

The Supermodularity of the Tax Competition Game

Grégoire Rota-Graziosi

January 31, 2016

CERDI-CNRS, Université d'Auvergne, France.

Email address: gregoire.rota-graziosi@udamail.fr

Abstract

We determine sufficient conditions on the concavity of the marginal production function to establish the supermodularity of a n asymmetric country tax competition, when countries maximize their tax revenue. We use the notion of extended concavity, more specifically this of ρ -concavity. The tax competition game is supermodular when the marginal production function (or equivalently the inverse demand for capital) of each country is log-concave and $1/2$ -convex. These conditions allow us to bound the degree of curvature of the inverse demand for capital. Applying some results from supermodular games, we deduce the existence and uniqueness of the Nash equilibrium. We show also that any increase in the number of tax-competing jurisdictions decreases tax rates and tax revenues and improves the net return of capital. Establishing similar sufficient conditions for the supermodularity of the tax competition game with welfare maximizers raises multiple issues. Besides the question of the nature of public spending (complement or substitute of private consumption), we discuss the role of capital by considering successively an elastic worldwide stock of capital and capital ownership.

Keywords: Tax competition; tax coordination; supermodularity; ρ -concavity and ρ -convexity.

JEL classification: H25; H77; H87; C72.

1 Introduction

Is tax competition harmful? Can tax coordination be Pareto improving? These questions among others have been addressed in the literature of tax competition initiated by the seminal articles of Zodrow and Mieszkowski (1986) and Wilson (1986). One of the main conclusions of the literature recently reviewed by Keen and Konrad (2013) is that international tax competition would trigger a “race to the bottom.” In other words, the Nash equilibrium of the standard tax competition game would be characterized by too low tax rates and consequently an underprovision of public goods with respect to the social optimum. This result, which is widely held beyond the academic circle (OECD, 1998, 2013) is far from obvious to establish in a general framework with $n(>2)$ asymmetric countries in interaction.

The “race to the bottom” may be viewed as the result of two properties of the tax competition game: a positive tax spillover and the strategic complementarity of tax rates. The first property means that any decrease in the tax rate of one country reduces the payoff of the other countries (positive tax spillover or equivalently). The second property (strategic complementarity of tax rates) characterizes the similarity of countries’ reaction in any change in the tax rate of one of them: A decrease in one country would induce a similar reaction from the other. In contrast, with negative tax spillovers (plain substitutability), any decrease in the tax rate of one country improves the payoff of the others. If tax rates are strategic substitutes, any tax rate change in one country implies an opposite reaction by the others and neither “race to the bottom” nor the race to the top may take place, the need for some tax coordination becoming dubious. On the empirical side, a significant number of works, recently reviewed in Leibrecht and Hochgatterer (2012) and Devereux and Loretz (2013), focus on the existence of tax competition and its nature. A large body of this literature establishes the existence of positive slopes of the tax reaction function¹ or equivalently the strategic complementarity of tax rates.² In contrast, the nature and the degree of tax spillovers have received considerably less notice. An exception is the recent study from the International Monetary Fund (IMF, 2014), which established positive tax spillovers based on panel data of corporate income tax for 103 countries for the period 1980 to 2013.

Effective tax rate versus statutory tax rate... Reduced form

The plain and strategic complementarities of tax rates are critical to understanding the “race to the bottom.” These properties are often implicitly assumed in the literature or derived from the specification of the frameworks proposed by the authors. We study here necessary conditions to obtain plain and strategic complementarities of tax rates. To remain as general as possible, we will use some notion of generalized concavity

¹For Devereux and Loretz (2013) this is the “most important empirically testable hypothesis.”

²However, some recent analyses (??) display downward sloped reaction functions (respectively, among US states and Swiss municipalities) leaving the question of the nature of tax competition open for further empirical investigations.

(see Vives (1990)) to apply the theory of supermodularity to tax competition. This theory introduced in economics by Topkis (1979) has been developed in particular by Vives (1990), Topkis (1998), and Milgrom and Roberts (1990) and remains mainly applied in industrial organization (see Topkis, 1998; Vives, 1999; Amir, 2005; Vives, 2005).³ Supermodular games display several nice properties: First, they encompass many analytical specifications, allowing appreciation of the robustness of the results; second, the existence of at least one pure-strategy Nash equilibrium is immediate, and many solution concepts yield the same prediction; finally, these games tend to be analytically appealing by significantly simplifying the analysis.⁴ These three qualities are particularly relevant in the context of tax competition, where the formalization of the problem may differ among authors and the existence of a Nash equilibrium remains an issue. By restricting our analysis to tax rate competition, we do not exploit fully the explanatory power of the supermodularity approach. Indeed, supermodular games are based on the lattice theory, which allows the study of multidimensional interactions among players with potential discrete strategic variables and then non-differentiable payoffs. Our analysis may be viewed as a very preliminary stage in the understanding of tax systems' competition.⁵

We establish that the log-concavity and the $1/2$ -convexity of marginal production are sufficient conditions for the strategic complementarity of tax rates and consequently for the supermodularity of the tax competition game, when countries maximize their tax revenue. Log-concavity and $1/2$ convexity are both special cases of ρ -concavity/convexity, which is a building block of the theory of generalized concavity (see). The notion of has been introduced in economics by Caplin???

Our result draws some parallel between the tax competition literature and industrial organization, where for instance the log-concavity of the demand function is a sufficient condition for the existence of Nash equilibrium in oligopoly competition (Amir, 1996). Assuming that tax revenue are countries' payoff function allows us to focus on the main characteristics of the tax competition game, avoiding some other issues such as the type of public good (substitute or complement to private consumption) or the distribution of capital between countries, which will be considered in the discussion about the limits of our analysis.

From the supermodularity of the tax competition game we deduce the existence of at least one pure-strategy Nash equilibrium. Our sufficient conditions We then distinguish tax coordination from tax cooperation. Following the literature on macroeconomic coordination failures, we consider that there is a tax coordination

³Cooper (1999) applies some lessons of supermodular games to macroeconomics. Ashworth and Bueno de Mesquita (2006) provide an introduction to monotone comparative statics in political science through an application to the appeasement problem. Costinot (2009) generalizes the Ricardian model of comparative advantage by using the notion of log-supermodularity.

⁴Vives (2005) wrote:

"The beauty of the approach is not its complexity but rather how much it simplifies the analysis and clarifies results. In fact, even the basic tools [of the theory of supermodular games] are not fully exploited by economists in current research."

⁵These tax systems encompass at least three general components: tax rates, tax bases, and tax laws enforcement (audit). See ? and Slermod and Gillitzer (2014) for an analysis of tax systems.

problem, when countries fail to coordinate on the Pareto dominant Nash equilibrium, while tax cooperation consists of reaching a Pareto superior outcome, which does not have to be a Nash equilibrium of the initial tax competition game. With these definitions and given the property of plain complementarity of tax rates we establish that tax coordination is unambiguously Pareto improving. We also show that neither cheap talk (costless communication) nor coalition can solve, respectively, the tax coordination or the tax cooperation issue. Finally, we highlight that any increase in the number of tax-competing countries reduces tax rates and tax revenue, and improves the net return of capital if and only if tax rates are strategic complements.⁶ All these results hold with welfare maximizers or in other types of tax competition (e.g., commodity or excise tax competition) as long as the tax competition game is supermodular.⁷

However, establishing general sufficient conditions for the supermodularity of the tax competition game in the presence of welfare maximizers raises multiple issues. The nature of public goods, in particular their degree of substitutability or complementarity with respect to private consumption, may determine the strategic property (complementarity or substitutability) of tax rates as de Mooij and Vrijburg (2012) emphasize.⁸ Besides this issue, we highlight the role of capital supply. First, we relax the implicit assumption of an inelastic worldwide stock of capital by considering saving decisions. This induces a positive relationship between interest rate and the total stock of capital. The supermodularity of the tax competition game still holds if the concavity of the saving function remains moderate. Second, capital ownership,⁹ its distribution, and its potential concentration in some countries yield consider a new kind of player in the tax competition game: Offshore finance centers or tax havens. Characterized by a zero capital tax rate and no real economic activity,¹⁰ tax havens are singular players displaying plain and eventually strategic substitutability of their tax rates. They may modify the international tax competition, which is not supermodular anymore.¹¹

The paper is structured as follows: Section 2 presents the tax competition game and the sufficient condition for its supermodularity. Section 3 displays some consequences of the supermodularity of the tax competition game in particular in terms of tax coordination and cooperation. Section 4 emphasizes the role of capital in the nature of tax competition. Section 5 concludes.

⁶In contrast, strategic substitutability of tax rates means an opposite relationship between the number of competing countries and the level of tax rates.

⁷For instance, it is straightforward to establish the supermodularity of the commodity tax competition proposed by Kanbur and Keen (1993) despite that reaction functions are not always differentiable there.

⁸Assuming quadratic production functions these authors show that tax rates are strategic complements (respectively strategic substitutes) if public and private goods are close substitutes (respectively close complements).

⁹Capital ownership would have to be taken into account if governments are welfare maximizers.

¹⁰We do not consider here secrecy, which is another characteristic of many tax havens and induces tax evasion.

¹¹It would even be possible to establish with an adequate formulation that a tax haven may involve an increase in the net return of capital, a higher level of worldwide capital supply, and an increase in tax revenues of the non-tax-haven countries.

2 Preamble: Generalized concavity

As a preamble, we present the notion of generalized concavity following Avriel et al. (1988).

Definition 1. Let h be a real-valued continuous function defined on the convex set $C \subset \mathbb{R}^n$, and denote by $I_h(C)$ the range of h ; that is, the image of C under h . The function h is said to be G -concave (G -convex) if there exists a continuous real-valued increasing function G defined on $I_h(C)$, such that $G(h(x))$ is concave (convex) over C .

We will use a subset of G -concave functions by considering ρ -concave functions as defined in Caplin and Nalebuff (1991).

Definition 2. Let h be a real-valued nonnegative continuous function defined on the convex set $C \subset \mathbb{R}^n$. The function h is ρ -concave (ρ -convex) if there exists a real number ρ , such that

$$\forall \lambda \in [0, 1], h(\lambda x^1 + (1 - \lambda)x^2) \geq \begin{cases} \lambda (h(x^1))^\rho + (1 - \lambda) (h(x^2))^{1-\rho} & \text{if } \rho \neq 0 \\ (h(x^1))^\lambda (h(x^2))^{1-\lambda} & \text{if } \rho = 0 \end{cases}$$

The concept of ρ -concavity is closely related to this of r -concavity, which is a restriction of the function G to exponential transformation (see lemma 1 in Balogh and Ewerhart 2015).¹² By focusing on ρ -concave functions in \mathbb{R} , which are twice continuously differentiable, we have the following definition:

Definition 3. Let h be a real-valued nonnegative continuous function twice continuously differentiable on the convex set $C \subset \mathbb{R}^+$. The function h is ρ -concave on C for some $\rho \in \mathbb{R}$ if and only if $\frac{1}{\rho} (h(x))^\rho$ is concave ($\log(h(x))$ is concave for $\rho = 0$) or equivalently if and only if

$$h(x) h''(x) + (\rho - 1) (h'(x))^2 \leq 0. \quad (1)$$

Strict ρ -concavity (strict logconcavity) is similarly defined with strict inequality and the notion of ρ -convexity with the opposite inequality. The case of standard concavity is equivalent to $\rho = 1$. Aumann (1975) and Caplin and Nalebuff (1991) introduced ρ -concavity to the economics literature. Anderson and Renault (2003) use this to determine efficiency and surplus bounds in the Cournot oligopoly. Ewerhart (2014) develops (α, β) -biconcavity, which corresponds to an exponential transformation of price and quantity in the Cournot model.

¹²A real-valued function h is r -concave if there exists a nonnegative real number r such that

$$\forall \lambda \in [0, 1], h(\lambda x^1 + (1 - \lambda)x^2) \geq \begin{cases} -\log \left\{ \lambda e^{-rh(x^1)} + (1 - \lambda) e^{-rh(x^2)} \right\}^{1/r} & \text{if } r \neq 0 \\ \lambda h(x^1) + (1 - \lambda) h(x^2) & \text{if } r = 0 \end{cases}$$

They deduce simple conditions on α and β for the existence and the uniqueness of a pure-strategy Nash equilibrium. Logconcavity is more widely used in economics (see Bagnoli and Bergstrom, 2005 for a review of applications).

We will need to consider the ρ -concavity and the ρ -convexity of the same function. Given previous definitions we have:

Lemma 1. *Let h be a real-valued continuous function twice continuously differentiable on the convex set $C \subset \mathbb{R}$. The function h is ρ' -concave and ρ'' -convex with $0 \leq \rho' \leq \rho''$.*

Proof. Immediate from (1). □

The parameter ρ may represent the degree of concavity (and convexity) of the function h . Indeed, if the function h is ρ -concave, then it is also ρ' -concave with $\rho < \rho'$. Similarly, the lower is ρ the more convex is the ρ -convex function h since a ρ -convex function h is also ρ' -convex function h with $\rho > \rho'$. Lemma 1 establishes that any function is ρ -concave and ρ' -convex.¹³ This property has been used by Anderson and Renault (2003) in a Cournot competition to bound the curvature of the demand.

3 The tax competition game

The basic framework of tax competition ascribed to Zodrow and Mieszkowski (1986) and Wilson (1986), is a one-period model featuring a single good produced by two factors: Labor, which is immobile across countries, and capital (k_i), which is perfectly mobile. The government of each jurisdiction chooses a tax rate on capital (the mobile production factor) to maximize a welfare function. In contrast to these seminal articles and a large part of the literature on tax competition, we consider tax revenue maximizers in this section. We will discuss potential extensions in section 4. However, we emphasize first that tax revenue is the cornerstone of any model of tax competition; second, establish the supermodularity of this game may be viewed as a preliminary step to a broader approach; and finally a large number of empirical works on tax competition study countries' tax revenue (Leibrecht and Hochgatterer, 2012).

We consider the following tax competition game, denoted by $\Gamma \equiv (S_i, R_i; i \in \mathbf{N})$, where S_i is the strategy set of country i , R_i is its payoff function, and n is the number of interacting countries. Each country maximizes simultaneously its tax revenue (R_i) with respect to its own tax rate, denoted by t_i , under the constraint provided by the market-clearing conditions (4). The strategy set of each country (S_i) is identical

¹³Even the linear function may be considered as ρ -concave and ρ' -convex with $0 \leq \rho < 1 < \rho'$.

and corresponds to the interval $S_i \equiv [0, 1]$.¹⁴ Country i 's payoff function is given by¹⁵

$$R_i(t) \equiv t_i k_i(t), \quad (2)$$

where t is the vector of tax rates ($t \equiv (t_1, \dots, t_N)$).

The production function in country i is denoted by $f_i(k_i)$ and differs among countries.¹⁶ We assume the following:

Assumption (1):

$$\begin{aligned} (i) \quad & f'_i(k_i) \geq 0 \geq f''_i(k_i), \\ (ii) \quad & \forall k_i > 0, \quad f'''_i(k_i) \geq 0 \end{aligned} \quad (3)$$

The production function of each country is increasing and concave in capital. We also consider that the third derivative of the production function is positive.¹⁷ These assumptions are necessary in Laussel and Le Breton (1998) for the existence of a Nash equilibrium.

Following the literature on tax competition, we assume that firms behave competitively in each country. Capital is priced at its net marginal productivity: $f'(k_i) - t_i = r$, where r denotes the net return to capital and is endogenous depending on t . Capital being perfectly mobile across countries and the total stock of capital being fixed equal to \bar{k} , the market-clearing conditions yield the following:

$$\left\{ \begin{array}{l} f'_i(k_i) - t_i = r, \quad \forall i \in \{1, \dots, n\} \\ \sum_{i=1}^n k_i = \bar{k} \end{array} \right. \quad (4)$$

From (4) and by application of the Implicit Function Theorem, we deduce some standard results, already established in the literature (Wildasin, 1988; Keen and Konrad, 2013):

$$\frac{\partial r}{\partial t_i} = -\frac{\frac{1}{f''_i(k_i)}}{\sum_{l=1}^n \frac{1}{f''_l(k_l)}} \in [-1, 0], \quad (5)$$

and

$$\frac{\partial k_i}{\partial t_i} = \frac{1}{f''_i(k_i)} \left(1 + \frac{\partial r}{\partial t_i} \right) \leq 0 \quad \text{and} \quad \frac{\partial k_j}{\partial t_i} = \frac{1}{f''_j(k_j)} \frac{\partial r}{\partial t_i} \geq 0. \quad (6)$$

The return to capital is decreasing in the tax rate of each country. The demand of capital in country i is a

¹⁴We consider that tax rates cannot be negative.

¹⁵The concavity of $R_i(t)$ is established in Appendix A.1.

¹⁶Fixed factors as explicit arguments of the production function are suppressed.

¹⁷This assumption may be derived by assuming that $f'''_i(k_i)$ does not change of sign for any value of capital (k_i) and by applying the result of Menegatti (2001).

decreasing function in the tax rate of this country (t_i) and an increasing function in the tax rate of the other country (t_j).

An important property of the game (Γ) is the positive tax spillover or equivalently the plain complementarity of tax rates following the taxonomy proposed by Eaton (2004).¹⁸ In other words, the payoff function is increasing (nondecreasing) in the strategic variables of the other players:

$$\frac{\partial R_i(t)}{\partial t_j} = t_i \frac{\partial k_i}{\partial t_j} = t_i \frac{1}{f_j''(k_j)} \frac{\partial r}{\partial t_i} > 0. \quad (7)$$

This property reflects the *tax base effect*: Any increase in the tax rate in country j reduces the net return of capital in this country and drives out capital from this country into country i ; this flow broadens the capital tax base of country i and increases its tax revenue.

Applying Definition 3, we consider two additional assumptions, which are critical to establishing the supermodularity of the tax competition game:

Assumption 2: The marginal production function of country i is log-concave, or equivalently

$$\frac{f_i'''(k_i)}{f_i'(k_i)} - \left(\frac{f_i''(k_i)}{f_i'(k_i)} \right)^2 \leq 0. \quad (8)$$

Assumption 3: The marginal production function of country i is 1/2-convex, or equivalently

$$\frac{f_i'''(k_i)}{f_i'(k_i)} - \frac{1}{2} \left(\frac{f_i''(k_i)}{f_i'(k_i)} \right)^2 \geq 0. \quad (9)$$

Assumptions 2 and 3 shape the curvature of the marginal production function, which corresponds also to the inverse demand for capital in country i . We may remark that these assumptions are closely related to sign the Schwarzian derivative of the production function $f_i(\cdot)$, denoted by $S(\cdot)$, and given by: $S(f_i(x)) = \frac{f_i'''(k_i)}{f_i'(k_i)} - \frac{3}{2} \left(\frac{f_i''(k_i)}{f_i'(k_i)} \right)^2$. The Schwarzian derivative discovered by Lagrange in 1781 (*Sur la construction des cartes géographiques*) is a way to appreciate the curvature of a function (see Ovsienko and Tabachnikov, 2008).

Given the unidimensionality of the strategy set, the supermodularity of the tax competition game derives from the strategic complementarity of tax rates as defined by Bulow et al. (1985): Any increase (decrease) in the tax rate of one country induces a similar variation in the tax rate of the other country. Our main result consists of giving sufficient conditions on the production function, which imply the supermodularity of the

¹⁸Plain complementarity is equivalent to positive spillovers, while plain substitutability corresponds to negative spillovers. We prefer Eaton's terminology for its clarity and its "complementarity" with the notions of strategic complementarity and substitutability used below.

tax competition game.

Proposition 1. *Under assumptions 1, 2, and 3, the tax competition game (Γ) is supermodular.*

Proof. See Appendix A.2. □

TO COMMENT

An alternative sufficient condition relies on the elasticities of the net and gross return of capital

Corollary 1. *Under assumptions 1 and 2, the tax competition game (Γ) is supermodular if*

$$1 + \epsilon_{(r+t_i)/t_i} \geq \epsilon_{r/t_i},$$

where $\epsilon_{r/t_i} = -\frac{t_i}{r} \frac{\partial r}{\partial t_i} > 0$ and $\epsilon_{(r+t_i)/t_i} = -\frac{t_i}{r+t_i} \left(1 + \frac{\partial r}{\partial t_i}\right) < 0$.

Proof. See Appendix A.3. □

The existence of Nash equilibrium is an immediate consequence of the supermodularity of the studied game.

We obtain the following Corollary:

Corollary 2. *Under assumptions 1, 2, and 3, there is always a pure-strategy Nash equilibrium of the tax competition game (Γ).*

Proof. See Topkis (1998), Theorem 4.2.1, page 181 or Vives (2005), Theorem 2.5, page 33. □

The existence of the Nash equilibrium follows directly from the analysis of Topkis (1998). Several authors have studied this issue in the tax competition context. For instance, Bucovetsky (1991), Wildasin (1991) or Wilson (1991), specified their objective functions in such a way that countries' best replies are linear and cross once, which implies the existence and the uniqueness of the Nash equilibrium. Laussel and Le Breton (1998) establish the existence of the Nash equilibrium in a more general framework, but still under some restrictive assumptions: (i) the convexity of the marginal production function, (ii) the linearity of the objective functions in public and private consumption, and (iii) the absence of capital owners in these functions. Some recent papers (e.g., Bayindir-Upmann and Ziad, 2005, or Taugourdeau and Ziad, 2011) attempted to enlarge the former analysis by dropping some of these assumptions. By establishing the supermodularity of the tax competition game, we circumvent some difficulties emphasized in previous articles to establish the existence of a Nash equilibrium.

Adopting the contraction approach,¹⁹ we establish a sufficient condition for the uniqueness of the Nash equilibrium when the game is supermodular. We also deduce another sufficient condition for the multiplicity of Nash equilibriums:

Proposition 2. *The Nash equilibrium of the tax competition game (Γ) is unique if*

$$\forall i \in \{1, \dots, n\}, \quad \frac{\partial r}{\partial t_i} < \frac{1}{4}. \quad (10)$$

Proof. See Appendix A.4. □

TO COMMENT

4 Some consequences of the supermodularity of the tax competition game

We display in this section some interesting results contributing to the debate about tax competition and coordination, which derive from the supermodularity of our specific game (Γ) and to any other tax competition game, where tax rates are strategic complements.²⁰ Thus, our results hold with different payoff functions (e.g., welfare functions) or different types of tax competition (e.g., commodity tax competition), as long as these games are supermodular.

First, we distinguish two notions often confused in the literature: Tax coordination and tax cooperation. The proposed distinction will clarify some consequences of the supermodularity of the tax competition game. Following the literature on macroeconomic games (Cooper, 1999), we say there is a tax coordination problem when countries fail to reach the Pareto dominant Nash equilibrium of the tax competition game (Γ) . This definition suggests that a tax coordination problem emerges when the two following conditions are met: *(i)* Nash equilibriums are multiple and *(ii)* they can be Pareto ranked.²¹ Tax cooperation consists of reaching a Pareto superior outcome, which does not need to correspond to a Nash equilibrium of the game (Γ) .²² For instance, Keen and Wildasin (2004) consider tax cooperation by applying the Motzkin's theorem to determine under which conditions a Pareto improving tax reform exists. Given our previous definitions, tax harmonization seems to be more a tax cooperation issue than a tax coordination one: Identical tax rates

¹⁹See Vives (1999, p. 46-48) for an application of this approach in industrial organization.

²⁰Most of these results have already been established and used in game theory and in industrial organization.

²¹An example of coordination failure is given by the stag hunt game, while the battle of sex game does not meet the second criterion.

²²The well-known prisoner dilemma illustrates a cooperation failure.

may occur at the Pareto dominant Nash equilibrium, but such case will be very fortuitous; it seems more realistic to consider Pareto improving tax harmonization, when it exists as an outcome, which is not a Nash equilibrium of the initial tax competition game (Γ).

A first Corollary derives from our definition of tax coordination:

Corollary 3. *Under assumptions 1 and 2, tax coordination is Pareto improving.*

Proof. From Milgrom and Roberts (1990) or Vives (1990), we know that the Nash equilibriums of the tax competition game (Γ) are Pareto ordered with a minimal and a maximal equilibrium because tax revenue is increasing in the tax rates of other countries (plain complementarity property). \square

Tax coordination, which consists of switching from one Nash equilibrium to a Pareto dominant one is unambiguously Pareto improving. This result yields to assume obviously and implicitly that countries are currently locked on a *bad* Nash equilibrium (low tax rate and low tax revenue), which would correspond to the harmful tax competition in OECD's terminology (OECD, 1998).

Part of the literature of game theory and industrial organization has studied several ways to coordinate or to cooperate. A first simple instrument available to countries is communicating on their respective tax policy. The game (Γ) is then extended by allowing some cheap talk or costless communication before any tax policy decision. Cheap talk would solve the tax coordination failure if and only if the tax competition game (Γ) displays two necessary credibility properties: The self-committing condition as defined by Farrell (1988) and the self-signaling condition, a stronger requirement emphasized by Aumann (1990). Baliga and Morris (2002) establish that games with plain and strategic complementarity do not fulfill these two properties. In consequence, cheap talk is ineffective to induce any tax coordination under assumptions 1 and 2: Each country always has an incentive to induce other countries to raise their respective tax rate.

Beyond communicating about their respective tax policy, some countries may form a coalition to avoid harmful tax competition. An example is the Enhanced Cooperation Agreements for European member states proposed in the treaties of Amsterdam (1997) and Nice (2003). Such schemes correspond to partial tax cooperation in our terminology. The cooperation among a subset of countries aims at reaching a Pareto superior situation for the participating countries at least, which is not a Nash equilibrium of the initial tax competition game (Γ). ? (1996, Theorem A.2, page 127) establish that the highest Nash equilibrium of a game with plain and strategic complements is (strongly) coalition proof. In other words, partial tax cooperation is not an equilibrium if the tax competition game (Γ) is supermodular. This result completes

previous analyses (Keen and Konrad, 2013) and contrasts with Konrad and Schjelderup (1999).²³ Besides communication and coalition, other ways to implement some tax coordination or tax cooperation may be a matter for future research such as commitment or strategic delegation.²⁴

Finally, an interesting consequence of the supermodularity of the tax competition game consists of appreciating the effect of the number of competing countries on Nash equilibrium tax rates. Adopting a similar approach to Corchon (1994) for aggregative games we obtain the following proposition:

Proposition 3. *Under assumptions 1 and 2, any increase in the number of the tax-competing countries reduces (i) tax rates and (ii) tax revenues, and increases the net return of capital.*

Proof. See Appendix A.4. □

Proposition 3 provides an alternative view to Bucovetsky (2009), who analyzes tax competition intensity.²⁵ It also completes the analysis of Hoyt (1991), who shows that a decrease in the number of identical competing jurisdictions increases the equilibrium tax rates by improving their respective market power. Finally, this result suggests also an indirect empirical test of the strategic complementarity of tax rates. Indeed, if tax rates had been strategic substitutes, then any increase in the number of countries or jurisdictions would imply an increase in tax rates (see Corchon, 1994 for the theoretical result in presence of strategic substitutes).

5 Limits to the supermodularity of the tax competition game: The role of capital supply

One obvious limit of our analysis is the choice of the payoff function: Tax revenue. Establishing similar conditions for the supermodularity of the tax competition game with welfare functions would imply several additional restrictive assumptions. For instance, the choice of the type of public consumption as substitute (public good) or complement (public input) of private income would affect the nature of interactions between countries. The degree of the marginal rate of substitution between private and public consumption would also modify the properties of the tax competition game, and tax rates may become strategic substitutes as highlighted by ? with quadratic production functions. However, as noted in the introduction, considering

²³Konrad and Schjelderup (1999) establish that partial cooperation is Pareto efficient if tax rates are strategic complements. However, these authors do not consider explicitly the internal and external stability properties of coalitions as in ?.

²⁴See ? for an application of commitment games to tax competition.

²⁵Bucovetsky (2009) considers welfare maximizers with quadratic production functions, Edgeworth independence between private and public consumption, and fixed capital supply. With this formalization any merger of countries implies an increase in the average tax rate.

public goods yields implicitly to study a fiscal competition framework, where public income and spending are taken into account, rather than a pure tax competition model.

Besides the type of public good, we focus on the role of capital supply, which would have to be taken into account with welfare maximizers. First, we address the issue of capital supply by relaxing the implicit assumption of its inelasticity at the level of the economy. We assume a positive relationship between total capital supply (\bar{k}) and its net return (r) as in Eichner and Runkel (2012). A common pool problem emerges: Any tax rate increase in one country lowers the net return of capital and consequently total capital supply. We consider a positive elasticity of capital supply, which may result from individual saving behaviors. Saving decisions and the choice of tax rate occur simultaneously, allowing us to avoid the time inconsistency problem as pointed out by Kydland and Prescott (1977). We have²⁶

$$\bar{k} \equiv S(r), \text{ with } S'(r) > 0 > S''(r).$$

The market-clearing conditions given in (4) are modified consequently:

$$\begin{cases} f'_i(k_i) - t_i = r, & \forall i \in \{1, \dots, n\} \\ \sum_{i=1}^n k_i = S(r) \end{cases} \quad (11)$$

where $\bar{k}(t) = \bar{k}(t_1, \dots, t_n)$ is decreasing in tax rate: $\forall i \in \{1, \dots, n\}, \frac{\partial \bar{k}(t)}{\partial t_i} < 0$. This yields an additional effect: the global tax base contracts in reaction to any increase of tax rate. Expression (5) becomes

$$\frac{\partial r}{\partial t_i} = - \frac{\frac{1}{f'_i(k_i)}}{\sum_{l=1}^n \frac{1}{f'_l(k_l)} - S'(r)} < 0, \quad \text{since } S'(r) > 0. \quad (12)$$

Thus, we obtain²⁷

$$\frac{\partial^2 R_i}{\partial t_i \partial t_j} \geq \Omega - t_i \frac{f''_i(k_i)}{f'_j(k_j)} \left(\frac{\partial r}{\partial t_i} \right)^3 S''(r) \geq 0.$$

The supermodularity of the tax competition game imposes then an additional constraint on the shape of the saving function. The latter must be not too concave to preserve the supermodularity of the tax competition game. We remark that not only is the strategic substitutability of tax rates possible for some countries maximizing their tax revenue, but also that some may have non-monotone best replies even with log-concave

²⁶Saving is a function of the interest rate and is increasing with the rate of net return ($S'(r) > 0$) as a result of the concavity of the utility function: $S'(r) = -1/U''$. Moreover, the saving function is concave owing to the non negative absolute risk aversion captured by $S''(r) = -U''' / (U'')^2$.

²⁷ $\Omega \equiv \left(\frac{1}{f'_j(k_j)} - t_i \frac{f''_i(k_i)}{(f'_i(k_i))^2} \right) \frac{\partial r}{\partial t_i} \left(1 + \frac{\partial r}{\partial t_i} \right) - \frac{1}{f'_j(k_j)} \left(\frac{\partial r}{\partial t_i} \right)^2 \left(t_i \frac{f'''_i(k_i)}{(f'_i(k_i))^2} + t_i \frac{f'''_j(k_j)}{(f'_j(k_j))^2} + \frac{r}{f'_i(k_i)} \right).$

marginal production function.

A second issue, we consider here, is capital ownership, which induces a pecuniary effect in an opposite way from the tax base effect. Indeed, any increase in the tax rate of one country reduces the worldwide net return of capital and hurts all capital owners. Consider the following payment function for capital owners, denoted by $H_i(t)$: $H_i(t) \equiv r(t)\theta_i\bar{k}$, where θ_i is the share of capital owned by inhabitants of country i . Assuming an inelastic capital supply (\bar{k}) for simplicity purpose, we have

$$\frac{\partial H_i(t)}{\partial t_j} = \theta_i\bar{k}\frac{\partial r(t)}{\partial t_j} < 0 \text{ and } \frac{\partial^2 H_i(t)}{\partial t_i\partial t_j} = \theta_i\bar{k}\frac{\partial^2 r(t)}{\partial t_i\partial t_j} \leq 0.$$

Introducing capital ownership in the objective function may not only cancel the properties of plain and strategic complementarities but also the monotonicity of the payoff function with respect to the action of the other countries and the monotonicity of best replies. Moreover adding capital ownership implies its distribution within and between countries. The tax competition game becomes more complex.

Offshore financial centers or tax havens characterized by a zero capital tax rate and no real economic activity,²⁸ may be represented by the objective function $H_i(t)$. They are specific players: They display plain substitute and may be characterized by strategic substitutability. Consequently, they modify drastically the nature of international tax competition as emphasized by Slemrod and Wilson (2009), Johannesen (2010), Keen and Konrad (2013), and Bucovetsky (2014). Tax havens provide an opportunity to capital owners to protect their interests by improving the net return of capital through a more intensive tax competition. Indeed let assume that a country i initially represented by the payoff function $R_i(t)$ becomes a tax haven characterized by the function $H_i(t)$, the optimal tax rate of this country, which was initially strictly positive becomes zero.²⁹ Given the strategic complementarity of the tax rates for all the other $n - 1$ countries, this variation means a decrease in their respective equilibrium tax rates and then an increase in the net return of capital (r).³⁰ If we assume an elastic capital supply with respect to r , the emergence of a tax haven induces a higher level of capital and even potentially an increase in tax revenues for some non-tax-haven countries. Finally, we remark that if the supermodularity of the tax competition game does not hold anymore in the presence of tax havens, then the “race to the bottom” may not exist, tax coordination as previously defined may become impossible, and coalition of a subgroup of countries (tax haven or not) may be Pareto improving. A formalization of previous relationships imposes a general analysis of games with strategic complements and substitutes, which

²⁸We do not consider here secrecy, which is another characteristic of many tax havens and tax evasion.

²⁹From the First Order Condition of the maximization of $R_i(t)$ with respect to t_i , we deduce that t_i is strictly positive as long as k_i is not equal to zero. Considering $k_i = 0$ is equivalent to saying that country i does not participate to the tax competition game. We excluded the case of negative tax rates.

³⁰Since the tax rate of the tax haven is a corner solution, the reaction of the other countries does not modify its own equilibrium policy.

remains for future research.

6 Conclusion

Is tax competition harmful? Can coordination be Pareto improving? Yes, when the tax competition game is supermodular. We established that the log-concavity of marginal production in each country is a sufficient condition for supermodularity of the tax competition game, when countries maximize their tax revenue. The existence of at least one Nash equilibrium is then deduced and in case of multiplicity they can be Pareto ranked. The “race to the bottom” corresponds to a coordination failure on a low tax rate and low tax revenue equilibrium. Thus, tax coordination is unambiguously Pareto improving. However, cheap talk is ineffective to allow this coordination. Moreover, tax cooperation is not possible through the coalition of some subset of countries since the highest Nash equilibrium is coalition-proof. Finally, any increase in the number of tax-competing countries reduces equilibrium tax rates and tax revenue and improves the net return of capital.

Considering elastic capital supply or capital ownership modifies the nature of the tax competition game. Supermodularity is still possible when the total stock of capital depends on its net return. However, this property may vanish completely when capital ownership is concentrated in some particular jurisdictions such as offshore financial centers or tax havens. Despite its simplicity: Tax revenue maximizer and capital tax rate competition, our formalization displays the potential complexity of any tax competition game. As mentioned in the introduction, this paper is a preliminary stage toward a deeper application of the supermodularity tools to tax system competition, in particular to address the multidimensionality of tax systems (see for instance Bucovetsky, 1991, who study capital and labor taxation, or Cremer and Gahvari 2000, who consider tax and audit rate as policy variables).

References

- Amir, R. (1996). Cournot oligopoly and the theory of supermodular games. *Games and Economic Behavior*, 15(2):132–148.
- Amir, R. (2005). Supermodularity and complementarity in economic: An elementary survey. *Southern Economic Journal*, 71(3):636–660.
- Anderson, S. P. and Renault, R. (2003). Efficiency and surplus bounds in cournot competition. *Journal of Economic Theory*, 113:253–264.
- Ashworth, S. and Bueno de Mesquita, E. (2006). Monotone comparative statics for models of politics. *American Journal of Political Science*, 50(1):214–231.
- Aumann, R. (1975). Values of Markets with a Continuum of Traders. *Econometrica*, 43(4):611–46.
- Aumann, R. (1990). Nash equilibria are not self-enforcing. In Gabszewicz, J., Richard, J.-F., and Wolsey, L., editors, *Economic Decision-Making: Games, Econometrics and Optimization*. Elsevier, Amsterdam.
- Avriel, M. A., Diewert, W. E., Schaible, S., and Zang, I. (1988). *Generalized Concavity*. Society for Industrial and Applied Mathematics, Philadelphia, PA.
- Bagnoli, M. and Bergstrom, T. (2005). Log-concave probability and its applications. *Economic Theory*, 26(2):445–469.
- Baliga, S. and Morris, S. (2002). Co-ordination, spillovers, and cheap talk. *Journal of Economic Theory*, 105(2):450–468.
- Balogh, T. and Ewerhart, C. (2015). On the origin of r-concavity and related concepts. Technical Report 187, University of Zurich, Department of Economics.
- Bayindir-Upmann, T. and Ziad, A. (2005). Existence of equilibria in a basic tax-competition model. *Regional Science and Urban Economics*, 35(1):1–22.
- Bucovetsky, S. (1991). Asymmetric tax competition. *Journal of Urban Economics*, 30(2):167–181.
- Bucovetsky, S. (2009). An index of capital tax competition. *International Tax and Public Finance*, 16(6):727–752.
- Bucovetsky, S. (2014). Honor among tax havens. *Journal of Public Economics*, 110(3):74–81.
- Bulow, J. I., Geanakoplos, J. D., and Klemperer, P. D. (1985). Multimarket oligopoly: strategic substitutes and complements. *Journal of Political Economy*, 93(3):488–511.
- Caplin, A. and Nalebuff, B. (1991). Aggregation and imperfect competition: On the existence of equilibrium. *Econometrica*, 59(1):25–59.
- Cooper, R. W. (1999). *Coordination Games. Complementarities and Macroeconomics*. Cambridge University Press, Cambridge, UK.

- Corchon, L. C. (1994). Comparative statics for aggregative games: the strong concavity case. *Mathematical Social Sciences*, 28(3):151–165.
- Costinot, A. (2009). An elementary theory of comparative advantage. *Econometrica*, 77(4):1165–1192.
- Cremer, H. and Gahvari, F. (2000). Tax evasion, fiscal competition and economic integration. *European Economic Review*, 44(9):1633–1657.
- de Mooij, R. A. and Vrijburg, H. (2012). Tax rates as strategic substitutes. Tinbergen Institute Discussion Papers 12-104/VI, Tinbergen Institute.
- Devereux, M. and Loretz, S. (2013). What do we know about corporate tax competition? *National Tax Journal*, 66(3):745–773.
- Eaton, B. C. (2004). The elementary economics of social dilemmas. *Canadian Journal of Economics*, 37(4):805–829.
- Eichner, T. and Runkel, M. (2012). Interjurisdictional spillovers, decentralized policymaking, and the elasticity of capital supply. *American Economic Review*, 102(5):2349–2357.
- Ewerhart, C. (2014). Cournot games with biconcave demand. *Games and Economic Behavior*, 85(C):37–47.
- Farrell, J. (1988). Communication, coordination and nash equilibrium. *Economics Letters*, 27(3):209–214.
- Hoyt, W. H. (1991). Property taxation, nash equilibrium and market power. *Journal of Urban Economics*, 30(1):123–131.
- IMF (2014). Spillovers in international taxation. Board paper, Washington: International Monetary Fund.
- Johannesen, N. (2010). Imperfect tax competition for profits, asymmetric equilibrium and beneficial tax havens. *Journal of International Economics*, 81(2):253–264.
- Kanbur, R. and Keen, M. (1993). Jeux sans frontieres: tax competition and tax coordination when countries differ in size. *American Economic Review*, 83(4):877–92.
- Keen, M. and Konrad, K. A. (2013). Chapter 5 - the theory of international tax competition and coordination. In Alan J. Auerbach, Raj Chetty, M. F. and Saez, E., editors, *handbook of public economics, vol. 5*, volume 5 of *Handbook of Public Economics*, pages 257–328. Elsevier.
- Keen, M. and Wildasin, D. (2004). Pareto-Efficient International Taxation. *American Economic Review*, 94(1):259–275.
- Konrad, K. A. and Schjelderup, G. (1999). Fortress building in global tax competition. *Journal of Urban Economics*, 46(1):156 – 167.
- Kydland, F. E. and Prescott, E. C. (1977). Rules rather than discretion: The inconsistency of optimal plans. *Journal of Political Economy*, 85(3):473–491.
- Laussel, D. and Le Breton, M. (1998). Existence of nash equilibria in fiscal competition models. *Regional*

- Science and Urban Economics*, 28(3):283–296.
- Leibrecht, M. and Hochgatterer, C. (2012). Tax competition as a cause of falling corporate income tax rates: A survey of empirical literature. *Journal of Economic Surveys*, 26(4):616–648.
- Menegatti, M. (2001). On the conditions for precautionary saving. *Journal of Economic Theory*, 98(1):189–193.
- Milgrom, P. and Roberts, J. (1990). Rationalizability, learning, and equilibrium in games with strategic complementarities. *Econometrica*, 58(6):1255–77.
- OECD (1998). *Harmful tax competition. An emerging global issuer*. OECD Publishing, Paris.
- OECD (2013). *Addressing base erosion and profit shifting*. OECD Publishing, Paris.
- Ovsienko, V. and Tabachnikov, S. (2008). What is the schwarzian derivative? *Notices of the AMS*, 56(1):34–36.
- Slemrod, J. and Wilson, J. D. (2009). Tax competition with parasitic tax havens. *Journal of Public Economics*, 93(11-12):1261–1270.
- Slermod, J. and Gillitzer, C. (2014). *Tax Systems*. The MIT Press.
- Taugourdeau, E. and Ziad, A. (2011). On the existence of nash equilibria in an asymmetric tax competition game. *Regional Science and Urban Economics*, 41(5):439–445.
- Topkis, D. (1998). *Supermodularity and Complementarity*. Princeton University Press, Princeton, New Jersey.
- Topkis, D. M. (1979). Equilibrium points in non-zero sum n-person submodular games. *SIAM Journal of Control and Optimization*, 17(6):7773–7787.
- Vives, X. (1990). Nash equilibrium with strategic complementarities. *Journal of Mathematical Economics*, 19(3):305–321.
- Vives, X. (1999). *Olipoly pricing. Old ideas and new tools*. The MIT Press, Cambridge, Massachusetts.
- Vives, X. (2005). Complementarities and games: New developments. *Journal of Economic Literature*, 43(2):437–479.
- Wildasin, D. E. (1988). Nash equilibria in models of fiscal competition. *Journal of Public Economics*, 35(2):229–240.
- Wildasin, D. E. (1991). Some rudimentary 'duopoly' theory. *Regional Science and Urban Economics*, 21(3):393–421.
- Wilson, J. D. (1986). A theory of interregional tax competition. *Journal of Urban Economics*, 19(3):296–315.
- Wilson, J. D. (1991). Tax competition with interregional differences in factor endowments. *Regional Science and Urban Economics*, 21(3):423–451.
- Zodrow, G. R. and Mieszkowski, P. (1986). Pigou, tiebout, property taxation, and the underprovision of

local public goods. *Journal of Urban Economics*, 19(3):356–370.

Appendix

A.1 Second Order Conditions

From (4) we deduce the following expressions, which will be useful for our main proofs:

$$\frac{\partial k_i}{\partial t_i} = \frac{1}{f_i''(k_i)} \left(1 + \frac{\partial r}{\partial t_i} \right), \quad (13)$$

$$\frac{\partial k_l}{\partial t_i} = \frac{1}{f_l''(k_l)} \frac{\partial r}{\partial t_i}, \quad (14)$$

$$\sum_{l=1}^n \frac{1}{f_l''(k_l)} \frac{\partial r}{\partial t_i} = -\frac{1}{f_i''(k_i)}. \quad (15)$$

The Second Order Condition of (2) is given by

$$\frac{\partial^2 R_i}{\partial t_i^2} = 2 \frac{\partial k_i}{\partial t_i} + t_i \frac{\partial^2 k_i}{\partial t_i^2} = \frac{2}{f_i''(k_i)} \left(1 + \frac{\partial r}{\partial t_i} \right) + t_i \frac{\partial^2 k_i}{\partial t_i^2}, \quad (16)$$

where

$$\frac{\partial^2 k_i}{\partial t_i^2} = -\frac{f_i'''(k_i)}{(f_i''(k_i))^2} \left(1 + \frac{\partial r}{\partial t_i} \right) + \frac{1}{f_i''(k_i)} \frac{\partial^2 r}{\partial t_i^2}. \quad (17)$$

We consider $\frac{\partial^2 r}{\partial t_i^2}$. We have

$$\frac{\partial^2 r}{\partial t_i^2} = -\frac{-\frac{f_i'''(k_i)}{(f_i''(k_i))^2} \frac{\partial k_i}{\partial t_i} \sum_{l=1}^n \frac{1}{f_l''(k_l)} + \frac{1}{f_i''(k_i)} \sum_{l=1}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^2} \frac{\partial k_l}{\partial t_i}}{\left(\sum_{l=1}^n \frac{1}{f_l''(k_l)} \right)^2},$$

or equivalently

$$\frac{\partial^2 r}{\partial t_i^2} = f_i'''(k_i) \left(\frac{\partial r}{\partial t_i} \right)^2 \frac{\partial k_i}{\partial t_i} \sum_{l=1}^n \frac{1}{f_l''(k_l)} - \frac{f_i'''(k_i)}{f_i''(k_i)} \left(\frac{\partial r}{\partial t_i} \right)^2 \frac{\partial k_i}{\partial t_i} - f_i''(k_i) \left(\frac{\partial r}{\partial t_i} \right)^2 \sum_{l=1, l \neq i}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^2} \frac{\partial k_l}{\partial t_i}.$$

Using (13), (14), and (15) the previous expression is equivalent to

$$\begin{aligned} \frac{\partial^2 r}{\partial t_i^2} &= \frac{f_i'''(k_i)}{f_i''(k_i)} \left(\frac{\partial r}{\partial t_i} \right)^2 \left(1 + \frac{\partial r}{\partial t_i} \right) \sum_{l=1}^n \frac{1}{f_l''(k_l)} \\ &\quad - \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \left(\frac{\partial r}{\partial t_i} \right)^2 \left(1 + \frac{\partial r}{\partial t_i} \right) - f_i''(k_i) \left(\frac{\partial r}{\partial t_i} \right)^3 \sum_{l=1, l \neq i}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^3}. \end{aligned} \quad (18)$$

or equivalently,

$$\frac{\partial^2 r}{\partial t_i^2} = \frac{f_i'''(k_i)}{f_i''(k_i)} \left(\frac{\partial r}{\partial t_i} \right)^2 \left(1 + \frac{\partial r}{\partial t_i} \right) \sum_{l=1, l \neq i}^n \frac{1}{f_l''(k_l)} - f_i''(k_i) \left(\frac{\partial r}{\partial t_i} \right)^3 \sum_{l=1, l \neq i}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^3} > 0,$$

since $f_i''(k_i) < 0 < f_i'''(k_i)$ and $1 + \frac{\partial r}{\partial t_i} > 0 > \frac{\partial r}{\partial t_i}$. We deduce then that

$$\frac{\partial^2 k_i}{\partial t_i^2} < 0 \text{ and } \frac{\partial^2 R_i}{\partial t_i^2} < 0.$$

A.2 Proof for Proposition 1: The strategic complementarity of tax rates

The game $\Gamma \equiv (T_i, R_i; i \in \mathbf{N})$ is supermodular if (1) T_i is a compact cube in Euclidean space; (2) R_i displays strategic complementarity in tax rates since the strategy set is one-dimensional. The first condition always holds since we consider $T_i = [0, 1]$. The second condition yields to sign of the cross derivative of tax revenue, which is given by

$$\frac{\partial^2 R_i}{\partial t_i \partial t_j} = \frac{\partial k_i}{\partial t_j} - t_i \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \frac{\partial k_i}{\partial t_j} \left(1 + \frac{\partial r}{\partial t_i} \right) + \frac{t_i}{f_i''(k_i)} \frac{\partial^2 r}{\partial t_i \partial t_j}.$$

Since $\frac{\partial k_i}{\partial t_j} = \frac{1}{f_i''(k_i)} \frac{\partial r}{\partial t_j} = \frac{1}{f_j''(k_j)} \frac{\partial r}{\partial t_i}$, we obtain

$$\frac{\partial^2 R_i}{\partial t_i \partial t_j} = \frac{1}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \left[1 - t_i \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \left(1 + \frac{\partial r}{\partial t_i} \right) \right] + \frac{t_i}{f_i''(k_i)} \frac{\partial^2 r}{\partial t_i \partial t_j}.$$

We now focus on the expression of $\frac{\partial^2 r}{\partial t_i \partial t_j}$. We have

$$\frac{\partial^2 r}{\partial t_i \partial t_j} = - \frac{- \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \frac{\partial k_i}{\partial t_j} \sum_{l=1}^n \frac{1}{f_l''(k_l)} + \frac{1}{f_i''(k_i)} \sum_{l=1}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^2} \frac{\partial k_l}{\partial t_j}}{\left(\sum_{l=1}^n \frac{1}{f_l''(k_l)} \right)^2},$$

which corresponds to

$$\frac{\partial^2 r}{\partial t_i \partial t_j} = - \frac{f_i'''(k_i)}{f_i''(k_i)} \frac{\partial k_i}{\partial t_j} \left(\frac{\partial r}{\partial t_i} \right)^2 - f_i''(k_i) \left(\frac{\partial r}{\partial t_i} \right)^2 \frac{f_j'''(k_j)}{(f_j''(k_j))^2} \frac{\partial k_j}{\partial t_j} - f_i''(k_i) \left(\frac{\partial r}{\partial t_i} \right)^2 \sum_{l=1, l \neq j}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^2} \frac{\partial k_l}{\partial t_j}.$$

From (14) and $\frac{\partial k_j}{\partial t_j} = \frac{1}{f_j''(k_j)} \left(1 + \frac{\partial r}{\partial t_j}\right) = \frac{1}{f_j''(k_j)} \left(1 + \frac{f_i''(k_i)}{f_j''(k_j)} \frac{\partial r}{\partial t_i}\right)$, we deduce

$$\frac{\partial^2 r}{\partial t_i \partial t_j} = -\frac{f_i'''(k_i)}{f_i''(k_i)} \frac{1}{f_j''(k_j)} \left(\frac{\partial r}{\partial t_i}\right)^2 - f_i''(k_i) \left(\frac{\partial r}{\partial t_i}\right)^2 \frac{f_j'''(k_j)}{(f_j''(k_j))^3} - \frac{(f_i''(k_i))^2}{f_j''(k_j)} \left(\frac{\partial r}{\partial t_i}\right)^3 \sum_{l=1}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^3},$$

and therefore

$$\begin{aligned} \frac{\partial^2 R_i}{\partial t_i \partial t_j} &= \frac{1}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \left[1 - t_i \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \left(1 + \frac{\partial r}{\partial t_i}\right)\right] \\ &\quad - \frac{t_i}{f_j''(k_j)} \left(\frac{\partial r}{\partial t_i}\right)^2 \left[\frac{f_i'''(k_i)}{(f_i''(k_i))^2} + \frac{f_j'''(k_j)}{(f_j''(k_j))^2} + f_i''(k_i) \frac{\partial r}{\partial t_i} \sum_{l=1}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^3}\right]. \end{aligned}$$

The log-concavity assumption (3) and the capital market clearing condition (4) involve We deduce

$$-\frac{f_i'''(k_i)}{(f_i''(k_i))^2} \geq -\frac{1}{f_i'(k_i)} = -\frac{1}{r+t_i} \text{ and } \frac{f_l'''(k_l)}{(f_l''(k_l))^3} \geq \frac{1}{f_l'(k_l) f_l''(k_l)} = \frac{1}{(r+t_l) f_l''(k_l)} \geq \frac{1}{r f_l''(k_l)}. \quad (19)$$

Given that $f_i''(k_i) \frac{\partial r}{\partial t_i} \sum_{l=1}^n \frac{1}{f_l''(k_l)} = -1$ and (19), we have

$$\frac{\partial^2 R_i}{\partial t_i \partial t_j} \geq \frac{1}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \left[1 - \frac{t_i}{r+t_i} \left(1 + \frac{\partial r}{\partial t_i}\right)\right] - \frac{t_i}{f_j''(k_j)} \left(\frac{\partial r}{\partial t_i}\right)^2 \left[\frac{f_i'''(k_i)}{(f_i''(k_i))^2} + \frac{f_j'''(k_j)}{(f_j''(k_j))^2} - \frac{1}{r}\right]. \quad (20)$$

The 1/2-convexity of function $f_i'(\cdot)$ involves

$$f_i'''(k_i) f_i'(k_i) - \frac{1}{2} (f_i''(k_i))^2 > 0 \Leftrightarrow \frac{f_i'''(k_i)}{(f_i''(k_i))^2} > \frac{1}{2f_i'(k_i)} \geq \frac{1}{2r}.$$

We deduce that

$$\frac{f_i'''(k_i)}{(f_i''(k_i))^2} + \frac{f_j'''(k_j)}{(f_j''(k_j))^2} - \frac{1}{r} \geq 0,$$

and consequently

$$\frac{\partial^2 R_i}{\partial t_i \partial t_j} \geq 0,$$

since $1 - \frac{t_i}{r+t_i} \left(1 + \frac{\partial r}{\partial t_i}\right) \geq 1 - \frac{t_i}{r+t_i} \geq 0$.

A.3 Proof for Corollary 1

An alternative sufficient condition relies on the elasticities of the net and gross return of capital with respect to t_i , denoted respectively by ϵ_{r/t_i} and $\epsilon_{(r+t_i)/t_i}$. We have: $\epsilon_{r/t_i} = -\frac{t_i}{r} \frac{\partial r}{\partial t_i} > 0$ and $\epsilon_{(r+t_i)/t_i} = -\frac{t_i}{r+t_i} \left(1 + \frac{\partial r}{\partial t_i}\right) < 0$. Assuming the following condition

$$1 + \epsilon_{(r+t_i)/t_i} \geq \epsilon_{r/t_i},$$

expression (20) yields

$$\begin{aligned} \frac{\partial^2 R_i}{\partial t_i \partial t_j} &\geq \frac{1}{f_j''(k_j)} \frac{\partial r}{\partial t_i} \left[1 - \frac{t_i}{r+t_i} \left(1 + \frac{\partial r}{\partial t_i}\right) + \frac{\partial r}{\partial t_i} \frac{t_i}{r} \right] - \frac{t_i}{f_j''(k_j)} \left(\frac{\partial r}{\partial t_i}\right)^2 \left[\frac{f_i'''(k_i)}{(f_i''(k_i))^2} + \frac{f_j'''(k_j)}{(f_j''(k_j))^2} \right] \\ &= \frac{1}{f_j''(k_j)} \frac{\partial r}{\partial t_i} [1 + \epsilon_{(r+t_i)/t_i} - \epsilon_{r/t_i}] - \frac{t_i}{f_j''(k_j)} \left(\frac{\partial r}{\partial t_i}\right)^2 \left[\frac{f_i'''(k_i)}{(f_i''(k_i))^2} + \frac{f_j'''(k_j)}{(f_j''(k_j))^2} \right] \geq 0. \end{aligned}$$

A.4 Proof for Proposition 2: Uniqueness of the Nash equilibrium

We follow the contraction approach to establish the uniqueness of the Nash equilibrium. We consider that the marginal production function is log-concave and 1/2-convex. By application of Proposition 1, the game Γ is supermodular and the uniqueness of the Nash equilibrium may be deduce from:

$$\frac{\partial^2 R_i}{\partial t_i^2} + \sum_{j=1, j \neq i}^n \left| \frac{\partial^2 R_i}{\partial t_i \partial t_j} \right| = \frac{\partial^2 R_i}{\partial t_i^2} + \sum_{j=1, j \neq i}^n \frac{\partial^2 R_i}{\partial t_i \partial t_j} < 0.$$

We have

$$\begin{aligned} \frac{\partial^2 R_i}{\partial t_i^2} + \sum_{j=1, j \neq i}^n \frac{\partial^2 R_i}{\partial t_i \partial t_j} &= \frac{2}{f_i''(k_i)} \left(1 + \frac{\partial r}{\partial t_i}\right) + t_i \frac{\partial^2 k_i}{\partial t_i^2} + \sum_{j=1, j \neq i}^n \left(\frac{1}{f_j''(k_j)} \frac{\partial r}{\partial t_i} + t_i \frac{\partial^2 k_i}{\partial t_i \partial t_j} \right) \\ &= \frac{1}{f_i''(k_i)} \left(2 + \frac{\partial r}{\partial t_i}\right) + \sum_{j=1}^n \left(\frac{1}{f_j''(k_j)} \frac{\partial r}{\partial t_i} + t_i \frac{\partial^2 k_i}{\partial t_i \partial t_j} \right). \end{aligned}$$

Applying (15), we obtain

$$\frac{\partial^2 R_i}{\partial t_i^2} + \sum_{j=1, j \neq i}^n \frac{\partial^2 R_i}{\partial t_i \partial t_j} = \frac{1}{f_i''(k_i)} \left(1 + \frac{\partial r}{\partial t_i}\right) + t_i \sum_{j=1}^n \frac{\partial^2 k_i}{\partial t_i \partial t_j}.$$

with

$$\frac{\partial^2 k_i}{\partial t_i \partial t_j} = -\frac{f_j'''(k_j)}{(f_j''(k_j))^3} \left(\frac{\partial r}{\partial t_i}\right)^2 + \frac{1}{f_j''(k_j)} \frac{\partial^2 r}{\partial t_i^2}.$$

From (18) and given (15) we have

$$\begin{aligned}\frac{\partial^2 r}{\partial t_i^2} &= \frac{f_i'''(k_i)}{f_i''(k_i)} \left(\frac{\partial r}{\partial t_i} \right)^2 \left(1 + \frac{\partial r}{\partial t_i} \right) \sum_{l=1}^n \frac{1}{f_l''(k_l)} - \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \left(\frac{\partial r}{\partial t_i} \right)^2 \left(1 + \frac{\partial r}{\partial t_i} \right) - f_i''(k_i) \left(\frac{\partial r}{\partial t_i} \right)^3 \sum_{l=1, l \neq i}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^3} \\ &= -\frac{f_i'''(k_i)}{(f_i''(k_i))^2} \frac{\partial r}{\partial t_i} \left(1 + \frac{\partial r}{\partial t_i} \right)^2 - f_i''(k_i) \left(\frac{\partial r}{\partial t_i} \right)^3 \sum_{l=1, l \neq i}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^3}.\end{aligned}$$

We deduce that

$$\begin{aligned}\frac{\partial^2 k_i}{\partial t_i \partial t_j} &= -\frac{f_j'''(k_j)}{(f_j''(k_j))^3} \left(\frac{\partial r}{\partial t_i} \right)^2 - \frac{1}{f_j''(k_j)} \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \frac{\partial r}{\partial t_i} \left(1 + \frac{\partial r}{\partial t_i} \right)^2 - \frac{f_i''(k_i)}{f_j''(k_j)} \left(\frac{\partial r}{\partial t_i} \right)^3 \sum_{l=1, l \neq i}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^3} \\ &= -\frac{f_j'''(k_j)}{(f_j''(k_j))^3} \left(\frac{\partial r}{\partial t_i} \right)^2 - \frac{1}{f_j''(k_j)} \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \frac{\partial r}{\partial t_i} \left(1 + 2 \frac{\partial r}{\partial t_i} \right) - \frac{f_i''(k_i)}{f_j''(k_j)} \left(\frac{\partial r}{\partial t_i} \right)^3 \sum_{l=1}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^3}.\end{aligned}$$

Given (15) we obtain

$$\begin{aligned}\sum_{j=1}^n \frac{\partial^2 k_i}{\partial t_i \partial t_j} &= -\sum_{j=1}^n \frac{f_j'''(k_j)}{(f_j''(k_j))^3} \left(\frac{\partial r}{\partial t_i} \right)^2 - \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \frac{\partial r}{\partial t_i} \left(1 + 2 \frac{\partial r}{\partial t_i} \right) \sum_{j=1}^n \frac{1}{f_j''(k_j)} \\ &\quad - f_i''(k_i) \left(\frac{\partial r}{\partial t_i} \right)^3 \sum_{l=1}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^3} \sum_{j=1}^n \frac{1}{f_j''(k_j)} \\ &= -\left(\frac{\partial r}{\partial t_i} \right)^2 \sum_{j=1}^n \frac{f_j'''(k_j)}{(f_j''(k_j))^3} + \frac{f_i'''(k_i)}{(f_i''(k_i))^3} \left(1 + 2 \frac{\partial r}{\partial t_i} \right) + \left(\frac{\partial r}{\partial t_i} \right)^2 \sum_{l=1}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^3} \\ &= \frac{f_i'''(k_i)}{(f_i''(k_i))^3} \left(1 + 2 \frac{\partial r}{\partial t_i} \right).\end{aligned}$$

We obtain

$$\frac{\partial^2 R_i}{\partial t_i^2} + \sum_{j=1, j \neq i}^n \frac{\partial^2 R_i}{\partial t_i \partial t_j} = \frac{1}{f_i''(k_i)} \left(1 + \frac{\partial r}{\partial t_i} \right) + t_i \frac{f_i'''(k_i)}{(f_i''(k_i))^3} \left(1 + 2 \frac{\partial r}{\partial t_i} \right).$$

From the log-conavity and the 1/2-convexity of the function $f'(\cdot)$ we have:

$$\frac{1}{f_i'(k_i) f_i''(k_i)} \leq \frac{f_i'''(k_i)}{(f_i''(k_i))^3} \leq \frac{1}{2f_i'(k_i) f_i''(k_i)},$$

which allows us to establish that

$$\begin{aligned}
\frac{\partial^2 R_i}{\partial t_i^2} + \sum_{j=1, j \neq i}^n \frac{\partial^2 R_i}{\partial t_i \partial t_j} &= \frac{1}{f_i''(k_i)} \left(1 + \frac{\partial r}{\partial t_i}\right) + t_i \frac{f_i'''(k_i)}{(f_i''(k_i))^3} + 2t_i \frac{f_i'''(k_i)}{(f_i''(k_i))^3} \frac{\partial r}{\partial t_i} \\
&\leq \frac{1}{f_i''(k_i)} \left(1 + \frac{\partial r}{\partial t_i}\right) + \frac{t_i}{2f_i'(k_i) f_i''(k_i)} + 2t_i \frac{\partial r}{\partial t_i} \frac{1}{f_i'(k_i) f_i''(k_i)} \\
&= \frac{1}{f_i''(k_i)} \left(1 + \frac{\partial r}{\partial t_i}\right) + \frac{2t_i}{f_i'(k_i) f_i''(k_i)} \left(\frac{1}{4} + \frac{\partial r}{\partial t_i}\right).
\end{aligned}$$

The sufficient condition: $\partial r / \partial t_i > -1/4$ follows.

A.5 Proof for Proposition 3.

(i) We proceed by contradiction. Consider the game $\Gamma \equiv (S_i, R_i; i \in \mathbf{N} + 1)$. We denote respectively the Nash equilibrium tax rates of the game Γ by $(t_i(N), t_{-i}(N))$, and Γ' by $(t_i(N+1), t_{-i}(N+1))$, where $t_{-i}(N+1)$ (respectively $t_{-i}(N)$) is the vector of equilibrium tax rates for the $n-1$ (resp. n) other countries j with $(j \neq i)$. We denoted by $\Phi_i(\cdot)$ the First Order Condition (FOC) of each game for country i :

$$\Phi^i(t_i(N), t_{-i}(N)) \equiv \partial R_i(t) / \partial t_i \text{ for game } \Gamma$$

$$\text{and } \Phi^i(t_i(N+1), t_{-i}(N+1)) \equiv \partial R_i(t) / \partial t_i \text{ for game } \Gamma'.$$

Proof. For any country i ($i \in \{1, \dots, N\} \cap \{1, \dots, N+1\}$) the FOCs of the two games hold.

$$\Phi^i(t_i(N), t_{-i}(N)) = \Phi^i(t_i(N+1), t_{-i}(N+1)) = 0.$$

Assume that $\forall i \in \{1, \dots, N+1\}$, $t_i(N) > t_i(N+1)$. The concavity of $R_i(\cdot)$ with respect to t_i implies that $\Phi^i(\cdot)$ is decreasing in its first argument or equivalently $\Phi_1^i(t_i, t_{-i}) \equiv \partial \Phi^i(t_i, t_{-i}) / \partial t_i = \partial^2 R_i(t) / \partial t_i^2 < 0$. Similarly, the strategic complementarity of tax rates implies that $\Phi^i(\cdot)$ is increasing in the other arguments $\Phi_2^i(t_i, t_{-i}) \equiv \partial \Phi^i(t_i, t_{-i}) / \partial t_{-i} = \partial^2 R_i(t) / \partial t_i \partial t_{j \neq i} > 0$. We have

$$\Phi_1^i(t_i(N+1), t_{-i}(N)) > \Phi_1^i(t_i(N), t_{-i}(N)) = \Phi_1^i(t_i(N+1), t_{-i}(N+1)),$$

which implies that $t_{-i}(N) < t_{-i}(N+1)$ due to strategic complementarity and is a contradiction. \square

(ii) We have

$$\begin{aligned} R_i(t_i(N), t_{-i}(N)) &= \max_{t_i \in [0,1]} R_i(t_i, t_{-i}(N)) \geq R_i(t_i(N+1), t_{-i}(N)) \\ &\geq R_i(t_i(N+1), t_{-i}(N+1)). \end{aligned}$$

where the first inequality results from the definition of the Nash equilibrium and the second from the fact that $t_{-i}(N) > t_{-i}(N+1)$ and the plain complementarity of tax rates ($\partial R_i(\cdot) / \partial t_j \geq 0$).

Additional Appendices (not for publication)

The elasticity of the net return of capital with respect to tax rate

We show that the log-concavity of the marginal production function (assumption 2) implies that the elasticity of the net return of capital with respect to tax rate, denoted by ϵ_{r/t_i} , is decreasing in t_i . Differentiating ϵ_{r/t_i} with respect to t_i yields

$$\frac{\partial \epsilon_{r/t_i}}{\partial t_i} = \frac{\partial^2 r}{\partial t_i^2} \frac{t_i}{r} + \frac{1}{r} \frac{\partial r}{\partial t_i} - \frac{t_i}{r^2} \left(\frac{\partial r}{\partial t_i} \right)^2$$

or equivalently

$$\frac{\partial \epsilon_{r/t_i}}{\partial t_i} = \frac{t_i}{r} \frac{\partial r}{\partial t_i} \left[\frac{1}{t_i} - \frac{f_i'''(k_i)}{(f_i''(k_i))^2} - \left(1 + 2 \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \right) \frac{\partial r}{\partial t_i} - f_i''(k_i) \sum_{l=1}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^3} \left(\frac{\partial r}{\partial t_i} \right)^2 \right].$$

From assumption (3) and expression (4) we deduce

$$-f_i''(k_i) \left(\frac{\partial r}{\partial t_i} \right)^3 \sum_{l=1}^n \frac{f_l'''(k_l)}{(f_l''(k_l))^3} \leq -\frac{1}{r} f_i''(k_i) \left(\frac{\partial r}{\partial t_i} \right)^3 \sum_{l=1}^n \frac{1}{f_i''(k_i)} = -\frac{1}{r} \left(\frac{\partial r}{\partial t_i} \right)^2.$$

We deduce that

$$\begin{aligned} \frac{\partial \epsilon_{r/t_i}}{\partial t_i} &\leq -\frac{f_i'''(k_i)}{(f_i''(k_i))^2} \frac{\partial r}{\partial t_i} \left(1 + 2 \frac{\partial r}{\partial t_i} \right) \frac{t_i}{r} - \left(\frac{\partial r}{\partial t_i} \right)^2 \frac{t_i}{(r)^2} + \frac{1}{r} \frac{\partial r}{\partial t_i} - \frac{t_i}{r^2} \left(\frac{\partial r}{\partial t_i} \right)^2 \\ &= \left[1 - \frac{f_i'''(k_i)}{(f_i''(k_i))^2} t_i \right] \frac{1}{r} \frac{\partial r}{\partial t_i} - 2 \left[\frac{1}{r} + \frac{f_i'''(k_i)}{(f_i''(k_i))^2} \right] \frac{t_i}{r} \left(\frac{\partial r}{\partial t_i} \right)^2 \leq 0, \end{aligned}$$

since $1 - \frac{f_i'''(k_i)}{(f_i''(k_i))^2} t_i \geq 1 - \frac{t_i}{r+t_i} \geq 0$.

Cheap talk

Corollary 4. *Under assumptions (1) and (2), cheap talk is neither self-committing nor self-signaling in the tax competition game (Γ).*

Proof. Following Baliga and Morris (2002), we consider the following definitions:

Definition 1: Action t_i is self-committing if

$$R^i(t_i, \tau_j(t_i)) \geq R^i(t'_i, \tau_j(t'_i)) \text{ for } \forall t'_i \in [0, 1].$$

By definition, only the leader's tax rate at the Stackelberg equilibrium is self-committing on $[0, 1]$.

Definition 2: Action t_i is self-signaling if

$$R^i(t_i, \tau_j(t_i)) \geq R^i(t'_i, t_j) \text{ for } \forall (t'_i, t_j) \in [0, 1]^2.$$

Given the plain complementarity of the game (Γ) , we have:

$$R^i(t_i, \tau_j(t_i)) < R^i(t_i, t'_i) \text{ for } t'_i > \tau_j(t_i),$$

which contradicts the self-signaling condition. □