Private Protection and Public Policing[1]

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Abstract

abstract

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1 Introduction

Previous research studies how private protection can be a substitute for public protection (policing) services. This paper looks carefully at situations in which public and private protection are complementary, that is, when private protection must be coordinated with public protection to be effective. For example, home alarms deter theft by being connected to a local police station: if the police do not respond to a home alarm, the home alarm on its own is virtually useless in halting a crime in action. We show that very different optimal policy recommendations are generated when public and private protection are substitutes versus when they are complements.

Different types of private protection interact differently with public protection. On the one hand, bars on windows or speciality locks do not require police intervention to deter crime - they are substitutable forms of protection. The benefit from these forms of private protection diminishes with lower crime rate. On the other hand, a private home or business alarm is a complement for public protection services. For instance, in the well publicized April 2015 Hatton Garden gem heist, 200 million GBP worth of jewels were stolen. The police dismissed the burglar alarm at the time of the heist, however, had they responded, they may have been able to catch the robbers in action: the alarm on its own was ineffective without police response. Interestingly, the theoretical literature on the interaction between private and public protection focuses exclusively on situations where public and private protection are substitutes.

Tulkens and Jacquemin (1971) and Clotfelter (1977) were the first to discuss the optimal combination of public and private protection when public protection is a substitute for private protection. Clotfelter (1978) and Hotte and Van Ypersele (2008) investigate externalities generated by private protection and its impact on public policy but they both do not consider what would happen if public and private protection were complements. Helsley and Strange (2005) introduced the concept of a political incentive externality - how private protection influences the objective function of the government. They argue that increased local private policing diverts crime to other targets and reduces the aggregate expenditure on traditional policing, revealing another form of substitutability. Helsley and Strange (1999) explore the competition between gated communities and highlights the potential for strategic complementarity or substitutability. However, they abstract from public protection investment. Lee and Pinto (2009) explore the interaction between private and public protection in a multi-jurisdictions setting. They use a general model with an appropriation technology that could allow for complementarity, but only explore the case where private and public protection are substitutes. Ben-Shahar and Harel (1995) look at the question of incentivizing private protection effort. They acknowledge the possibility that private protection may be a substitute or complement for government effort, but they also assume away complementarity.

The empirical literature has found support for both substitutability and complementarity. In support of substitutability, Bartel (1975) finds that increased public protective expenditures reduces the demand for private protection. Friedman, Hakim, and Spiegel (1987) also discuss the shifts between public protection and private security by emphasizing the role of community size. Philipson and Posner (1996) find that when public anti-crime activities improved, the proportion of homes with burglar alarms drop significantly.

There is also evidence of complementarity between public and private protection. Focusing on environmental regulation, Langpap and Shimshack (2010) find that private enforcement crowds in public monitoring (complementarity), but crowds out public sanctions (substitutability). Ayres and Levitt (1998) investigate the impact of Lojack, a system designed for recovering stolen cars. A secret radio transmitter is placed in a car to enable the police to track it once it has been stolen. Because thieves cannot discover the transmitters, 95% of the Lojack-equipped stolen cars are recovered. As a result, this system reduces the profitability of car thefts along with the incidence of them.

We aim to look carefully at situations in which the complementarity may take place and to assess this in a less anecdotal, more comprehensive way using Canadian data. We construct a theoretical model to generate new insights, and to impose some structure for the empirical work. The model is a general equilibrium search model in the spirit of Decreuse, Mongrain and van Ypersele (2014). On the supply side of the market of crime, potential perpetrators of criminal activities choose whether or not to commit a crime. On the demand side, households invest in private protection by choosing both the size of their investment and potentially the type of protection. Types of protection vary with respect to their degree of substitutability or complementarity with public protection. The public sector is composed of an enforcement authority who decides on the allocation of resources. There are potentially two externalities: 1) a diversion externality (negative) where the investment by one household diverts crime to other households. This generally leads to over-investment in private protection, and excessively low crime rates; and 2) a deterrence externality (positive) where households do not consider the effect of their investment on criminal's decisions. This generally leads to under-investment in private protection, as well as an excessively high crime rate. We concentrate on the second type of externality, the deterrence externality, but all results are symmetrically opposite with the first type of externality.

In the section two, we develop the theoretical model with one form of public protection and one form of private protection, allowing for either gross complementarity or gross substitutability. We define the relationship between public and private protection as gross substitutes when public investment helps households who are not investing (or investing less) in private protection relatively more than those who are investing (or investing more) in private protection. Similarly, if public investment helps households who are investing (or investing more) in private protection relatively more than those who are not investing (or investing less), then public and private protection are gross complements. In this case, an increase in public protection increases the incentive to invest in private protection. We also identify when public and private protection can be net substitutes or net complements. That is, when government increases public protection, it reduces the crime rate. With a lower crime rate, there is less incentive to invest in any form of private protection. This endogenously introduces substitutability between private and public investment: forms of protection that are gross substitutes are also net substitutes, but some forms of protection that are gross complements could be net substitutes.

We characterize the constrained first best (CSB) outcome, where government chooses both the level of private and public protection investment. The first best outcome is trivial as it features no crime at all. We then compare the CSB outcome with a second best (SB) outcome, where the government chooses the level of investment in public protection and households choose their own private investment. In the SB outcome, we show that the crime rate is too high because the deterrence externality is not taken in account by households. The government can then respond two ways: 1) the manipulation response: the government takes actions intended to manipulate the level of investment in private protection to reduce crime. If public and private protection are complements, the government over-provides public enforcement to manipulate households to provide more private investment. If, instead, protections are substitutes, the government under-provides public protection, or 2) the compensating response: the government tries to reduce crime on its own by increasing investment in public protection. If private protection is complementary, there is a double benefit of investing in public protection: it reduces crime directly and indirectly via the increase in private investment. If protections are substitutes, it is more difficult to reduce crime because private investment in protection decreases.

In the third section, we look at a richer environment were two forms of private protection are available at the same time - one complement and one substitute. In order to link the theoretical model to the empirical model, we look at two particular forms of protection which are the most prevalent as recorded in the victimization survey of the Canadian General Social Survey (GSS): private alarms, which are gross complements for public protection, and bars on windows, which are gross substitutes. This will allow us to look at the distributional impact of public policy as these two forms of private protection are more intensively used by different income bracket households.

In the fourth section (to be completed), we use a linearized versions of the relevant predictions from the theoretical model in an empirical model. We supplement the victimization survey of the Canadian General Social Survey (GSS) that provides information on victimization and on private protection investments and household characteristics (income, dwelling type, family structure) with Census data on neighbourhood characteristics that may affect crime. We also use the Uniform Crime Reporting Survey, and match these three surveys using Forward Citation Areas. All proofs are in the Appendix.

2 Basic Model

The economy is composed of two types of agents and a government. There is a unitary mass of individuals who may be inclined to commit property crime. Each of those individuals chooses between attempting robbery or remaining honest. For expositional reason, we refer to those individuals as dishonest, even if a proportion choose to commit no crime. Dishonest individuals have heterogenous cost r of committing a crime. We assume that r is uniformly distributed between zero and one. Dishonest individuals wealth is normalized to zero. Let c be the proportion of dishonest individuals who actually commits a crime. Two components determine a criminal's payoff:

first, matching between a thief and a potential victim must happen for any crime to take place. We assume a constant return matching technology, where a criminal finds a victim with constant probability, ρ. Second, thieves who are successfully matched with a victim steal a certain fraction of the household's wealth. This fraction is determined in part by the household's investment in private protection. Thieves who are unsuccessful at finding a suitable victim get a payoff normalized to zero.

On the other side on the matching market are the households who are the potential victims. There is a mass H of households for whom the cost of committing a crime is arbitrarily large. Households are heterogenous in wealth w, which is uniformly distributed between zero and H. Each household is matched with a criminal with probability $c\rho/H$. A household loses a proportion $\ell_i(u)$ of their wealth, where $i = 0$ if the household is unprotected and $i = 1$ if the household invested in private protection. Private protection reduces potential losses, where the difference in expected losses is given by $\Delta(u) = \rho[\ell_0(u) - \ell_1(u)] > 0$. Losses are also affected by public protection u. Public enforcement reduces private losses, so $\ell_i'(u) < 0$. We assume that $\ell_i''(u) > 0$. If $\Delta'(u) > 0$ public protection will help relatively more privately protected households than non-protected ones. This will generate gross complementarity between public and private protection. When $\Delta'(u) < 0$, unprotected household benefit relatively more, so public and private protection will become gross substitutes. Obviously, the neutral case is given when $\Delta'(u) = 0$. Later on in the paper, we will formally develop substitutability versus complementarity, but for the moment these technologies are taken as given. Private protection carries a fixed cost F. We modelled private investment as a discrete choice to match the information contain in the Canadian GSS on victimization.

Government chooses the level of public protection, which has a unitary cost. The government maximizes the sum of honest agents' welfare. The timing is simple: the government chooses u, and then households choose whether or not to invest in private protection. Finally, dishonest individuals choose whether or not to commit a crime.

2.1 Private Protection Investment

A household with wealth w invests in private protection if the marginal benefit is larger or equal to the investment cost F. Since the marginal benefit is strictly increasing with wealth, households with wealth $w \geq \bar{w}(c,u)$ will invest in private protection where:

$$\bar{w}(c,u) = \frac{FH}{c\Delta(u)}. \tag{1}$$

An increase in the matching probability ρ, or an increase in the number of active criminals c increases investment in private protection. The sign of $\bar{w}_u(c,u)$ depends on the sign of $\Delta'(u)$. When $\Delta'(u) > 0$, public enforcement promotes investment in private protection since $\bar{w}_u(c,u) < 0$: public and private protection are gross complements. When $\Delta'(u) < 0$, public enforcement reduces private protection, $\bar{w}_u(c,u) > 0$: public and private protection are gross substitutes. Note that these gross effects are for a given crime rate. In equilibrium, the crime rate will depend on public protection so the net effect may be different.

2.2 Criminality Decision

Dishonest individuals attempt to commit a robbery if and only if the expected return is larger than the cost r. Let $R(\bar{w}(c,u),u)$ be the expected return of a robbery on a random household. Dishonest individuals with cost $r < \bar{r}$ attempt to commit a robbery, where $\bar{r} = \rho R(\bar{w}(c,u),u)$. The expected return of a robbery is given by:

$$R\big(\bar{w}(c,u),u\big) = \ell_0(u)\int_0^{\bar{w}(c,u)} \frac{w}{H}dw + \ell_1(u)\int_{\bar{w}(c,u)}^{H} \frac{w}{H}dw. \tag{2}$$

Lemma 1 describes the equilibrium investment in private protection and the dishonest individuals criminality decisions.

Lemma 1: *The equilibrium number of dishonest individuals attempting a robbery $c(u)$ and the equilibrium protection income threshold $\bar{w}(u)$ are determined by the following two equations:*

$$c(u) = \rho R\big(\bar{w}(u),u\big) = \rho\ell_0(u)\int_0^{\bar{w}(u)} \frac{w}{H}dw + \rho\ell_1(u)\int_{\bar{w}(u)}^{H} \frac{w}{H}dw; \tag{3}$$

$$\bar{w}(u) = \frac{FH}{c(u)\Delta(u)}. \tag{4}$$

Crime rate is then given by $\rho c(u)/H$.

The effect of a change in public protection has an ambiguous effect on private protection. As stated in Lemma 2 below, public protection displaces private protection when the elasticity of $\Delta(u)$ is smaller than the elasticity of $R(\bar{w}(u), u)$. This condition differs from the gross substitution condition $\Delta'(u) < 0$ stated before because crime rate is no longer given. An increase in public protection reduces crime, which reduces the incentive to invest in private protection. Only if public and private protections are sufficiently gross complements will private protection respond positively to public protection.

Lemma 2: *Public and private protections are net complements if and only if:*

$$\frac{u\ \Delta'(u)}{\Delta(u)} > \frac{-u}{R(\bar{w}(u), u)} \frac{\partial R(\bar{w}(u), u)}{\partial u}, \tag{5}$$

otherwise both forms of protections as net substitutes. Denote by $\bar{\Delta}'(u)$*, the value of* $\Delta'(u)$ *such that the condition above is satisfied with equality.*

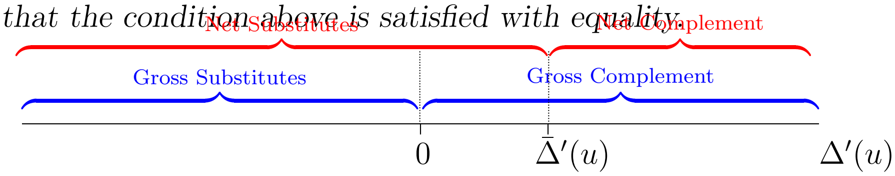

Figure 1: Net and Gross Substitute/Complement

When both forms of protection are gross substitutes, they are also net substitutes. Figure 1 highlight the difference between net and gross substitute/complement. The opposite is not necessarily true. Lemma 2 takes as given that crime rate is decreasing with public enforcement, which is true in most cases. When private and public protections are strong substitutes however, an increase in public enforcement could potentially lead to a large reduction in private protection such that the return to robbery increases. Conceptually interesting, this special case is not realistically relevant. We assume away this possibility.[1]

2.3 Constrained First Best

Obviously, the best possible outcome would be a world without crime and no protection spending. However, such an outcome is not useful in term of comparison.

[1] A sufficient condition for the equilibrium number of dishonest individuals attempting a robbery $c(u)$ to be decreasing with public protection is that $-\ell_1'(u) > -\ell_0'(u)/2$.

We define instead the constrained first best (CFB) as the outcome obtained by a government who maximizes the sum of all honest agents utility by choosing the level of public protection u and the set of households who invest in private protection $\tilde{w}$. Note that a government could potentially manipulate $\bar{w}(u)$ via taxes or subsidies to get to the desired $\tilde{w}$. This objective function is most commonly used in the law and economic literature.[2] The CFB problem face by the government is:

$$\max_{\{u,\tilde{w}\}} \left\{ H\frac{H}{2} - \rho c(\tilde{w},u)\ell_0(u)\int_0^{\tilde{w}} \frac{w}{H}dw - \rho c(\tilde{w},u)\ell_1(u)\int_{\tilde{w}}^{H} \frac{w}{H}dw - F\int_{\tilde{w}}^{H} dw - u \right\},$$

where $c(\tilde{w},u) = \rho R(\tilde{w},u)$ is the number of dishonest individuals attempting a robbery given $\tilde{w}$ and public protection u.

Definition 1: *The CFB investment in private protection $\tilde{w}^\star$ and in public enforcement $u^\star$ are respectively given by:*

$$\underbrace{\frac{\Delta(u^\star)}{H}\left[\frac{FH}{c(\tilde{w}^\star,u^\star)\Delta(u^\star)} - \tilde{w}^\star\right]}_{\text{Net private MB of lower losses (PvPPvG)}} = \underbrace{\rho\frac{\partial R(\tilde{w}^\star,u^\star)}{\partial \tilde{w}}}_{\text{Social MB of lowering crime (PvPPuG)}} \quad ; \qquad (6)$$

$$\underbrace{-\rho c(\tilde{w}^\star,u^\star)\frac{\partial R(\tilde{w}^\star,u^\star)}{\partial u}}_{\text{Private MB of lower losses (PuPPvG)}} - \underbrace{\rho R(\tilde{w}^\star,u^\star)\frac{\partial c(\tilde{w}^\star,u^\star)}{\partial u}}_{\text{Social MB of lower crime (PuPPuG)}} = \underbrace{1}_{\text{MC of public protection}} \quad ; \quad (7)$$

The first equation represents the optimal allocation of private protection. On the left hand side, we find the sum of all net private marginal benefits associated with lower losses net of the cost of investment in private protection. This is a privately provided private good (PvPPvG). On the right hand side is the public good aspect of private protection. More investment in private protection leads to lower crime rates. This is commonly referred to in the literature as the deterrence effect of private protection and is a privately provided public good (PvPPuG). As stated in Proposition 1 below, there is under-provision of private protection in equilibrium because households do not take in consideration the effect of their own protection decision on the crime rate. The investment in public protection is also governed by public and private benefit aspects. On the left hand side of the second equation is the private benefit of public investment, or the publicly provided private good (PuPPvG). When public investment increases, all households face lower expected losses for a given crime rate.

[2]See Hotte and van Ypersele (2008) for a discussion.

On the right hand side is the public good aspect of the public enforcement arising from a lower crime rate, and is a classic publicly provided public good (PuPPvG).

Proposition 1: *For any given level of investment in public protection, the decentralize income threshold $\bar{w}(u)$ for investing in private protection is higher than the CFB level $\tilde{w}$. Consequently, the decentralized equilibrium features under-provision of private protection.*

2.4 Second Best

If the government was able to directly subsidize private protection, the CFB could be achieved. The government would offer a subsidy equal to the social value of the deterrence effect, which is simply the right hand side of equation (**??**). It would then set public enforcement optimally, which is given by equation (**??**). A more challenging problem is what to do when a government has only one instrument, namely public protection? In such a case, the government problem is:

$$\max_{\{u\}} \frac{H^2}{2} - \rho c(u)\ell_0(u) \int_0^{\bar{w}(u)} \frac{w}{H} dw - \rho c(u)\ell_1(u) \int_{\bar{w}(u)}^{H} \frac{w}{H} dw - F \int_{\bar{w}(u)}^{H} dw - u.$$

The solution to this problem will be referred a second best (SB) solution. The goal is to understand in what manner this solution differs from the CFB solution and what is the role of complementarity and substitutability of the private and public investment.

Definition 2: *The SB investment in public enforcement u^* is given by:*

$$\underbrace{-\rho c(u^*) \frac{\partial R\big(\bar{w}(u^*), u^*\big)}{\partial u}}_{\text{Private MB of lower losses at } \bar{w}(u^*) > \tilde{w} \text{ (PuPPvG)}} - \underbrace{\rho^2 R\big(\bar{w}(u^*), u^*\big) \frac{\partial R\big(\bar{w}(u^*), u^*\big)}{\partial u}}_{\text{Social MB of lower crime for } \bar{w}(u^*) > \tilde{w} \text{ (PuPPuG)}} - \underbrace{F \frac{d\bar{w}(u^*)}{du}}_{\text{MB of manipulating private protection (PvPPuG)}} = 1 \tag{8}$$

The first two terms in Definition 2 combined with the marginal cost public investment represent the same trade off as in the first best situation. However, it is evaluated at a sub-optimal investment in private protection. The remaining term in Definition

2, represent the fact that the government may gain by altering public enforcement to manipulate private investments in a more desirable way. Using Definition 2, we can characterize differences in public enforcement, private protection and crime rates between the constrained first best and the second best. Proposition 1 already reveals that for a given level of public enforcement, there is less private protection investment in the second best compared to the desired level. We start by looking at one expression derived from the proof of Proposition 2 found bellow. It simply describes the difference between the SB first order condition on u and the similar first order condition for the CFB, both evaluated at the CFB solution $u^\star$. The expression is composed of three important part:

$$\underbrace{\rho\left[c(\bar{w}(u^\star),u^\star)\frac{-\partial R\big(\bar{w}(u^\star),u^\star\big)}{\partial u}-c(\tilde{w}^\star,u^\star)\frac{-\partial R(\tilde{w}^\star,u^\star)}{\partial u}\right]}_{\text{Compensation for under-investment in private protection via (PuPPvG)}}$$
$$+\underbrace{\rho^2\left[R(\bar{w}(u^\star),u^\star)\frac{-\partial R\big(\bar{w}(u^\star),u^\star\big)}{\partial u}-R(\tilde{w}^\star,u^\star)\frac{-\partial R(\tilde{w}^\star,u^\star)}{\partial u}\right]}_{\text{Adjustment for differences in crime rate elasticity (PuPPuG)}}$$
$$-\underbrace{F\,\frac{d\bar{w}\big(u^\star\big)}{du}}_{\text{Manipulation of private protection (PvPPuG)}}\lesseqgtr 0. \tag{9}$$

Whenever the expression above is positive, the government should provide more public enforcement in the SB compared to what should be done in the CFB. That is, the government should over-invest in enforcement. When the expression is negative, the government should under-invest. The last term represents the manipulation incentives faced by the government. Since private protection is too low, the government wish to increase private protection investment. When public and private protection are net complements, the government would like to over-invest in protection to stimulate private protection. When public and private protection are net substitutes, the government would like to under-invest in protection to force households to invest more in private protection.

The first two terms are much more delicate to interpret. We refer to the first term as compensation for under-investment in private protection. Because households do not internalize the positive externality associated with private protection, too few households invest. The government adjusts public investment to take this into account, and alter its provision of publicly provided private good. At first glance, we may think the government definitively wants to over-invest in public protection to compensate for the lack of private protection, but it is not necessarily the case. Obviously, criminality $c(\bar{w}(u^{\star}), u^{\star})$ is increasing with low private protection efforts (high W). If the efficacy of public protection at reducing private losses was constant, then the compensation effect would call for more protection. However, the efficacy of public protection at reducing private losses is only constant in a very particular case. Note that $\frac{-\partial R(W,u)}{\partial u} = -\ell_0'(u) + \Delta'(u)\int_W^H (w/H)dw > 0$. Imagine that private and public protection are gross substitutes. Then, $\frac{-\partial R(W,u)}{\partial u}$ is also increasing with W. Public protection help relatively more the unprotected, and there is a lot of them. Consequently, the marginal reduction in private losses associated with increases in public efforts is higher when private protection investments are low. Now, imagine that private and public protection are gross complement. When investment in private protection is low (high W), public protection effort does little to reduce private losses. Because of complementarity, public protection help relatively more the protected households, but there are few of them. Consequently, $\frac{-\partial R(W,u)}{\partial u}$ is now decreasing with W when $\Delta'(u) > 0$. If it is sufficiently decreasing, it is possible that the marginal reduction in private losses associated with the increase in public investment become lower when private protection investments are low. The government would then prefer to lower its public protection effort because it just not worthed, it just help too few. Lemma 3 bellow reveals the net complementarity is needed for this to be the case. The second term in (**??**) above represents a similar type of adjustment, but this time the government adjusts for the fact that criminals respond differently to public enforcement when private protection is lower. Because $c(w,u) = \rho R(w,u)$, the two adjustments are perfectly symmetric. **Lemma 3:** *When public and private protection are net substitutes, the manipulation incentives ask for under-investment in public protection, but the adjustment incentives call for over-investment. The reverse is true when public and private protections are net complements.*

Proposition 2: *When public and private protection are net neutral, the optimal investment in public protection is the same in the SB and in the CFB, but crime rate is higher for the CFB. Given a first degree approximation, when public and private protection are net complements, the optimal investment in public protection is larger in the SB and in the CFB, while the opposite is true when both form of protection are net substitutes. In both of those cases crime rate is high for the CFB.*

As seen in Lemma 3, the manipulation effect and the compounded compensation effects are acting in opposite directions, except for the case of net neutrality where bot effect are nil. With net neutrality, the government is unable to manipulate behaviour, and at the same time the marginal return on public protection effort is independent of private protection investment level. There is simply nothing to do differently for the government, but crime rate is higher because private protection is lower. Starting form net neutrality, if investments becomes more complementary, the manipulation effect calls for more public investments, but the compensation effect suggest to shy away from public investment because the return is too low. The value of manipulating versus adjusting determines the appropriate response from the enforcement authority. If we concentrate on the first order effects, by taking a linear approximation Proposition 2 shows that the manipulation effect dominates.

3 Bars and Alarms

The preceding exercise allows us to derive some results about optimal public enforcement policy in a tractable fashion. In this section, we link the different forms of substitutability and complementarity with observable forms of private protection. To fully capture the interaction between public and private enforcement, we now introduce two different forms of private protection. Because of the technological differences, but also because of their prevalence and the information available in victimization survey of the Canadian General Social Survey (GSS), we look at alarms as one of the private investments, with the other being bars on windows or specialty locks.

3.1 Data and Stylized Facts – In construction!!!!

This section is currently based on Decreuse, Mongrain and Van Ypersele (2015). A new version with a more adapted data selection and different focus will be available soon.

The Canadian GSS provide us with cross-sectional individual data on victimization. This survey is conducted every five years, or cycles. We consider years 1999, 2004, and 2009, which correspond to cycles 13, 18, and 23. The survey samples dwellings. The respondent may be anyone belonging to the household. We only keep the following types of households: singles or couples with or without children.

This excludes less traditional households with additional relatives and non relatives at home. Such households lead to never-ending questions as to the identity of wage-earners and the respondent's statute in the household. We also exclude observations where the respondent is a child. Household income is not well reported in such cases, and time use is missing for the parents. There are five category of investments in private protection. We abstract for the one where individuals are asked if there are changing their habits to lower their exposure to crime. Not only the question is vague, but the variable almost independent of individual's characteristics. We also abstract form guns and dogs as a method of protection because of their limited use. In the survey, respondents declare whether they have ever installed an alarm and if they have ever installed locks or bars. The variable PR proxies the magnitude of household investment. It is equal to 0 when neither bars/locks nor an alarm have ever been installed. It is equal to 1 when only bars/locks have been installed, to 2 when only an alarm, and to 3 when both have been installed. The quantitative values taken by this variable are somewhat arbitrary. Thus the other variable $PA = 1$ when there is an alarm and $PA = 0$ otherwise. For robustness purposes, we also consider the dummy variable $PA2$, which takes the value one when the household has installed an alarm or bars/locks, and $PA3$, which takes the value one when the household has an alarm, bars/locks and the respondent holds a gun. Tables A1 and A2 in Appendix II show that all these variables similarly vary across household income.

The household characteristics are as follows. A set of dummy variables describes the household type: couple, single woman, single man. Then come the number of children, the household age (defined as the mean age in five-year classes of the leading members of the household) and education (defined as the highest level of education of the leading members). We include a dummy equal to 1 when the leading members of the household are all occupied full time (because they follow an educational program or because they work). We also consider individual lifetime victimization. This includes the events presented above (break-in, property damage, theft outside property), but also events like assault, sexual or moral harassment, rape, etc. We exclude the events presented above and that occurred during the previous 12 months. We finally include the following dwelling characteristics: three dummy variables describing the dwelling type (detached house, semi-detached and apartment), the age of the household in the dwelling, and a dummy equal to one if the household owns the dwelling.

Household income is declared in 10 classes. We attribute the class mean to each income class. We then divide by the contemporaneous census metropolitan area (CMA) average and compute the log of this ratio. Thus household income is specific to each CMA. This is in line with the model where criminals are mobile within CMAs but not across them.

3.2 Model with Bars and Alarms

A household who does not invest in any form of protection suffers a loss of $\ell_0(u)$ if targeted as in the previous section. Technology A requires police response to be effective. A household alarm system is the perfect example of such technology. The indicator function $a \in \{0,1\}$ determines if a household invests in such a technology. If a household invests in protection A, the loss is then given by $\ell_0(u) - A(u)$, meaning that $\Delta_A(u) = \rho A(u)$, where $A'(u) > 0$. The motivation is that public enforcement reduces expected losses for all households, but alarm-protected households receive an additional benefit from public responses to alarms. The second form of private protection B, is assumed to have no interaction with public protection. Bars on windows or specialty locks are good examples. The indicator function $b \in \{0,1\}$ determines if the individual invests in this second technology. Investments in this form of protection reduces losses by $\ell_0(u) - B$.[3] Consequently, $\Delta_B(u) = \rho B$. Investment in technology $i \in \{A, B\}$ costs F_i. We assume that alarms are more costly, so $F_A > F_B$. A thief matched with a household imposes a loss $L(a, b; u)w$. The loss function is defined the following way:

$$L(a,b;u) = \begin{cases} \ell_0(u) & \text{if } a = 0 \text{ and } b = 0; \\ \ell_0(u) - A(u) & \text{if } a = 1 \text{ and } b = 0; \\ \ell_0(u) - B & \text{if } a = 0 \text{ and } b = 1; \\ \ell_0(u) - A(u) - B + D & \text{if } a = 1 \text{ and } b = 1, \end{cases} \tag{10}$$

where $D < B$ represents the fact that the marginal benefit of installing a given protection measure is lower when the alternative measure is already installed. In other words, there may be redundancy between the two forms of protection. We assume that $A(u) \in \left[B\frac{F_A}{F_B} - D\frac{F_A - F_B}{F_B}, B\frac{F_A}{F_B}\right]$ for all relevant values of u. Whenever

[3] One could argue that bars or locks may in fact help unprotected houses relatively more. As we saw in the previous section however, gross substitutes and gross neutral are both cases of net substitutability.

this condition is not satisfied, no household invests solely in protection A or solely in protection B, however, these cases are empirically irrelevant.

Denote by $N_i(c, u)$ the mass of individual who invest a positive amount in technology $i \in \{A, B\}$ to be determined later.

3.3 Investment in Private Protection

A household with wage w, maximizes utility $u(a, b; w)$, by choosing investment a and b, where:

$$u(a, b; w) = w - \frac{\rho c}{H} L(a, b; u) w - aF_A - bF_B. \tag{11}$$

Definition 3: *Define $\bar{w}_{\{0,a\}}(c, u)$ as the revenue such that an individual is indifferent between having protection of type A and having no protection. Define $\bar{w}_{\{b,ab\}}(c, u) > \bar{w}_{\{0,a\}}(c, u)$ as the revenue such that an individual is indifferent between having protection of type A in addition to protection of type B and having protection only of type B. Similarly, we can define $\bar{w}_{\{0,b\}}(c)$ and $\bar{w}_{\{a,ab\}}(c) > \bar{w}_{\{0,b\}}(c)$ for the investment in protection B. The different revenues cutoffs are given by:*

$$\bar{w}_{\{0,a\}}(c, u) = \frac{HF_A}{\rho c A(u)} \quad \text{and} \quad \bar{w}_{\{b,ab\}}(c, u) = \frac{HF_A}{\rho c [A(u) - D]};$$

$$\bar{w}_{\{0,b\}}(c) = \frac{HF_B}{\rho c B} \quad \text{and} \quad \bar{w}_{\{a,ab\}}(c) = \frac{HF_B}{\rho c (B - D)};$$

Definition 4: *Define $\bar{w}_{\{a,b\}}(c, u) = \frac{H[F_A - F_B]}{\rho c [A(u) - B]}$ as the income level such that investing only in A is equivalent to investing only in B.*

Lemma 4: *Investment in private protection as a function of income is given by:*

$$\begin{cases} \text{none} & \text{if } w \leq \bar{w}_{\{0,b\}}(c); \\ \text{only } B & \text{if } \bar{w}_{\{0,b\}}(c) < w \leq \bar{w}_{\{a,b\}}(c, u); \\ \text{only } A & \text{if } \bar{w}_{\{a,b\}}(c, u) < w \leq \bar{w}_{\{a,ab\}}(c); \\ \text{both } A \text{ and } B & \text{if } w > \bar{w}_{\{a,ab\}}(c). \end{cases}$$

Total investments are $N_A(c, u) = \frac{1 - \bar{w}_{\{a,b\}}(c,u)}{H}$, and $N_B(c, u) = \frac{\bar{w}_{\{a,b\}}(c,u) - \bar{w}_{\{0,b\}}(c) + 1 - \bar{w}_{\{a,ab\}}(c)}{H}$.

To understand Lemma 4, we refer to Figure 2. The red line corresponds to the lower contour of the sum of the loss associated with theft and total expenditures on private protection. Naturally, low income individuals do not invest in protection. Above a certain point, individual are wealthy enough to invest in the cheaper form of investment, so investment B. Above that, individuals are ready to chose the more efficient, but more expensive investment A. The wealthiest individual invest in both.

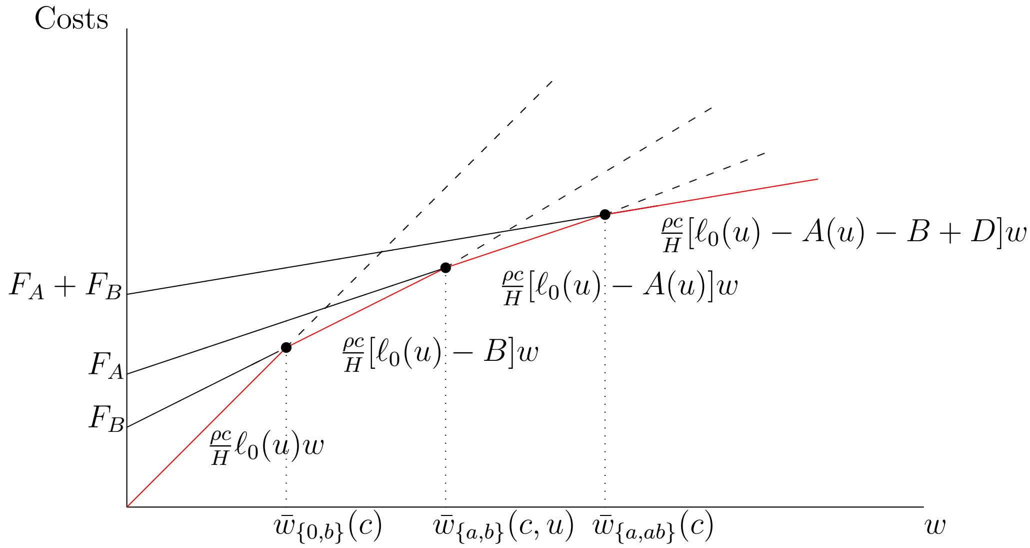

Figure 2: Private Protection Investments.

Taking crime rate as constant, public protection and private investment in protection A are gross complements. As in the previous section, the driving force of this complementarity is the fact that public spending benefits relatively more the individuals who invest in A compared to those who did not invest in A. In mathematical terms, $\Delta'_A(u) > 0$. Interestingly, public protection and private investment in B are gross substitutes. Investments in public protection u displaces private investment in protection B, even if it does not directly influence the return on such investments, because an increase in u increases investment in protection A relative to protection B. As public investment on protection increases, $\bar{w}_{\{a,b\}}(c,u)$ moves to the left on Figure 2. Lemma 5 below summarizes the effect of a change in u on the prevalence of both types of private protections.

Lemma 5: *The effects of a change in public protection u on the prevalence of private* *protection A and B is the following:*

$$\frac{\partial N_A(c,u)}{\partial u} = \frac{(F_A - F_B)A'(u)}{\rho c[A(u)-B]^2} > 0 \text{ and } \frac{\partial N_B(c,u)}{\partial u} = -\frac{(F_A - F_B)A'(u)}{\rho c[A(u)-B]^2} < 0.$$

We will now discuss the impact of a change in the number of criminals, c, on investment on private protection A and B. An increase in the number of criminals augments the expected benefit of investing in both forms of private protection, but it also increase the relative expected benefit of investing in protection A.

Lemma 6: *Higher crime rate $\rho c/H$ stimulates investments in private protection A and B if and only if $A(u) > \left[\frac{B-D}{2B-D}\right] B\frac{F_A}{F_B} + \frac{B^2}{3B-D}$.*

Initially, low-to-mid income individuals invest only in protection B, while mid-to-high income individuals invest only in protection A. High income individuals invest in both. An increase in the number of criminals induces additional low income households, who were previously not investing in protection, to investment in B and more high income individuals, who were previously investing only in A, to invest in both. More mid-income individuals invest in A instead of B as investment A is more productive than B, however, it is also more costly. Regardless, total private protection increases. All households who switch from B to A choose a more productive but more costly form of protection.

3.4 Equilibrium Crime Rate and Private Protection

From Lemma 4, we can define $a(w;c,u) \in \{0,1\}$ and $b(w;c,u) \in \{0,1\}$ as the private protection investment choices by an individual with wealth w, given public policy u and crime intensity c. A dishonest individual who chooses thievery has an expected payoff of $\rho R\left(\bar{W}(c,u),u\right)$ where $\bar{W}(c,u)$ is a vector of investment decisions in private protection A and B. The expected returns is given by:

$$\begin{aligned} R\left(\bar{W}(c,u),u\right) =& \ell_0(u)\frac{H}{2} - A(u)\int_{\bar{w}_{\{a,b\}}(c,u)}^{H}\frac{w}{H}dw \\ & - B\int_{\bar{w}_{\{0,b\}}(c,u)}^{\bar{w}_{\{a,b\}}(c,u)}\frac{w}{H}dw - (B-D)\int_{\bar{w}_{\{a,ab\}}(c,u)}^{H}\frac{w}{H}dw \qquad (12) \end{aligned}$$

Lemma 7: *The equilibrium number of dishonest individuals attempting a robbery $c(u)$ and the equilibrium protection income threshold vector $\bar{W}(u)$ are determined by the following four equations:*

$$c(u) = \rho R\big(\bar{W}(u), u\big);$$

$$\bar{w}_{\{0,b\}}(u) = \frac{HF_B}{\rho c(u) B}, \quad \bar{w}_{\{a,b\}}(u) = \frac{H[F_A - F_B]}{\rho c(u)[A(u) - B]} \text{ and } \bar{w}_{\{a,ab\}}(u) = \frac{HF_B}{\rho c(u)(B - D)}$$

Crime rate is then given by $\rho c(u)/H$.

Note that with this particular loss function, crime is always decreasing with public enforcement.[4] Since public protection and private protection B are gross substitutes, it is not surprising to learn that they are also net substitutes. Below is the condition for net complementarity for public protection and private protection A. The condition is similar to the condition with only one form of protection except that net complementarity is easier to achieve. The right hand side of the condition below shows that as public protection increases, crime rate is lower and so there is less incentive to invest in any form of protection. In other words, this represents the net effect. On the right hand side is the gross complementarity effect. It is multiplied by a number larger than one due to the fact that, at the margin, households are already investing in some form of protection. They are much more responsive to a change in public protection.

Lemma 8: *Public protection and private protection B are always net substitutes, while public protection and private protection A are net complements if and only if:*

$$\frac{uA'(u)}{A(u)} \left(\frac{A(u)}{A(u) - B}\right) \left[1 + \frac{R(\cdot)}{H} \left[B\bar{w}_{\{0,b\}}(u)^2 + (B - D)\bar{w}_{\{a,ab\}}(u)^2\right]\right] > \frac{-u}{R\left(\bar{W}(u), u\right)} \frac{\partial R\left(\bar{W}(u), u\right)}{\partial u}$$

3.5 Constrained First Best

We define the vector of private protections $\tilde{W}$ a government would choose if it were able to force private investment decisions. The CFB problem face by the government

[4] The effect of u on $c(u)$ is given by $\frac{dc(u)}{du} = \frac{\rho H}{2} \frac{A'(u)\left(1+\bar{w}_{\{a,b\}}(u)^2/H^2\right) - \ell_0'(u)}{1+\frac{\rho}{Hc(u)}\left[B\bar{w}_{\{0,b\}}(u)^2+[A(u)-B]\bar{w}_{\{a,b\}}(u)^2+(B-D)\bar{w}_{\{a,ab\}}(u)^2\right]}$. This corresponds to a case where the condition of Lemma 2 is satisfied.

is:

$$\max_{\{u,\tilde{w}\}} \left[H - \rho c(\tilde{W},u)\ell_0(u)\right]\frac{H}{2} + \rho c(\tilde{W},u)A(u)\int_{\tilde{w}_{\{a,b\}}}^{H}\frac{w}{H}dw$$
$$+\rho c(\tilde{W},u)B\int_{\tilde{w}_{\{0,b\}}}^{\tilde{w}_{\{a,b\}}}\frac{w}{H}dw + \rho c(\tilde{W},u)(B-D)\int_{\tilde{w}_{\{a,ab\}}}^{H}\frac{w}{H}dw$$
$$-\int_{\tilde{w}_{\{a,b\}}}^{H}F_A dw - \int_{\tilde{w}_{\{0,b\}}}^{\tilde{w}_{\{a,b\}}}F_B dw - \int_{\tilde{w}_{\{a,ab\}}}^{H}F_B dw - u.$$

Definition 5: *The CFB investment in private protection vector $\tilde{W}^{\star}$ and in public enforcement $u^{\star}$ are respectively given by:*

$$\underbrace{\frac{c(\tilde{W}^{\star},u^{\star})B}{H}\left[\frac{F_B H}{\rho c(\tilde{W}^{\star},u^{\star})B} - \tilde{w}^{\star}_{\{0,b\}}\right]}_{\text{Net private MB of lower losses (PvPPvG } w_{\{0,b\}})} = \underbrace{\rho R(\tilde{W}^{\star},u^{\star})\frac{\partial R(\tilde{w}^{\star},u^{\star})}{\partial \tilde{w}_{\{0,b\}}}}_{\text{Social MB of loweri crime (PvPPuG } w_{\{0,b\}})} \quad ; \tag{13}$$

$$\underbrace{\frac{c(\tilde{W}^{\star},u^{\star})[A(u)-B]}{H}\left[\frac{(F_A-F_B)H}{\rho c(\tilde{W}^{\star},u^{\star})[A(u)-B]} - \tilde{w}^{\star}_{\{a,b\}}\right]}_{\text{Net private MB of lower losses (PvPPvG } w_{\{a,b\}})} = \underbrace{\rho R(\tilde{W}^{\star},u^{\star})\frac{\partial R(\tilde{w}^{\star},u^{\star})}{\partial \tilde{w}_{\{a,b\}}}}_{\text{Social MB of loweri crime (PvPPuG } w_{\{a,b\}})} \quad ; \tag{14}$$

$$\underbrace{\frac{c(\tilde{W}^{\star},u^{\star})(B-D)}{H}\left[\frac{F_B H}{\rho c(\tilde{W}^{\star},u^{\star})B} - \tilde{w}^{\star}_{\{a,ab\}}\right]}_{\text{Net private MB of lower losses (PvPPvG } w_{\{a,ab\}})} = \underbrace{\rho R(\tilde{W}^{\star},u^{\star})\frac{\partial R(\tilde{w}^{\star},u^{\star})}{\partial \tilde{w}_{\{a,ab\}}}}_{\text{Social MB of loweri crime (PvPPuG } w_{\{a,ab\}})} \quad ; \tag{15}$$

$$\underbrace{-\rho c(\tilde{W}^{\star},u^{\star})\frac{\partial R(\tilde{W}^{\star},u^{\star})}{\partial u}}_{\text{Private MB from lower losses (PuPPvG)}} - \underbrace{\rho R(\tilde{w}^{\star},u^{\star})\frac{\partial c(\tilde{W}^{\star},u^{\star})}{\partial u}}_{\text{Social MB of lower crime (PuPPuG)}} = \underbrace{1}_{\text{MC of public protection}} \quad ; \tag{16}$$

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5 Appendix I: Proof

Proof of Lemma 1: The number of criminals $c(u)$ is given by $c(u) = \int_0^{\bar{r}} dr$, and so $c(u) = \rho R(\bar{w}(c,u),u)$. By substituting (**??**) in (**??**), we obtain equation (**??**) that determined $c(u)$. We can then substitute back to get $\bar{w}(u) = \frac{FH}{c(u)\Delta(u)}$. QED

Proof of Lemma 2: Comparative static on implicit equations (**??**) and (**??**) reveals that:

$$\frac{d\bar{w}(u)}{du} = -\left(\frac{\bar{w}(u)}{u}\right)\left[\frac{\frac{u}{R(\bar{w}(u),u)}\frac{\partial R(\bar{w}(u),u)}{\partial u} + \frac{u\Delta'(u)}{\Delta(u)}}{1 + \frac{\bar{w}(u)}{R(\bar{w}(u),u)}\frac{\partial R(\bar{w}(u),u)}{\partial w}}\right]. \tag{17}$$

Consequently, $\frac{d\bar{w}(u)}{du} < 0$ if and only if $\frac{u\Delta'(u)}{\Delta(u)} > \frac{-u}{R(\bar{w}(u),u)}\frac{\partial R(\bar{w}(u),u)}{\partial u}$. QED

Proof of Proposition 1: The term inside the bracket on the LHS of (**??**) is the same as the investment decision $\bar{w}(u)$, but where $c(w,u)$ is evaluated at $\tilde{w}$ instead of $\bar{w}(u)$. We will denote this variable by $\bar{w}(u)|_{\tilde{c}}$ Since the RHS is always positive, then $\bar{w}(u)|_{\tilde{c}} > \tilde{w}$. We will now show by contradiction that $\bar{w}(u) > \tilde{w}$. Imagine that $\tilde{w} > \bar{w}(u)$, then it would imply that $\bar{w}(u)|_{\tilde{c}} > \bar{w}(u)$, but this can be true only when $\bar{w}(u)|_{\tilde{c}} < \bar{w}(u)$. Consequently, $\bar{w}(u) > \tilde{w}$ for all combination of u and $c(u)$. QED

Origin of Definition 2: The first order condition for the SB problem with respect to u is given by:

$$\begin{aligned} -\rho c(u^*)\frac{\partial R\big(\bar{w}(u^*),u^*\big)}{\partial u} - 1 - \rho R\big(\bar{w}(u^*),u^*\big)\frac{dc(u^*)}{du} \\ -\frac{c(u^*)\Delta(u^*)}{H}\left[\bar{w}(u^*) - \frac{FH}{c(u^*)\Delta(u^*)}\right]\frac{d\bar{w}(u^*)}{du} = 0. \end{aligned} \tag{18}$$

Since $\bar{w}(u^*) = \frac{FH}{c(u^*)\Delta(u^*)}$, the term on the second line is zero. By rewriting $\frac{dc(u^*)}{du}$ using the fact that $c(u) = \rho R(\bar{w}(u),u)$ and decomposing in partial derivative $\frac{dc(u^*)}{du} = \rho\frac{\partial R(\bar{w}(u^*),u^*)}{\partial u} + \rho\frac{\partial R(\bar{w}(u^*),u^*)}{\partial w}\frac{d\bar{w}(u^*)}{du}$, we get equation (**??**). QED

Proof of Lemma 3: The sign of the manipulation incentives comes directly form Lemma 2. The sign of the adjustment incentives is positive or negative depending on wether $-R(w,u)R_u(w,u)$ is positive or negative. The slope is given by $-\rho^2 R_w(\cdot)R_u(\cdot) - \rho^2 R(\cdot)R_{uw}(\cdot)$, which can be written as $\frac{\rho R(\cdot)\Delta(u)}{u}\left[\frac{uR_u(\cdot)}{R(\cdot)} + \frac{u\Delta'(u)}{\Delta(u)}\right]$. QED

Proof of Proposition 2: We start by evaluating the SB first order condition at $u^\star$. Note that the equality is no longer guaranteed:

$$-2\rho c(u^\star)\frac{\partial R\big(\bar{w}(u^\star),u^\star\big)}{\partial u} - 1 - \rho c(u^\star)\frac{\partial R\big(\bar{w}(u^\star),u^\star\big)}{\partial \bar{w}}\frac{d\bar{w}(u^\star)}{du} \lesseqgtr 0. \tag{19}$$

To properly differentiate between $\bar{w}(u^\star)$ and $\tilde{w}^\star$, we replace $c(u)$ by $\rho R(w,u)$. Subtracting the first order condition for the CFB problem we get the following:

$$2\rho\left[R\big(\bar{w}(u^\star),u^\star\big)\frac{-\partial R\big(\bar{w}(u^\star),u^\star\big)}{\partial u} - R(\tilde{w}^\star,u^\star)\frac{-\partial R(\tilde{w}^\star,u^\star)}{\partial u}\right] \\ -F\ \frac{d\bar{w}\big(c(u^\star),u^\star\big)}{du} \lesseqgtr 0. \tag{20}$$

Using a linear approximation of the first term, we get:

$$-2\frac{\Delta(u^\star)R\big(\bar{w}(u^\star),u^\star\big)}{H}\left[\bar{w}(u^\star)-\tilde{w}^\star\right]\frac{\bar{w}(u^\star)}{u}\left[\frac{u^\star\Delta'(u^\star)}{\Delta(u^\star)}+\varepsilon_{R,u}\right] \\ -F\ \frac{d\bar{w}\big(c(u^\star),u^\star\big)}{du} \lesseqgtr 0, \tag{21}$$

where $\varepsilon_{R,u} = \frac{u^\star}{R\left(\bar{w}(u^\star),u^\star\right)}\frac{\partial R\left(\bar{w}(u^\star),u^\star\right)}{\partial u}$. Using Lemma 2, the expression above becomes:

$$\frac{\bar{w}(u^\star)}{u^\star H}\left[\frac{u^\star\Delta'(u^\star)}{\Delta(u^\star)}+\varepsilon_{R,u}\right]\left[\frac{HF}{1+\varepsilon_{R,\bar{w}}} - 2\Delta(u^\star)R\big(\bar{w}(u^\star),u^\star\big)\left[\bar{w}(u^\star)-\tilde{w}^\star\right]\right] \lesseqgtr 0, \tag{22}$$

where, $\varepsilon_{R,\bar{w}} = \frac{u^\star}{R\left(\bar{w}(u^\star),u^\star\right)}\frac{\partial R\left(\bar{w}(u^\star),u^\star\right)}{\partial \bar{w}}$. Equivalently, we get:

$$\Delta(u^\star)R\big(\bar{w}(u^\star),u^\star\big)\frac{\bar{w}(u^\star)}{u^\star H}\left[\frac{u^\star\Delta'(u^\star)}{\Delta(u^\star)}+\varepsilon_{R,u}\right]\left[2\tilde{w}^\star - \frac{1+2\varepsilon_{R,\bar{w}}}{1+\varepsilon_{R,\bar{w}}}\bar{w}(u^\star)\right] \lesseqgtr 0. \tag{23}$$

From the first order condition (**??**) of the CFB, we can show that:

$$\left[\frac{FH}{c(\tilde{w}^\star,u^\star)\Delta(u^\star)} - \tilde{w}^\star\right] = \tilde{w}^\star; \tag{24}$$

We can rewrite this condition as $\bar{w}(u^\star)|_{c(\tilde{w}^\star,u^\star)} = 2\tilde{w}^\star$, where $\bar{w}(u^\star)|_{c(\tilde{w}^\star,u^\star)} = \frac{FH}{c(\tilde{w}^\star,u^\star)\Delta(u^\star)}$. We aslo have that $\bar{w}(u^\star)|_{c(\tilde{w}^\star,u^\star)} > \bar{w}(u^\star)$, so $\bar{w}(u^\star) < 2\tilde{w}^\star$. Consequently, the term $2\tilde{w}^\star - \frac{1+2\varepsilon_{R,\bar{w}}}{1+\varepsilon_{R,\bar{w}}}\bar{w}(u^\star)$ in the expression above is always positive. We can then conclude that when public protection is a net complement to private protection, $\frac{u^\star\Delta'(u^\star)}{\Delta(u^\star)} + \varepsilon_{R,u} > 0$, the SB investment in public protection is larger than the CFB investment. When private and public investments are net substitutes, the SB investment is lower than the CFB investment. QED

Proof of Lemma 4: Looking at Figure 1 is helpful in understanding the proof. Given the assumption made about $A(u)$, we have that $\bar{w}_{\{0,b\}}(c) < \bar{w}_{\{0,a\}}(c,u) < \bar{w}_{\{a,ab\}}(c)$. Moreover, it must be the case that $\bar{w}_{\{a,b\}}(c,u) \in [\bar{w}_{\{0,b\}}(c), \bar{w}_{\{a,ab\}}(c)]$. Individuals with $w < \bar{w}_{\{0,b\}}(c)$ do not invest in any protection. Individuals with income between $\bar{w}_{\{0,b\}}(c)$ and $\bar{w}_{\{a,b\}}(c,u)$ prefer to invest in protection B over protection A and they prefer investing in protection B as opposed to not investing at all. This implies that $\bar{w}_{\{0,a\}}(c,u)$ is irrelevant in the decision making. Above $\bar{w}_{\{a,b\}}(c,u)$, $F_B + \frac{\rho c}{H}[\ell_0(u) - B]w$ is dominated by $F_A + \frac{\rho c}{H}[\ell_0(R) - A(u)]w$. Consequently, individuals between $\bar{w}_{\{a,b\}}(c,u)$ and $\bar{w}_{\{a,ab\}}(c)$ invest only in A. Individuals with income above $\bar{w}_{\{a,ab\}}(c)$ invest in both. QED

Proof of Lemma 5: All results are simple comparative statics of $N_A(c,u)$ and $N_B(c,u)$. QED

Proof of Lemma 6: The effect of c on $N_B(c,u)$ is:

$$\frac{\partial N_B(c,u)}{\partial c} = \frac{1}{\rho c^2}\left[\frac{F_B}{B} + \frac{F_B}{B-D} - \frac{F_A - F_B}{A(u) - B}\right].$$

The sign of the expression above is given by $A(U) - \left[\frac{B-D}{2B-D}\right] B\frac{F_A}{F_B} - \frac{B^2}{2B-D}$. QED

Proof of Lemma 7: The proof is identical to proof of Lemma 1.

Proof of Lemma 8: We start by looking at the total effect of u on the vector $\bar{W}(u)$ using Cramer's rule.

$$\frac{d\bar{w}_{\{0,b\}}(u)}{du} = \frac{\rho H \bar{w}_{\{0,b\}}(u)}{2c(u)} \frac{A'(u)\left(1 + \bar{w}_{\{a,b\}}(u)^2/H^2\right) - \ell_0'(u)}{1 + \frac{\rho}{Hc(u)}\left[B\bar{w}_{\{0,b\}}(u)^2 + [A(u) - B]\bar{w}_{\{a,b\}}(u)^2 + (B-D)\bar{w}_{\{a,ab\}}(u)^2\right]} > 0$$

$$\frac{d\bar{w}_{\{a,b\}}(u)}{du} = \bar{w}_{\{a,b\}}(u) \frac{\frac{A'(u)}{A(u)-B}\left[1 + \frac{\rho}{Hc(u)}\left[B\bar{w}^2_{\{0,b\}} + (B-D)\bar{w}^2_{\{a,ab\}}\right]\right] + \frac{\rho}{c(u)}\frac{\partial R\left(\bar{W}(u),u\right)}{\partial u}}{1 + \frac{\rho}{Hc(u)}\left[B\bar{w}_{\{0,b\}}(u)^2 + [A(u) - B]\bar{w}_{\{a,b\}}(u)^2 + (B-D)\bar{w}_{\{a,ab\}}(u)^2\right]}$$

$$\frac{d\bar{w}_{\{a,ab\}}(u)}{du} = \frac{\rho H \bar{w}_{\{a,ab\}}(u)}{2c(u)} \frac{A'(u)\left(1 + \bar{w}_{\{a,b\}}(u)^2/H^2\right) - \ell_0'(u)}{1 + \frac{\rho}{Hc(u)}\left[B\bar{w}_{\{0,b\}}(u)^2 + [A(u) - B]\bar{w}_{\{a,b\}}(u)^2 + (B-D)\bar{w}_{\{a,ab\}}(u)^2\right]} > 0$$

The impact of public investment in private protection A is give by $\frac{dN_A(u)}{du} = -\frac{1}{H}\frac{d\bar{w}_{\{a,b\}}(u)}{du}$, and the impact of public investment in private protection B is give by $\frac{dN_B(u)}{du} = \frac{1}{H}\left[\frac{d\bar{w}_{\{a,b\}}(u)}{du} - \frac{d\bar{w}_{\{0,b\}}(u)}{du} - \frac{d\bar{w}_{\{a,ab\}}(u)}{du}\right]$, which is always negative. **QED**

6 Appendix II: Tables