 | 5.650 | 1.168 | 5.651 | 1.123 |
| 3.0 | 6.050 | 1.247 | 6.051 | 1.206 | 3.0 | 6.050 | 1.247 | 6.051 | 1.206 |
| 3.5 | 6.342 | 1.309 | 6.342 | 1.272 | 3.5 | 6.342 | 1.309 | 6.342 | 1.272 |

# Log utility

- The behaviour in an infinite economy can be illustrated by using log utility
- With $T$ periods the objective function is

$$U = \ln(Ak_0^\alpha - k_1) + \beta \sum_{t=1}^{T} \delta^t \ln(Ak_t^\alpha - k_{t+1}),\ k_T = 0$$

- The solution for the Commited consumer can be written as

$$k_1 = \frac{\beta \sum_{i=1}^{T} \alpha^i \delta^i}{1 + \beta \sum_{i=1}^{T} \alpha^i \delta^i} A k_0^\alpha$$

- And for $1 < t < T - 1$

$$k_t = \frac{\sum_{i=1}^{T+1-t} \alpha^i \delta^i}{1 + \sum_{i=1}^{T+1-t} \alpha^i \delta^i} A k_{t-1}^\alpha$$

# Log utility

- The solution of the Naive is a repetition of the first period for the committed
- This gives directly

$$k_t = \frac{\beta \sum_{i=1}^{T-t+1} \alpha^i \delta^i}{1 + \beta \sum_{i=1}^{T-t+1} \alpha^i \delta^i} A k_{t-1}^{\alpha}$$

- It can be seen directly that the path for the Committed and the Naive differ
- The question is whether this gives a motive for taxation
- In a finite economy the answer is clearly yes
- But what if the economy is infinite?

# Limit behaviour

- Consider the economy with the Naive consumer
- The process for capital is

$$k_t = \frac{\beta \sum_{i=1}^{T-t+1} \alpha^i \delta^i}{1 + \beta \sum_{i=1}^{T-t+1} \alpha^i \delta^i} A k_{t-1}^{\alpha}$$

- In the limit as $T \to \infty$,

$$k_t = \frac{\beta \frac{\alpha\delta}{1-\alpha\delta}}{1 + \beta \frac{\alpha\delta}{1-\alpha\delta}} A k_{t-1}^{\alpha}$$

- Observe that this is again the choice of an exponential consumer discounting at $\tilde{\delta} = \frac{\beta}{\frac{1}{\delta} - 1 + \beta}$
- The logic of Chamley-Judd will apply to this economy (care needed in interpretation!)

# An overlapping generations economy

- The effect of quasi-hyperbolic preferences is not significant when life is infinite
- This suggests that an analysis of tax policy should focus on a model with finite life
- The natural setting is then an overlapping generations economy
- This ensures the effect of the present-bias is continually present
- We now construct an overlapping generations economy with quasi-hyperbolic preferences

# An overlapping generations economy

- Each consumer lives for three periods
- They work when young and when middle-aged
- One unit of labour is supplied in each period of working life
- They are retired when old
- The path of saving is chosen with quasi-hyperbolic preferences
- All markets are competitive

# Committed

- The Committed consumer faces the optimization

$$
\begin{aligned}
U_t &= \ln(w_t - s_t^t) + \beta\delta \ln(w_{t+1} + [1 + r_{t+1}]\, s_t^t - s_{t+1}^t) \\
&\quad + \beta\delta^2 \ln([1 + r_{t+2}]\, s_{t+1}^t)
\end{aligned}
$$

- The solution is

$$
\begin{aligned}
s_t^t &= \frac{[1 + r_{t+1}]\, w_t \left[\beta\delta + \beta\delta^2\right] - w_{t+1}}{[1 + r_{t+1}] \left[1 + \beta\delta + \beta\delta^2\right]} \\
s_{t+1}^t &= \frac{\beta\delta^2 \left[[1 + r_{t+1}]\, w_t + w_{t+1}\right]}{1 + \beta\delta + \beta\delta^2}
\end{aligned}
$$

- The time path for capital is governed by

$$
k_t = \frac{\beta\delta^2 \left[[1 + r_{t-1}]\, w_{t-2} + w_{t-1}\right]}{1 + \beta\delta + \beta\delta^2} + \frac{[1 + r_t]\, w_{t-1} \left[\beta\delta + \beta\delta^2\right] - w_t}{[1 + r_t] \left[1 + \beta\delta + \beta\delta^2\right]}
$$

# Naive

- The first-period solution for the Naive is the same as the Committed
- In the second period (with $s_t^t$ given)

$$\max_{\{s_{t+1}^t\}} U_t = \ln\left(w_{t+1} + [1 + r_{t+1}]\, s_t^t - s_{t+1}^t\right) + \beta\delta \ln\left([1 + r_{t+2}]\, s_{t+1}^t\right)$$

- This provides the solution

$$s_{t+1}^t = \left(\frac{\beta\delta}{1+\beta\delta}\right)\left(\frac{\beta\delta + \beta\delta^2}{1 + \beta\delta + \beta\delta^2}\right)\left[[1 + r_{t+1}]\, w_t + w_{t+1}\right].$$

- The time path for capital is

$$k_t = \frac{\beta\delta + \beta\delta^2}{1 + \beta\delta + \beta\delta^2}\left(\frac{\beta\delta\,[1 + r_{t+1}]\, w_{t-2}}{1 + \beta\delta} + w_{t-1}\right) - \frac{w_t}{[1 + r_{t+1}]\,[1 + \beta\delta + \beta\delta^2]}$$

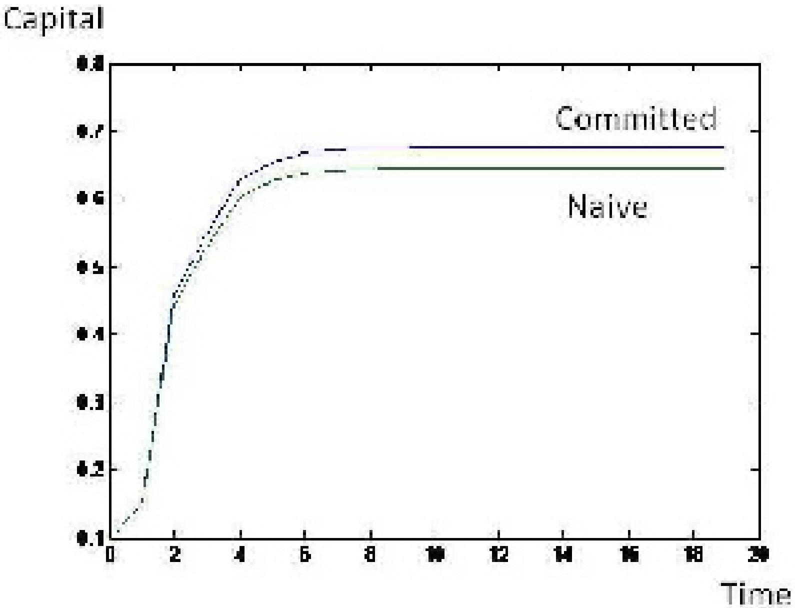
# Capital accumulation and the steady state

- Start the economy at time 0 with an initial stock of capital $k_0$
- At time 0 there are only young consumers
- The growth path for capital can then be constructed as

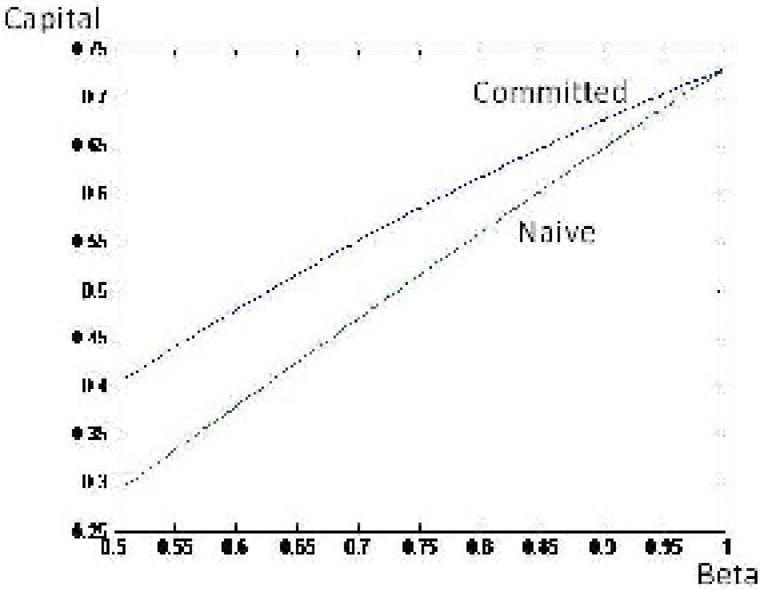
$$
\begin{aligned}
k_1 &= s_0^0 \\
k_2 &= s_1^0 + s_1^1 \\
k_2 &= s_2^1 + s_2^2 \\
&\vdots
\end{aligned}
$$

- This dynamic system can be simulated forward

# Capital accumulation and the steady state



# Capital accumulation and the steady state



# Taxation

- Taxation of capital and labour income can be introduced by the change of variables

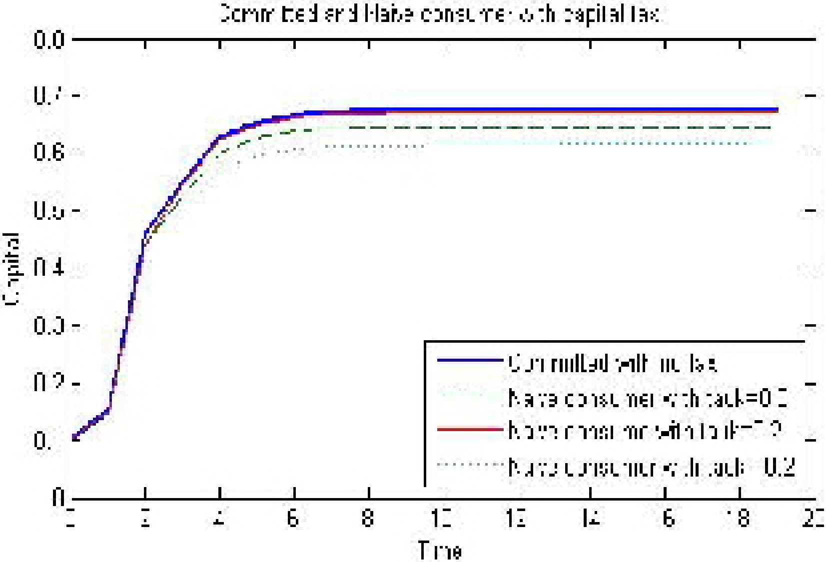
$$w_t \rightarrow [1 - \tau_l]\, w_t, \quad r_t \rightarrow [1 - \tau_k]\, r_t$$

- Impose a balanced budget for the government in each period

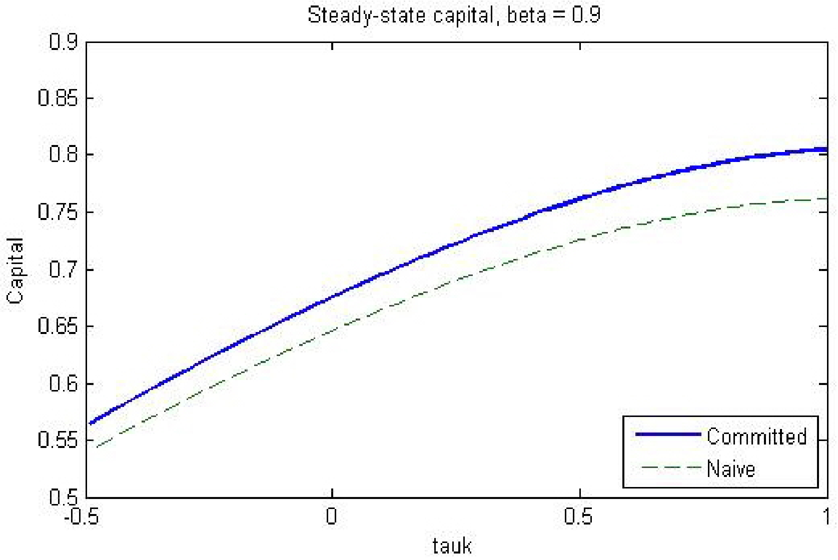
$$\tau_l w_t + \tau_k r_t k_t = 0$$

- Use this to eliminate $\tau_l$ and express growth in terms of $\tau_k$ alone
- The effects of tax policy can then be addressed
- How does the choice of $\tau_k$ affect the growth path for the naive consumer?

# Taxation



# Taxation

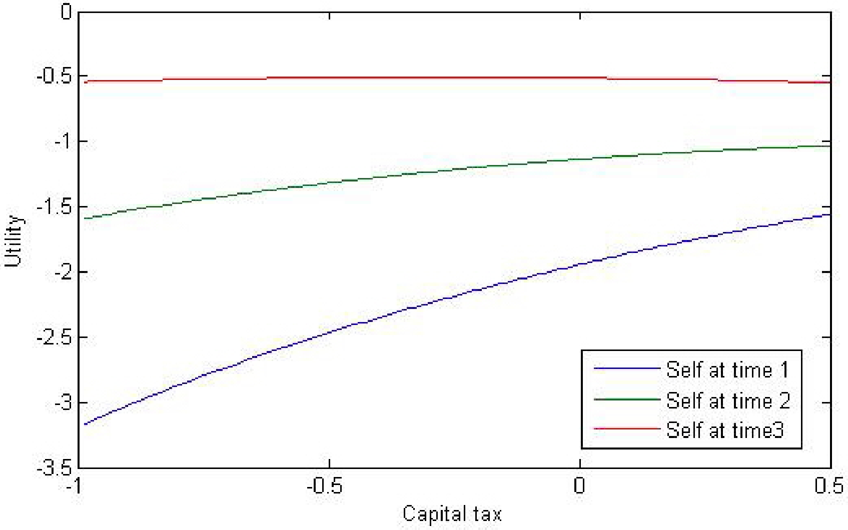


Steady-state capital, beta = 0.9
Capital
tauk
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
-0.5
0
0.5
1
Committed
Naive

# Taxation

- How can the tax policies be evaluated from a welfare perspective?
- The multiple selves of the Naive raise questions
- One approach is to use the evaluation of lifetime utility from time of birth
- An alternative is to seek unanimity on policy
  - Do the multiple selves agree on the policy?
- Welfare is now plotted for the multiple selves of the Naive

# Taxation



0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
Utility
-1
-0.5
0
0.5
Capital tax
Self at time 1
Self at time 2
Self at time3

# Conclusions

- Quasi-hyperbolic preferences distort savings patterns
- But in the long run the initial effect is diminished
- With successive generations the preferences have an effect
- This can motivate a tax intervention which will be unanimously supported by all the multiple selves