Accounting for Needs in Cost Sharing[*]

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Abstract

We develop a framework to formally account for needs when devising rates for utility services.

We first show that interdependence between agents should be explicitly accounted for: if rates depends only upon agents own consumption and needs, budget balance and equal treatment of equal are not compatible.

We then characterize two polar opposite rate-setting families of solutions. Conditional Equality solutions emphasizes responsibility for usage beyond needs while Egalitarian Equivalent solutions stress compensation for differences in needs.

Within these two families, we provide characterizations of several underlying cost-sharing rules to govern the management of the production externality when coupled by the relevant responsibility/compensation transfers. We then present corresponding rate schedules that make use of aggregate—and realistic—information to summarize distributional aspects.

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1 Introduction

Some public utilities, like water and wastewater services, are essential to achieving a decent standard of living. In a society where households differ in terms of their basic needs for utility services, these should be taken into account when setting utility rates. In practice, commendable efforts have been made in this regard, with rate schedules typically taking the form of multi-part tariffs (block pricing), including discounts given to households with higher needs (for the case of water supply in the US, see AWWA, 2012). These discounts can take the form of a rebate to low-income households, which is subsidized by a higher overall rate structure. Alternatively, increasing-block rate schedules subsidize the lowest block through rate premiums for large users, hence affording all households a low rate to meet basic needs. In the case of water services, this also addresses the issue of resource conservation. Nevertheless, while these practices recognize the fact that some households should be subsidized, the design of such subsidies, both in shape and in magnitude, is largely left to rule-of-thumb considerations.[1]

Also, while it remains an empirical matter whether pricing water actually leads to economic efficiency in practice, it is widely recognized that full cost recovery is essential to the sustainability of the infrastructure (Massarutto, 2007; AWWA, 2012; Canadian Water and Wastewater Association, 2015) and is “a key preoccupation” of many OECD countries (OECD, 2010).[2] Our concern is with the fair division of those costs among users and, specifically, with accounting for consumers’ needs.

We develop a framework to formally take matters of partial responsibility into account when devising rates for utility services, which we will assume to be water services, to fix ideas.[3] Each agent is summarized by its water consumption and its basic water needs, which may differ from one agent to the next. For instance, one can think of agents as being households of possibly different sizes. We take the view that agents are not responsible for their needs, but are fully responsible for their consumption beyond those needs.

Our approach builds on the axiomatic framework of liberal egalitarianism,

[1]For example, the M1 Manual of the American Water Works Association, a highly regarded reference by North American water utilities, gives surprisingly little guidance on how to determine rate blocks: “Generally, rate blocks should be set at logical break points.” (AWWA, 2012, p.107)

[2]In the context of water services, Massarutto (2007) identifies three important benefits of recovering costs through the pricing structure: to “ensure the viability of water management systems”, to “maintain asset value over time”, and to “guarantee the remuneration of inputs”.

[3]Our analysis applies to all utilities necessary for a decent standard of living, including electricity services.

which aims at compensating differences in “non-responsibility” characteristics while rewarding difference in characteristics under the agents’ control. Classically, agents are deemed responsible for their effort but have no control over their talents. Here, agents have no control over their basic water needs—say, 50 liters of clean water per day (Gleick, 1996)—but are responsible for their consumption beyond that amount. Thus, water consumption is a ’hybrid’ characteristic of sorts: the portion required to meet basic needs falls into the non-responsibility category, whereas the remainder falls into that of responsibility.

A general theme of that literature is that the two desiderata of compensation and reward are incompatible (Bossert, 1995; Bossert and Fleurbaey, 1996; Cappelen and Tungodden, 2006). Accordingly, one must set less ambitious goals for redistributive policies. This is typically done by giving priority to one ideal, compensation or reward, while weakening the scope of the other (Fleurbaey 2008, and references therein), leading to the *Egalitarian Equivalent* and *Conditional Equality* solutions, respectively. Likewise, we characterize two polar families of solutions: *Conditional Equality* solutions emphasize responsibility for excessive usage (Theorem 1) while *Egalitarian Equivalent* solutions stress compensation for differences in needs (Theorem 4).

Contrasting with previous results, the solutions we obtain are not unique because they depend on two additional dimensions that the literature is currently not equipped to handle: how to account for ’hybrid’ characteristics and how to account for cost externalities. Regarding the former, each family of solutions will produce different solutions whether one measures responsibility in terms of consumption beyond needs, $q-\bar{q}$, or in terms of its fraction relative to one’s own needs, $(q-\bar{q})/\bar{q}$, for example. We call these views *absolute responsibility* and *relative responsibility*, respectively. When agents’ welfare can be evaluated by the means of a (common) utility function—*i.e.* when agents differ only in their needs—, and the responsibility measure is chosen as to reflect the actual welfare of the agents—a more sophisticated exercise—Conditional Equality solutions are actually compatible with a much stronger compensation requirement than when responsibility is computed arbitrarily (Theorem 3). This implies that, when differences in needs summarize the relevant differences across agents, a sufficient knowledge of the utility function can afford greater compatibility between the desiderata of compensation and reward, a sharp contrast with existing results in the literature on liberal egalitarianism.

Even with a specific view on responsibility, much freedom remains regarding how to account for cost externalities within each family of solutions. Indeed, the partial responsibility approach determines how much of the costs should be associated to meeting basic needs, and should therefore be financed so as to compensate differences in needs. How to split the remainder, for which agents are deemed responsible, falls into the realm of cost-sharing theory. In principle, any cost-sharing rule can be associated with each family of solutions and with each responsibility view. However, given the nature of the service at hand, we posit an axiom, *independence of higher* (resp. *lower*) *responsibility*, which is particularly desirable when costs are convex (resp. concave). This characterizes a unique solution: the serial (resp. decreasing-serial) cost-sharing variant for each family of solutions (Moulin and Shenker, 1992; resp. de Frutos, 1998), Propositions 2-5.

Lastly, we show how one can implement the above schemes with realistic informational assumptions; i.e., without making explicit interpersonal comparisons of needs and consumption, which would prove very difficult and possibly counterproductive for all but small populations. In particular, we use household size as a proxy for needs and denote by $\bar{q}_s$ the needs of a household of size s. Using aggregate information to summarize distributional aspects, we design rate schedules that, otherwise, explicitly depend on the sole individual characteristics of households.

For instance, consider affine costs of the form $C(Q) = F + cQ$, with $F, c > 0$, where Q is the aggregate demand of the population.[4] When responsibility is measured by absolute responsibility, $q - \bar{q}_s$, the *decreasing serial conditional equality* solution[5] yields the following rate schedule for households of size s:

$$\frac{F + c\bar{Q}}{N} + c\left(q - \bar{q}_s\right), \tag{1}$$

where $\bar{Q}$ is the quantity needed to cover the needs of the entire population, and N is the total number of households. In addition to splitting the fixed cost equally, this rate schedule splits the cost of the population's needs equally before pricing consumption at marginal cost minus a rebate equal to the cost of meeting one's own needs.

The rate schedule changes significantly under the relative responsibility view. Assuming responsibility is identically distributed across types, we obtain the

[4]Such a cost structure is typical of water services, which exhibit high fixed costs (infrastructure) and low marginal costs (electricity for pumping and chemicals for treatment).

[5]As mentioned, the decreasing serial cost-sharing rule is the more appropriate for concave costs.

following rate schedule for households of size s:

$$\frac{F}{N} + \frac{c}{\bar{q}_s / \left(\bar{Q}/N\right)} q \tag{2}$$

The result is still a two-part tariff but one where only the fixed cost is split equally. No rebate is granted, and consumption is priced at a rate that is inversely proportional to one's needs.

As mentioned, the family of egalitarian equivalent solutions is based on utility comparisons with households having a hypothetical reference level of needs, $\bar{q}_0$, chosen by the planner. Under the *decreasing serial egalitarian equivalent* solution, which emphasizes compensating differences in needs, the rate schedule for households of size s is as follows:

$$\frac{F}{N} + cq + \left[u\left(q, \bar{q}_s\right) - u\left(q, \bar{q}_0\right)\right] - \frac{1}{N} \sum_t \int_{z=0}^{\infty} \left[u\left(z, \bar{q}_t\right) - u\left(z, \bar{q}_0\right)\right] n_t\left(z\right) dz,$$

where $u\left(q, \bar{q}_s\right)$ is the utility of a household of size s and where $n_s\left(q\right)$ is the density of households that are consuming q units in the distribution of size-s households. The cost-sharing portion of the schedule, $\frac{F}{N} + cq$, splits the fixed cost equally and prices consumption at marginal cost. Needs are completely absent from that component. However, they enter in the remaining redistributive portion to ensure that heterogeneity in needs does not drive differences in welfare.

The remainder is organized as follows. The next section offers a brief discussion of the related literature. Section 3 presents the formal model. In Section 4, we take the cost-sharing rule as given in order to focus on our contribution; namely, the introduction of essential needs in cost-sharing problems. We then introduce a specific property of the rate function, which aims at protecting small users while still holding them accountable, and show how doing so calls for adopting a specific underlying cost-sharing rule: the well-known *serial rule* (Section 5). Finally, we show in Section 6 how these abstract *formulae* actually boil down to specific two-part tariffs for which we provide an explicit and complete determination using only coarse information on characteristics of the population.

2 Related Literature

Liberal egalitarianism. Our work expands the literature on liberal egalitarianism in two ways. First, we extend the theory to settings with externalities. To our knowledge, the only other effort in this direction is Billette de Villemeur and Leroux (2011), which tackles the issue of global climate change and the design of transfer schemes between countries to account for their responsibility in current emissions and, possibly, their non-responsibility in past emissions.

Our second contribution has to do with our consideration of a characteristic—water consumption—for which one is both partly responsible and partly non-responsible. Ooghe and Peichl (2014) and Ooghe (2015) very recently introduced the notion of 'partial control' over some characteristics to handle different degrees of responsibility in any given characteristic. According to this 'soft cut', an agent may be responsible for, say, only 30% of his intellectual skills, the remainder being attributable to inborn abilities or environmental factors. Our view of consumption as a hybrid characteristic differs from theirs in that we deem households fully non-responsible for their needs, but fully responsible for any additional consumption. A portion of consumption is aimed at satisfying a household's needs—for which it is not responsible—whereas the remaining consumption is viewed as discretionary.

Needs. Economists have been aware for quite some time that the welfare interpretation of income inequality measures are problematic (see among others Garvy, 1954; David, 1959; Morgan, 1962). How to account for differences in ability and needs is still the topic of lively discussion in public economics, in particular in the literature on taxation, but not only (e.g., Mayshar and Yitshaki, 1996, Trannoy, 2003, Duclos et al. 2005, Duclos and Araar 2007). Ebert (1997) adopts an axiomatic approach to discuss the comparison of income distributions when the population consists of heterogeneous households. Observing that economic growth had done very little for the poorer half of the third world population, some economists at the World Bank have pointed out the importance of looking at basic needs (Streeten and Burki, 1978; Streeten, 1979; Hicks and Streeten, 1979). Similarly, rather than being concerned with the "affordability" of services to low-income households, as do most approaches to rate setting, we focus on the material—as opposed to financial—needs of households.

Fair division. Despite mounting empirical evidence suggesting that needs are a relevant ingredient of fairness (Konow, 2001; Traub et al, 2005; Schwettman, 2012), the literature on fair division has only recently considered basic needs in a formal fashion. Specifically, although in a setting different from ours, Bergantiños et al. (2012) and Manjunath (2012) modify the classical rationing problem—where a fixed social endowment must be divided among several recipients—to account for a minimal requirement. There, agents are indifferent between receiving less than this minimal share and receiving nothing.

Because full cost recovery is an objective, the relevant strand of the fair division literature is that of cost sharing. Yet, the literature on cost sharing does not explicitly address the issue of basic needs. The closest interpretation are sharing rules that protect small users when costs are convex (Moulin and Shenker, 1992) or guarantee that small users will indeed be rewarded from reducing their consumption to the tune of their effort (de Frutos, 1998). This relates to the notion of affordability rather than to the fact that agents have material needs, as we do here. Nonetheless, these two sharing rules complement our approach (Section 5).

3 Accounting for Needs

The Model. Let $\{1,...,n\}$ be the set of agents. Agent i consumes a quantity $q_i \geq 0$ of water. Serving all of the agents' demands costs $C\left(\sum_i q_i\right) \geq 0$ that must be paid for by the agents' water bills, x_i:

$$\sum_{i=1}^{n} x_i = C\left(Q\right),$$

where $Q = \sum_{i=1}^{n} q_i$. The cost function, C, is assumed to be increasing.[6] We denote by Γ the class of cost functions.

Each agent $i \in N$ must fulfill her basic needs in terms of water use, denoted $\bar{q}_i \geq 0$. We adopt a quasi-linear setup. Agent i's utility level is defined by:

$$U_i\left(q_i, \bar{q}_i, x\right) = u_i\left(q_i, \bar{q}_i\right) - x_i,$$

where x_i is agent i's payment for water use. The utility function u_i, which is

[6] We use the following convention: By 'increasing' we mean 'strictly increasing'. We use the term 'non-decreasing' when the monotonicity is not strict. Similarly, by 'positive' we mean 'strictly positive', and use 'nonnegative' when zero is not excluded. Etc.

possibly agent specific, is defined on $\mathbb{D} \equiv \left\{(x, y) \in \mathbb{R}_+^2 | x \geq y\right\}$.[7] It is assumed to be increasing in q_i and decreasing in $\bar{q}_i$. We denote by Υ the class of utility functions. When agents consume exactly their needs, they share a common utility level $\underline{u}$ that, without any loss of generality, we can set to zero. Formally,

$$u_i\left(\bar{q}_i, \bar{q}_i\right) \equiv 0, \qquad \forall i \in N.$$

Assigning responsibility. Our aim is to design a pricing rule that will take individual responsibilities into account. In order to do so, we must define the sphere of responsibility of the agents. We take the view that agent are not responsible for their essential needs, $\bar{q}_i$, but that are responsible for any additional water consumption. The extent of responsibility can be measured in many different ways. For the sake of generality, we define a real-valued function, $r(q_i, \bar{q}_i)$, defined on $\mathbb{D}$, which is increasing in water consumption q_i, non-increasing in needs $\bar{q}_i$, and normalized to zero when $q_i = \bar{q}_i$. When no confusion is possible, we abuse notations slightly by denoting $r_i = r(q_i, \bar{q}_i)$. We denote by R the class of responsibility functions.

A *consumption-needs profile* (or *a profile*) is a list of n consumption-needs pairs that we shall denote $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$, abusing notations slightly.[8]

Rate functions and cost-sharing rules. Our contribution is to account for needs in cost sharing. We aim at washing out the impact of differences in needs on consumers' well-being, because we consider that agents are not responsible for their needs. In turn, this calls for redefining the notion of responsibility towards the total cost. Once this is done, we can then share the total cost according to the responsibility profile, $\mathbf{r} \equiv (r_1, r_2, ..., r_n)$. In doing so, *cost-sharing rules* (ξ) will allow us to highlight the distinction between the handling of the production externality—governed by the shape of the cost function—and the redistribution problem that follows from taking essential needs into account. We ultimately provide pricing formulae for water utilities, that we shall refer to as *rate functions* (x).

Formally, let $\mathcal{C}(\mathbf{q}, \bar{\mathbf{q}})$ stand for the portion of the cost for which the population is considered to be responsible, once needs are accounted for. The

[7]Because we consider $\bar{q}_i$ to represent agent i's basic needs, it is a lower bound to her consumption.

[8]We shall adopt the convention that boldface type refers to the vector of the relevant variables. E.g., $\mathbf{q} = (q_1, ..., q_n)$ and so on.

principles of liberal reward and compensation will guide us in defining $\mathcal{C}(\mathbf{q},\bar{\mathbf{q}})$. A cost-sharing rule is a mapping that splits this portion of the cost across users: $\xi : \mathbb{R}^n \times \Gamma \to \mathbb{R}^n$, such that $\sum_i \xi_i(\mathbf{r},\mathcal{C}) = \mathcal{C}(\mathbf{q},\bar{\mathbf{q}})$. By contrast, a rate function takes all the information in the economy into account and is a mapping $x : \mathbb{D}^n \times R \times \Upsilon \times \Gamma \to \mathbb{R}^n$ such that $\sum_{i\in N} x_i(\mathbf{q},\bar{\mathbf{q}},r,u,C) = C(Q)$ where $C(Q)$ is the total cost to be covered.

Section 6 will be devoted to obtaining explicit formulae based on illustrative examples. Until then, fix the cost function, C, the common utility function, u, and the responsibility function, r. As a result, we abuse notations slightly and write $x(\mathbf{q},\bar{\mathbf{q}})$ instead of the more cumbersome $x(\mathbf{q},\bar{\mathbf{q}},r,u,C)$.

4 Fair Treatment

4.1 Interdependence and Anonymity

Since one is not responsible of others actions or characteristics, a natural (although naive) view of fairness in the context of rate setting consists in asking for a user's bill to be a function of her own individual characteristics only: q_i and $\bar{q}_i$. In this context, a minimal equality requirement is that two agents with identical needs face the same pricing schedule:

Axiom. ***(Equal Rate Schedule for Equal Needs, ERSEN)***
A user's bill depends solely on individual characteristics: $x_i : (\mathbf{q},\bar{\mathbf{q}}) \mapsto x_i(q_i,\bar{q}_i)$. *Moreover, the functions* $q_i \mapsto x_i(q_i,\bar{q}_i)$ *and* $q_j \mapsto x_j(q_j,\bar{q}_j)$ *must be identical whenever* $\bar{q}_i = \bar{q}_j$.

As it turns out, ERSEN is not only simplistic, it is downright unfeasible:

Theorem 1. *No rate function satisfies* ***ERSEN*** *unless the cost function is linear.*[9]

Proof. In Appendix A.1. □

By requiring that a user's bill depend solely on individual characteristics, ERSEN ignores the interdependence that exists between agents through the cost function. Theorem 1 makes it clear that, if agents interdependence is not accounted for, budget balance and equal treatment of equals are generically incompatible.

[9]Proposition 1 follows from the nonlinearity of the cost function, and therefore holds true even in the traditional cost-sharing set-up where needs are absent.

It follows that we must depart from the simplistic view according to which agents can ignore the impact they have on others, as it is assumed to be the case under perfect competition, for instance. We therefore adopt a more comprehensive view in which water bills depend explicitly on the entire profile of consumption and needs.

The fairness requirement we shall adopt is that the rate function satisfies *anonymity*. Formally, we shall require that, for any permutation of the agents $\pi : N \to N$:

$$x_{\pi(i)}\left(\mathbf{q}_\pi;\bar{\mathbf{q}}_\pi\right) = x_i\left(\mathbf{q};\bar{\mathbf{q}}\right) \qquad \text{for all } i \in N,$$

where $\mathbf{q}_\pi$ (resp. $\bar{\mathbf{q}}_\pi$) is the vector of consumption (resp. needs) after permutation of the agents along π.

Remark 1. Anonymity implies the equal treatment of equals: $(q_i, \bar{q}_i) = (q_j, \bar{q}_j) \implies x_i\left(\mathbf{q};\bar{\mathbf{q}}\right) = x_j\left(\mathbf{q};\bar{\mathbf{q}}\right)$. Two users with identical needs and identical consumption must pay the same bill.

4.2 The Reward Principle: Responsibility Axioms

The general idea behind the reward principle is that conservative users should be rewarded in the form a lower water bill. Of course, if needs are accounted for, whether consumption is moderate or not is not measured by considering only actual consumption, but on the basis of $r\left(q_i, \bar{q}_i\right)$.

A minimal requirement in terms of responsibility is that the portion of costs that exceeds the needs of the population, $C(Q) - C(\bar{Q})$, be fully distributed to users. This leads us to introduce a cost-sharing rule, ξ, that will split the cost $C(Q) - C(\bar{Q})$ according to the profile of responsibility characteristics, $\mathbf{r}$. Keeping with the desideratum of anonymity, we shall consider only symmetric cost-sharing rules:

$$\xi\left(\mathbf{r}, C - C(\bar{Q})\right) \text{ is a symmetric function of the variables } r_i,\ i \in N.$$

The function ξ embodies how we want to hold agents accountable for their consumption.[10] Given ξ, the following axioms specify how responsibility is assigned, and are presented in decreasing order of stringency.

Axiom. ***(Shared Responsibility, SR)***

[10]If needs were not an issue, we would be back to the classical cost-sharing framework where $\xi\left(\mathbf{q}, C\right)$ alone defines the shares to be paid (see Moulin, 2002, for a thorough survey).

For any profile $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$ *:*

$$x_k(\mathbf{q}, \bar{\mathbf{q}}) - x_k(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = \xi_k(\mathbf{r}, C - C(\bar{Q})),$$

for all $k \in N$*.*

A less demanding axiom consists in sharing the costs $C(Q) - C(\bar{Q})$ according to ξ only when all agents have equal needs.

Axiom. ***(Shared Responsibility for Uniform Needs, SRUN)***
For any profile $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$ *such that* $\bar{q}_i = \bar{q}_j$ *for all* $i, j \in N$*,*

$$x_k(\mathbf{q}, \bar{\mathbf{q}}) - x_k(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = \xi_k(\mathbf{r}, C - C(\bar{Q})),$$

for all $k \in N$*.*

Finally, an even less demanding axiom consists in sharing costs according to ξ only when needs are identical and equal to a reference level, $\bar{q}_0 \in \mathbb{R}_+$.

Axiom. ***(Shared Responsibility for Reference Needs, SRRN)***
Define $\bar{q}_0 \in \mathbb{R}_+$ *a reference level of needs. For any profile* $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$ *such that* $\bar{q}_i = \bar{q}_0$ *for all* $i \in N$*,*

$$x_k(\mathbf{q}, \bar{\mathbf{q}}_0) - x_k(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0) = \xi_k(\mathbf{r_0}, C - C(n\bar{q}_0)),$$

for all $k \in N$*, where* $\bar{\mathbf{q}}_\mathbf{0} = (\bar{q}_0, \bar{q}_0, ..., \bar{q}_0)$ *and* $r_{0,i} = r(q_i, \bar{q}_0)$ *for all* $i \in N$*.*

4.3 The Compensation Principle: No Responsibility for One's Needs

Throughout the paper, we take the view that agents are not responsible for their needs. This bears consequences on the way the cost of meeting the needs of the population, $C(\bar{Q})$, is distributed across agents. But this may also play a role in how the remaining cost, $C(Q) - C(\bar{Q})$, for which agents are considered collectively responsible, is priced to the single agent.

Ideally, needs should have no impact on welfare:

Axiom. ***(Group Solidarity, GS)***
For any $i \in N$ *and any profiles* $(\mathbf{q}, \overline{\mathbf{q}})$ *and* $(\mathbf{q}, \overline{\mathbf{q}}')$ *such that* $\overline{q}'_i \neq \overline{q}_i$ *and* $\overline{q}'_j = \overline{q}_j$ *all* $j \in N \backslash \{i\}$ *, then*

$$[u_i(q_i, \bar{q}'_i) - x'_i] - [u_i(q_i, \bar{q}_i) - x_i] = [u_j(q_j, \bar{q}'_j) - x'_j] - [u_j(q_j, \bar{q}_j) - x_j],$$

all $j \in N$, *where* $x = x(\mathbf{q}, \overline{\mathbf{q}})$ *and* $x' = x(\mathbf{q}, \overline{\mathbf{q}}')$.

Another approach in demanding that needs should not drive difference in welfare consists in requiring that when agents bear an equal responsibility, their welfare should be equal:

Axiom. ***(Equal Welfare for Equal Responsibility, EWER)***

$$r_i = r_j \implies u_i(q_i, \bar{q}_i) - x_i = u_j(q_j, \bar{q}_j) - x_j,$$

where $x = x(\mathbf{q}, \overline{\mathbf{q}})$.

We shall also consider a weaker axiom, which consists in requiring equality of welfare only if all agents bear an equal responsibility:

Axiom. ***(Uniform Welfare for Uniform Responsibility, UWUR)***
If $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$ *is such that,*

$$r_i = r_j, \qquad \text{for all } i, j \in N$$

then

$$u_i(q_i, \bar{q}_i) - x_i = u_j(q_j, \bar{q}_j) - x_j, \qquad \text{for all } i, j \in N$$

where $x = x(\mathbf{q}, \overline{\mathbf{q}})$.

An even weaker axiom consists in having the same requirement only if this common level of responsibility is equal to a reference level:

Axiom. ***(Uniform Welfare for Reference Responsibility, UWRR)***
Let $r_0 \in \mathbb{R}_+$ *be a reference responsibility level. If* $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$ *is such that,*

$$r(q_i, \bar{q}_i) = r_0, \qquad \text{for all } i \in N$$

then

$$u_i(q_i, \bar{q}_i) - x_i = u_j(q_j, \bar{q}_j) - x_j, \qquad \text{for all } i, j \in N$$

where $x = x(\mathbf{q}, \overline{\mathbf{q}})$.

Finally, the following axiom states that when this reference responsibility level is set to zero, all should end up with the same level of welfare.

Axiom. ***(Uniform Welfare for Minimal Consumption, UWMC)***
If $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$ *is such that,*

$$q_i = \bar{q}_i, \qquad \text{for all } i \in N,$$

then

$$u_i(q_i, \bar{q}_i) - x_i = u_j(q_j, \bar{q}_j) - x_j, \qquad \text{for all } i, j \in N,$$

where $x = x(\mathbf{q}, \bar{\mathbf{q}})$.

4.4 Pricing Mechanisms

We now turn to the design of pricing mechanisms. The principles of responsibility and compensation will determine how to allocate the cost of meeting the needs of the population, $C(\bar{Q})$, but not only. As we shall see, these principles will also interact with how the cost $C(Q) - C(\bar{Q})$, is to be split. The two portions of the cost cannot be considered in isolation.

Conditional Equality: SR+UWRR

Turning first to rate functions that prioritize holding agents responsible for their consumption, we identify the strongest compensation axioms compatible with **SR.** We find that **UWRR** and **SR** jointly characterize a family of rate functions that is parametrized by the choice of a reference responsibility level, r^0:

Theorem 2. *A rate function satisfies* ***SR*** *and* ***UWRR*** *if and only if it is a Conditional Equality solution: For some reference level* $r^0 > 0$*, and all* $i \in N$*,*

$$x_i^{CE}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C(\bar{Q})}{n} + \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j \in N} u_j\left(q_j^0, \bar{q}_j\right)$$

where q_i^0 *is defined by* $r\left(q_i^0, \bar{q}_i\right) = r^0$.

Proof. In Appendix A.2. □

A special variant of the Conditional Equality solution consists in choosing zero responsibility as a reference: $\mathbf{q_0} = \bar{\mathbf{q}}$. This implies charging households the same fee to meet their own needs, whatever these needs may be. Should they choose to consume more, they would bear the consequences according to the cost sharing rule in effect.

Corollary 1. *The unique rate function satisfying* ***SR*** *and* ***UWMC*** *is the following:*

$$x_i^{CE0}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i(\mathbf{r}, C - C(\bar{Q})) \qquad \text{for all } i \in N.$$

A limit of x^{CE0} is that compensation for needs is established on the basis of a single scenario which is actually never observed. However, it possesses the advantage of not requiring knowledge of the utility function.

Theorem 2 is generically tight because x^{CE} generically does not satisfy the stronger compensation axiom **UWUR**. The only exception is when the agents share a common utility function and the responsibility function, r, reflects the utility derived by the agents:

Proposition 1. x^{CE} *does not satisfy* ***UWUR*** *unless*
(1) all agents share a common utility function, i.e. $u_i = u$, *all* $i \in N$ *for some function* $u : \mathbb{D} \to \mathbb{R}$ *increasing in its first argument and decreasing in the second*
(2) the responsibility function co-varies with agents utility, i.e. $r = \rho \circ u$, *for some increasing function* $\rho : \mathbb{R} \to \mathbb{R}_+$.

Proof. In Appendix A.3. □

However, when the agents share a common utility function and the responsibility function is set so as to reflect that utility, **SR** is even compatible with the stronger compensation axiom **EWER**. Together, they characterize a unique solution:

Theorem 3. *When the agents differs only in their needs so that they share a common utility function* u *and the responsibility function is defined as* $r = \rho \circ u$, *for some increasing function* $\rho : \mathbb{R} \to \mathbb{R}_+$, *a rate function satisfies* ***EWER*** *and* ***SR*** *if and only if*

$$x^{CE} = \frac{C\left(\bar{Q}\right)}{n} + \xi_i(\mathbf{r}, C - C(\bar{Q})) \qquad \text{for all } i \in N.$$

Proof. In Appendix A.4. □

The above result apply only to specific circumstances: all agents are supposed to share a common utility function so that all their differences are supposed to derive from their sole difference in needs. Yet, a remarkable feature of the above characterization is that it does not require specifying a reference responsibility level, although it obviously requires knowledge of the (common) utility function.

Theorem 3 is a tight characterization because **SR** is incompatible with the strongest solidarity axiom, **GS**, as Theorem 4 below implies.

Egalitarian Equivalence: GS+SRRN

We now turn to rate functions that prioritize negating the impact of differences in needs on welfare. Axiom **GS** embodies this desideratum. We show that **GS** together with **SRRN** determine a family of rate functions that is parametrized by a reference level of needs, $\bar{q}_0$:

Theorem 4. *A rate function satisfies **GS** and **SRRN** if and only if it is an Egalitarian Equivalent solution: For a given reference level of needs,* $\bar{q}_0 > 0$*,*

$$\begin{aligned} x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}) &= \frac{C(n\bar{q}_0)}{n} + \xi_i\left(\mathbf{r}_0, C - C\left(n\bar{q}_0\right)\right) \\ &\quad + \left[u_i\left(q_i, \bar{q}_i\right) - u_i\left(q_i, \bar{q}_0\right)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u_k\left(q_k, \bar{q}_k\right) - u_k\left(q_k, \bar{q}_0\right)\right], \end{aligned}$$

where $\mathbf{r}_0 = \left(r\left(q_1, \bar{q}_0\right), r\left(q_2, \bar{q}_0\right), ..., r\left(q_n, \bar{q}_0\right)\right)$.

Proof. In Appendix A.5. □

x^{EE} measures responsibility relative to the common reference level, $\bar{q}_0$: $r_{i,0} = r\left(q_i, \bar{q}_0\right)$ and splits costs accordingly. Differences between actual needs and the reference level are compensated for so as to preserve the relative welfare distribution.

The characterization is tight, in the sense that the Egalitarian Equivalent does not satisfy stronger responsibility axioms. This can be shown by considering a profile $(\mathbf{q}, \bar{\mathbf{q}}_1)$ such that $\bar{q}_1 \neq \bar{q}_0$ to obtain that SRUN is not satisfied. The formal proof of which can be found in Appendix A.6.

Remark 2. The cost-sharing portion of the transfer, $(1/n)\, C(n\bar{q}_0) + \xi_i\left(\mathbf{r}_0, C - C\left(n\bar{q}_0\right)\right)$, is driven by the consumption profile of the agents and by the cost structure, but is actually independent of individual needs. By contrast, the redistributive component of the water bill, $\left[u_i\left(q_i, \bar{q}_i\right) - u_i\left(q_i, \bar{q}_0\right)\right] - (1/n)\sum_{k=1}^{n}\left[u_k\left(q_k, \bar{q}_k\right) - u_k\left(q_k, \bar{q}_0\right)\right]$, is based on the benefits the agents derive from water consumption and is independent of costs.

Remark 3. Whenever needs summarize all relevant differences across agents so that they share a common utility function u, whatever the value of $\bar{q}_0$, the Egalitarian Equivalent solution fully addresses the issue of differences in needs whenever consumption is uniform. Formally, if $q_1 = q_2 = ... = q_n$, then $u_i(q_i, \bar{q}_i) - x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}) = u_j(q_j, \bar{q}_j) - x_j^{EE}(\mathbf{q}, \bar{\mathbf{q}})$ for all i, j. In other words, under EE, any differences in utility levels are attributable to differences in consumption.

Remark 4. To appreciate the difference between the Egalitarian Equivalent solution, x^{EE}, and the Conditional Equality solution, x^{CE}, consider the case where the reference level of needs is the average level of needs of the population: $\bar{q}_0 = \bar{Q}/n$. It follows that:

$$x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}) = x_i^{CE}(\mathbf{q}, \bar{\mathbf{q}}_0) + [u_i(q_i, \bar{q}_i) - u_i(q_i, \bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n}[u_k(q_k, \bar{q}_k) - u_k(q_k, \bar{q}_0)]$$

for all $i \in N$. In this particular case, the Egalitarian Equivalent solution applies additional redistribution associated with the impact of needs on welfare levels.

Remark 5. However, it does not follow that the Egalitarian Equivalent solution is always more redistributive. Indeed, the parameter $\bar{q}_0$ dictates both the portion of the cost to be shared in an egalitarian fashion and how differences in needs are accounted for. In particular, when $\bar{q}_0 = 0$ rather than $\bar{Q}/n$, the portion of costs to be split equally under x^{EE} is nil—$C(n\bar{q}_0)/n = 0$—and users are held responsible for their whole consumption. By contrast, x^{CE} always shares equally the portion of costs corresponding to the needs of the population: $C(\bar{Q})$.

5 Protecting small users while holding them responsible

5.1 Convex Costs

We introduce an axiom that aims to protect parsimonious users from the cost externality caused by wasteful users: An agent who increases her responsibility level cannot result in consumers with lower responsibility paying a higher amount.

Axiom (Independence of Higher Responsibility, IHR)**.** *For all* $(\mathbf{q}, \bar{\mathbf{q}})$ *and* $(\mathbf{q}', \bar{\mathbf{q}}')$ *such that* $\bar{\mathbf{q}}' = \bar{\mathbf{q}}$ *and* $\mathbf{r}' \geq \mathbf{r}$*. For all* $i \in N$*, define* $L(i) = \{j \in N \text{ s.t. } r_j \leq r_i\}$

the set of users with lower responsibility than i. Then,

$$\begin{aligned} & \left\{ r'_j = r_j \text{ for all } j \in L(i) \right\} \\ \Longrightarrow \quad & \left\{ \xi_j \left(\mathbf{r}', C - C\left(\bar{Q}'\right)\right) = \xi_j \left(\mathbf{r}, C - C\left(\bar{Q}\right)\right) \text{ for all } j \in L(i) \right\}. \end{aligned}$$

Remark 6. Note that for a given profile $(\mathbf{q}, \bar{\mathbf{q}})$, such that $q_i > q_j$ and $\bar{q}_i > \bar{q}_j$ for some i and j, then one can find two functional forms $\tilde{r}$ and $\hat{r}$ such that

$$\tilde{r}(q_i, \bar{q}_i) \geq \tilde{r}(q_j, \bar{q}_j) \quad \text{and} \quad \hat{r}(q_i, \bar{q}_i) < \hat{r}(q_j, \bar{q}_j).$$

Hence, the identity of consumers with a smaller responsibility depends on how responsibility is measured; i.e., upon the specific functional form for r.

Serial Conditional Equality

Recall that $r(\cdot, \bar{q}_i)$ maps an agent's consumption to her responsibility level, given her needs. Define the inverse of this function, $g_i(\cdot) = (r)^{-1}(\cdot, \bar{q}_i)$, which maps a responsibility level to the corresponding consumption level given the needs of the agent.

Proposition 2. *The unique rate function satisfying* ***UWMC****,* ***SR*** *and* ***IHR*** *is the following:*

$$x_i^{SCE0}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C\left(\hat{Q}^i\right)}{(n-i+1)} - \sum_{k=1}^{i-1} \frac{C\left(\hat{Q}^k\right)}{(n-k)(n-k+1)} \qquad \text{for all } i \in N,$$

where, for all $k \in N$,

$$\hat{Q}^k = \sum_{i=1}^{k-1} q_i + \sum_{i=k}^{n} g_i(r_k),$$

where the set of agents is ordered so as to have $r_1 \leq r_2 \leq ... \leq r_n$.

Proof. In Appendix B.1. □

Remark 7. x^{SCE0} amounts to applying the serial cost-sharing rule to responsibility levels to split the associated costs. In fact,

$$x_i^{SCE0}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{1}{n} C\left(\bar{Q}\right) + \sum_{k=1}^{i} \frac{1}{n-k+1} \left[C\left(\hat{Q}^k\right) - C\left(\hat{Q}^{k-1}\right) \right],$$

with $\hat{Q}^0 = \bar{Q}$. This is of notable interest because the serial cost-sharing rule is known for its strong incentives properties (Moulin and Shenker, 1992).

Notice that a higher responsibility level leads to a higher bill: $r_i \geq r_j$ implies $x_i^{SCE0}(\mathbf{q}, \bar{\mathbf{q}}) \geq x_j^{SCE0}(\mathbf{q}, \bar{\mathbf{q}})$ because

$$\hat{Q}^{k+1} - \hat{Q}^k = \sum_{i=k+1}^{n} [g_i(r_{k+1}) - g_i(r_k)] \geq 0.$$

Serial Egalitarian Equivalence

Proposition 3. *The unique rate function satisfying **GS**, **SRRN** and **IHR** is the following:*

$$\begin{aligned} x_i^{SEE}(\mathbf{q}, \bar{\mathbf{q}}) &= \frac{C\left(\tilde{Q}^i\right)}{(n-i+1)} - \sum_{k=1}^{i-1} \frac{C\left(\tilde{Q}^k\right)}{(n-k)(n-k+1)} \\ &\quad + [u_i(q_i, \bar{q}_i) - u_i(q_i, \bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n} [u_k(q_k, \bar{q}_k) - u_k(q_k, \bar{q}_0)] \end{aligned}$$

for all $i \in N$, where $\tilde{Q}^k = \sum_{l=1}^{k} q_l + (n-k) q_k$ with the set of agents ordered so as to have $q_1 \leq q_2 \leq ... \leq q_n$.

Proof. In Appendix B.2. □

Remark 8. The expression for x^{SEE} is independent of the form of responsibility.

Remark 9. x^{SEE} amounts to applying the serial cost-sharing rule directly to consumption, along with transfers to compensate for differences in needs. In fact,

$$\begin{aligned} x_i^{SEE}(\mathbf{q}, \bar{\mathbf{q}}) &= \frac{1}{n} C\left(n \inf_j q_j\right) + \sum_{k=1}^{i-1} \frac{1}{n-k} \left[C\left(\tilde{Q}^{k+1}\right) - C\left(\tilde{Q}^k\right)\right] \\ &\quad + [u_i(q_i, \bar{q}_i) - u_i(q_i, \bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n} [u_k(q_k, \bar{q}_k) - u_k(q_k, \bar{q}_0)]. \end{aligned}$$

Note that the compensation terms may affect the well-known incentives properties of the serial cost-sharing rule.

At first blush, the expressions of x^{SCE0} and x^{SEE} may seem similar, with x^{SEE} having an additional compensation term. However, note that agents are ordered according to their consumption under x^{SEE} but are ordered according to their responsibility level under x^{SCE0}. Also, the Q^k's that enter in the cost-sharing portion stand for different aggregate consumption levels. In particular, SEE applies the serial cost-sharing rule directly on consumption levels, with the consideration for needs solely entering the compensation portion. By contrast, SCE applies the serial cost-sharing rule to responsibility levels which, by design, take individual needs into account.

5.2 Concave Costs

With increasing marginal cost, we wished to protect users with smaller responsibility levels from bearing a high marginal cost due to the presence of 'large users'. By contrast, when the technology exhibits increasing returns to scale, we want 'small users' to fully benefit from a further reduction in their consumption. It follows that larger users never benefit from the effort of smaller users in reducing their consumption.

Axiom (Independence of Lower Responsibility, ILR)**.** *For all* $(\mathbf{q}, \bar{\mathbf{q}})$ *and* $(\mathbf{q}', \bar{\mathbf{q}}')$ *such that* $\bar{\mathbf{q}}' = \bar{\mathbf{q}}$ *and* $\mathbf{r}' \leq \mathbf{r}$. *For all* $i \in N$, *define* $H(i) = \{j \in N \text{ s.t. } r_j \geq r_i\}$ *the set of users with higher responsibility level than* i. *Then,*

$$\begin{aligned} & \left\{r'_j = r_j \text{ for all } j \in H(i)\right\} \\ \Longrightarrow \quad & \left\{\xi_j\left(\mathbf{r}', C - C\left(\bar{Q}'\right)\right) = \xi_j\left(\mathbf{r}, C - C\left(\bar{Q}\right)\right) \text{ for all } j \in H(i)\right\}. \end{aligned}$$

Decreasing Serial Conditional Equality

Proposition 4. *The unique rate function satisfying* ***UWMC****,* ***SR*** *and* ***ILR*** *is the following:*

$$x_i^{DSCE0}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C\left(\check{Q}^i\right)}{i} - \sum_{k=i+1}^{n} \frac{C\left(\check{Q}^k\right)}{k(k-1)} \qquad \text{for all } i \in N,$$

where, for all $k \in N$,

$$\check{Q}^k = \sum_{l=1}^{k} g_l(r_k) + \sum_{l=k+1}^{n} q_l,$$

where the set of agents is ordered so as to have $r_1 \leq r_2 \leq ... \leq r_n$.

Proof. In Appendix B.3. □

Remark 10. x^{DSCE0} amounts to applying the decreasing serial cost-sharing rule to responsibility levels in order to split the associated costs. In fact,

$$x_i^{DSCE0}(\mathbf{q},\bar{\mathbf{q}}) = \frac{1}{n}C\left(\check{Q}^n\right) - \sum_{k=i}^{n-1}\frac{1}{k}\left[C\left(\check{Q}^{k+1}\right) - C\left(\check{Q}^k\right)\right]$$

with $\check{Q}^1 = Q$. Like the serial rule, the decreasing serial cost-sharing rule is also known for its strong incentives properties (de Frutos, 1998).

Note that a higher responsibility level indeed leads to a higher bill: $q_i^r \geq q_j^r$ implies $x_i^{DSCE0}(\mathbf{q},\bar{\mathbf{q}}) \geq x_j^{DSCE0}(\mathbf{q},\bar{\mathbf{q}})$ because

$$\check{Q}^{k+1} - \check{Q}^k = \sum_{l=1}^{k}\left[g_l\left(r_{k+1}\right) - g_l\left(r_k\right)\right] \geq 0.$$

Decreasing Serial Egalitarian Equivalence

Proposition 5. *The unique rate function satisfying* ***SRRN****,* ***GS*** *and* ***ILR*** *is the following: For all* $i \in N$*,*

$$x_i^{DSEE}(\mathbf{q},\bar{\mathbf{q}}) = \frac{C\left(\check{Q}^i\right)}{i} - \sum_{k=i+1}^{n-1}\frac{C\left(\check{Q}^k\right)}{k\left(k-1\right)} + \left[u_i\left(q_i,\bar{q}_i\right) - u_i\left(q_i,\bar{q}_0\right)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u_k\left(q_k,\bar{q}_k\right) - u_k\left(q_k,\bar{q}_0\right)\right]$$

where $\check{Q}^k = kq_k + \sum_{l=k+1}^{n} q_l$ *for all* $k = 1,...,n$*, with the set of agents ordered so as to have* $q_1 \leq q_2 \leq ... \leq q_n$*.*

Proof. In Appendix B.4. □

Remark 11. x^{DSEE} amounts to applying the decreasing serial cost-sharing rule directly to consumption, along with transfers to compensate for differences in needs. In fact,

$$\begin{aligned} x_i^{DSEE}(\mathbf{q},\bar{\mathbf{q}}) &= \frac{1}{n}C\left(n\sup_j q_j\right) - \sum_{k=i}^{n-1}\frac{1}{k}\left[C\left(\check{Q}^{k+1}\right) - C\left(\check{Q}^k\right)\right] \\ &\quad + \left[u_i\left(q_i,\bar{q}_i\right) - u_i\left(q_i,\bar{q}_0\right)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u_k\left(q_k,\bar{q}_k\right) - u_k\left(q_k,\bar{q}_0\right)\right] \end{aligned}$$

6 Accounting for responsibility in practice

In practice, making explicit interpersonal comparisons of needs and consumption would be very difficult and possibly counterproductive. Nevertheless, we show how one can implement the above schemes with realistic informational assumptions.[11]

6.1 Pricing using aggregate distributions

We now represent the population by a distribution. Assume that needs summarize all relevant differences so that agents share a common utility function u. Assume further that there is a finite number of types in the needs dimension due to, say, household size, and let $\bar{q}_s$ denote the needs of a household of size $s \in S$. Let $n_s(q)$ be the density of type-s households with consumption level q and let $N_s(q)$ be the associated cumulative distribution: $N_s(q) = \int_{z=0}^{q} n_s(z)\, dz$. Define $n(q) = \sum_{s \in S} n_s(q)$ and $N(q) = \sum_{s \in S} N_s(q)$. We slightly abuse notation and write $r(q, s)$ instead of $r(q, \bar{q}_s)$ whenever it is unambiguous. Given the responsibility function r, define $n_s^r(\rho)$ the density of type-s households with responsibility level ρ. Let $N_s^r(\rho)$ be the associated cumulative distribution: $N_s^r(\rho) = \int_{z=0}^{\rho} n_s^r(z)\, dz$ and define $N^r(\rho) = \sum_{s \in S} N_s^r(\rho)$. We now define the following continuous counterparts to the quantities $\hat{Q}, \tilde{Q}, \check{Q}$ and $\check{Q}$, respectively corresponding to the SCE0, SEE, DSCE0 and DSEE schemes:

$$\begin{aligned}
\text{SCE0} &: \quad \hat{Q}(r) = \sum_{s \in S} \left[\int_0^{+\infty} g_s(\inf\{r, z\})\, n_s^r(z)\, dz \right] \\
\text{SEE} &: \quad \tilde{Q}(q) = \int_0^{\infty} \inf\{q, z\} n(z)\, dz \\
\text{DSCE0} &: \quad \check{Q}(r) = \int_{z=0}^{\infty} \sum_{s \in S} g_s(\sup\{r, z\})\, n_s^r(z)\, dz \\
\text{DSEE} &: \quad \check{Q}(q) = \int_{z=0}^{\infty} \sup\{q, z\} n(z)\, dz
\end{aligned}$$

with $g_s(\cdot) \equiv r^{-1}(\cdot, \bar{q}_s)$ and where N and N_s denote the total number of households and the total number of type-s households, respectively.

With this notation, the expressions for x^{SCE0}, x^{SEE}, x^{DSCE0}, and x^{DSEE}

[11] Computations can be found in Appendix C

take the following forms:

$$\begin{aligned}
x^{SCE0}(\rho) &= \frac{C(\bar{Q})}{N} + \int_{z=0}^{\rho} \frac{1}{N - N^r(z)} C'\left(\hat{Q}(z)\right) \frac{d\hat{Q}(z)}{d\rho} dz \\
x^{SEE}(q,s) &= \frac{C(N \inf \mathbf{q})}{N} + \int_{z=0}^{q} C'\left(\tilde{Q}(z)\right) dz \\
&\quad + \left[u(q,\bar{q}_s) - u(q,\bar{q}_0)\right] - \frac{1}{N} \sum_{t\in S} \int_{z=0}^{\infty} \left[u(z,\bar{q}_t) - u(z,\bar{q}_0)\right] n_t(z)\, dz \\
x^{DSCE0}(\rho) &= \frac{1}{N} C\left(\check{Q}_{\sup}\right) - \int_{z=\rho}^{\sup \vec{\rho}} \frac{1}{N^r(z)} C'\left(\check{Q}(z)\right) \frac{d\check{Q}(z)}{dz} dz \\
x^{DSEE}(q,s) &= \tfrac{1}{N} \quad C(N \sup \mathbf{q}) - \int_{z=q}^{\sup \mathbf{q}} C'\left(\check{Q}(z)\right) dz \\
&\quad + \left[u(q,\bar{q}_s) - u(q,\bar{q}_0)\right] - \frac{1}{N} \sum_{t\in S} \int_{z=0}^{\infty} \left[u(z,\bar{q}_t) - u(z,\bar{q}_0)\right] n_t(z)\, dz
\end{aligned}$$

where $\check{Q}_{\sup} = \check{Q}(\sup \vec{\rho})$ with $\sup \vec{\rho}$ the largest responsibility level in the population.

6.2 Illustrative Examples

To illustrate, we now consider two specific forms for r. In the *absolute responsibility view*, $r(q,s) = q - \bar{q}_s$, whereas in the *relative responsibility view*, $r(q,s) = (q - \bar{q}_s)/\bar{q}_s$. If s indeed denotes household size, the former holds households equally responsible for consumption above needs regardless of their size. By contrast, the latter view holds larger households less responsible than smaller households for an identical consumption level above needs. In other words, needs also impact the way consumption beyond them is considered.

Decreasing Returns to Scale : Quadratic Costs

Assume that costs are given by the following quadratic function: $C(Q) = cQ^2/2$. Under absolute responsibility, the serial conditional equality rule with zero responsibility as a reference yields:

$$x^{SCE0}(q,s) = \frac{1}{N}\frac{cQ^2}{2} + cQ\left(q - \bar{q}_s - \frac{Q - \bar{Q}}{N}\right).$$

In words, users share the total cost equally and are rewarded or penalized for deviation from the average responsibility level . These deviations are valued at marginal cost.

Under relative responsibility, however, marginal consumption is not priced equally across household types. When responsibility is equally distributed across types, we obtain the following expression:

$$x^{SCE0}(q,s) = \frac{1}{N}\frac{cQ^2}{2} + cQ\frac{\bar{Q}}{N}\left(\frac{q-\bar{q}_s}{\bar{q}_s} - \frac{Q-\bar{Q}}{\bar{Q}}\right).$$

Again, x^{SCE0} charges everyone the average cost and prices deviations from the average responsibility, but this time at the marginal cost of responsibility if needs were equal to $\bar{Q}/N$. Observe that if $\bar{q}_s > \bar{Q}/N$ consumption is priced at less than the marginal cost while the consumption of households with lower-than-average needs ($\bar{q}_s < \bar{Q}/N$) is priced above marginal cost.[12]

The serial egalitarian equivalent solution takes on the following form:

$$x^{SEE}(q,s) = cQ\left(q - \frac{Q}{2N}\right) \tag{3}$$

$$+ \left[u(q,\bar{q}_s) - u(q,\bar{q}_0)\right] - \frac{1}{N}\sum_{t\in S}\int_{z=0}^{\infty}\left[u(z,\bar{q}_t) - u(z,\bar{q}_0)\right]n_t(z)\,dz. \tag{4}$$

Recall that the expression for SEE is independent of the responsibility view (e.g., absolute or relative responsibility). However, payments now depend upon the utility function. This calls for an observation. Suppose that a household's type is simply its size and that $\bar{q}_s = \tilde{q}\times s$ for some reference per-person level of needs, $\tilde{q}$. Given a consumption level, q, it seems natural for the total bill to be lower for larger households. For this to be the case, it must be that $u(q,\tilde{q}\times s)$ is decreasing in s, according to Expression (4). This implies that household utility cannot be written as a simple sum of the utility of its members, $s\times v_{\tilde{q}}(q/s)$, where $v_{\tilde{q}}$ is some increasing and concave function. Indeed, we would have:

$$\frac{d}{ds}\left[s\times v_{\tilde{q}}(q/s)\right] = v_{\tilde{q}}\left(\frac{q}{s}\right) - \frac{q}{s}v_{\tilde{q}}'\left(\frac{q}{s}\right) \geq 0,$$

by the concavity of $v_{\tilde{q}}$. Thus, one must refrain from modeling households as a sum of individual utility functions.[13]

[12] This is unlike the case of absolute responsibility above, where the marginal cost of responsibility was identical across households and equal to the marginal cost.

[13] This is reminiscent of the Repugnant Conclusion in population ethics (Blackorby et al., 2005). The latter is a consequence of the pure utilitarian criterion, which deems any population always worse off than a larger one sharing the same resources, even if the population size is

Increasing Returns to Scale: Affine Costs

Assume costs are of the form $C(Q) = F + cQ$, with $F, c \in \mathbb{R}_+$. When responsibility is measured by absolute responsibility, the decreasing serial conditional equality rule yields:

$$x^{DSCE0}(q, \bar{q}_s) = \frac{F + c\bar{Q}}{N} + c(q - \bar{q}_s).$$

In addition to splitting the fixed cost equally, DSCE0 also splits the cost of the population's needs equally before charging users at marginal cost with a rebate equal to the cost of meeting their needs.

Under the relative responsibility view, and if responsibility is identically distributed across types, we obtain:

$$x^{DSCE0}(q, \bar{q}_s) = \frac{F}{N} + c\frac{1}{\bar{q}_s / \left(\bar{Q}/N\right)} q.$$

As with absolute responsibility, DSCE0 splits the fixed cost equally. No rebate is granted, however, but consumption is priced at a rate that is inversely proportional to one's needs.

We now turn to DSEE.

$$\begin{aligned} x^{DSEE}(q, \bar{q}_s) = {} & \frac{F}{N} + cq \\ & + \left[u(q, \bar{q}_s) - u(q, \bar{q}_0)\right] - \frac{1}{N}\sum_{t \in S}\int_{z=0}^{\infty}\left[u(z, \bar{q}_t) - u(z, \bar{q}_0)\right] n_t(z)\, dz \end{aligned}$$

The cost-sharing portion of DSEE splits the fixed cost equally and prices consumption at marginal cost. Needs are completely absent from that component. However, the redistributive portion of DSEE ensures that heterogeneity in needs does not drive differences in welfare.

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such that individuals have barely enough to survive (see also Fleurbaey et al., 2014).

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A Appendix: Section 4 Proofs

A.1 Proof of Proposition 1

Consider two demand levels, $q, q' \in \mathbb{R}_+$. Assume $\bar{\mathbf{q}} \in \mathbb{R}^n_+$ is such that $\bar{q}_1 = \bar{q}_2$. Consider $\mathbf{q}, \mathbf{q}' \in \mathbb{R}^n_+$ such that $q_1 = q_2 = q$, $q'_1 = q'_2 = q'$ and $\mathbf{q}_{-12} = \mathbf{q}'_{-12}$. Lastly, define two intermediate profiles, $\mathbf{q}^1$ and $\mathbf{q}^2$ such that $\left(q^1_1, q^1_2\right) = (q', q)$, $\left(q^2_1, q^2_2\right) = (q, q')$, and $\mathbf{q}^1_{-12} = \mathbf{q}^2_{-12} = \mathbf{q}_{-12}$. By **ERSEN**, $x_1(\mathbf{q}, \bar{\mathbf{q}}) = x_2(\mathbf{q}, \bar{\mathbf{q}})$ and $x_1(\mathbf{q}', \bar{\mathbf{q}}) = x_2(\mathbf{q}', \bar{\mathbf{q}})$. Moreover, $q^1_1 = q^2_2$, thus by anonymity, $x_1\left(\mathbf{q}^1, \bar{\mathbf{q}}\right) = x_2\left(\mathbf{q}^2, \bar{\mathbf{q}}\right)$ and, with an unchanged consumption, $x_1\left(\mathbf{q}^2, \bar{\mathbf{q}}\right) = x_2\left(\mathbf{q}^1, \bar{\mathbf{q}}\right) = x_1(\mathbf{q}, \bar{\mathbf{q}}) = x_2(\mathbf{q}, \bar{\mathbf{q}})$. Hence, by budget balance,

$$\begin{aligned} x_1\left(\mathbf{q}^1, \bar{\mathbf{q}}\right) - x_1(\mathbf{q}, \bar{\mathbf{q}}) &= C(Q + q' - q) - C(Q), \text{ and} \\ x_2\left(\mathbf{q}^2, \bar{\mathbf{q}}\right) - x_2(\mathbf{q}, \bar{\mathbf{q}}) &= C(Q + q' - q) - C(Q). \end{aligned}$$

Moreover,

$$x_1(\mathbf{q}', \bar{\mathbf{q}}) - x_1(\mathbf{q}, \bar{\mathbf{q}}) = x_2(\mathbf{q}', \bar{\mathbf{q}}) - x_2(\mathbf{q}, \bar{\mathbf{q}}) = \frac{1}{2}\left[C(Q + 2(q' - q)) - C(Q)\right].$$

Because $q'_1 = q^1_1$, and $q'_2 = q^2_2$, we have $x_1(\mathbf{q}', \bar{\mathbf{q}}) = x_1\left(\mathbf{q}^1, \bar{\mathbf{q}}\right)$ and $x_2(\mathbf{q}', \bar{\mathbf{q}}) = x_2\left(\mathbf{q}^2, \bar{\mathbf{q}}\right)$. As a result, $C(Q + q' - q) - C(Q) = \frac{1}{2}\left[C(Q + 2(q' - q)) - C(Q)\right]$ that is

$$C(Q + 2(q' - q)) + C(Q) = 2C(Q + q' - q).$$

This rewrites as $f(2x) + f(0) = 2f(x)$ where $f \equiv C(Q + \cdot)$ and $x = q' - q$. This equality must hold for all x and thus defines a functional equation in f. This is a well-known Cauchy equation (Aczél, 1967), which requires f—and therefore C—to be linear in its argument.

A.2 Proof of Theorem 2

Let $r^0 \in \mathbb{R}_+$ be the reference responsibility level and $\left(\mathbf{q^0}, \bar{\mathbf{q}}\right) \in \mathcal{P}$ be such that,

$$r\left(q^0_i, \bar{q}_i\right) = r^0, \qquad \text{for all } i \in N.$$

By **UWRR**,

$$u_i\left(q^0_i, \bar{q}_i\right) - x_i\left(\mathbf{q^0}, \bar{\mathbf{q}}\right) = u_j\left(q^0_j, \bar{q}_j\right) - x_j\left(\mathbf{q^0}, \bar{\mathbf{q}}\right), \qquad \text{for all } i, j \in N.$$

Hence,

$$x_i\left(\mathbf{q^0},\bar{\mathbf{q}}\right) = u_i\left(q_i^0,\bar{q}_i\right) - \frac{1}{n}\sum_{j\in N}\left[u_j\left(q_j^0,\bar{q}_j\right) - x_j\right],$$
$$= \frac{C\left(Q^0\right)}{n} + u_i\left(q_i^0,\bar{q}_i\right) - \frac{1}{n}\sum_{j\in N}u_j\left(q_j^0,\bar{q}_j\right),$$

where $Q^0 \equiv \sum_{j\in N} q_j^0$.

Applying **SR** between profiles $\left(\mathbf{q^0},\bar{\mathbf{q}}\right)$ and $(\bar{\mathbf{q}},\bar{\mathbf{q}})$ yields:

$$x_i(\mathbf{q^0},\bar{\mathbf{q}}) - x_i(\bar{\mathbf{q}},\bar{\mathbf{q}}) \quad = \quad \xi_i\left(\mathbf{r}^0, C - C(\bar{Q})\right).$$

Hence, by symmetry of ξ,

$$x_i(\bar{\mathbf{q}},\bar{\mathbf{q}}) = x_i\left(\mathbf{q^0},\bar{\mathbf{q}}\right) - \frac{C\left(Q^0\right) - C\left(\bar{Q}\right)}{n}.$$

Applying **SR** between profiles $(\bar{\mathbf{q}},\bar{\mathbf{q}})$ and $(\mathbf{q},\bar{\mathbf{q}})$ yields:

$$x_i(\mathbf{q},\bar{\mathbf{q}}) - x_i(\bar{\mathbf{q}},\bar{\mathbf{q}}) \quad = \quad \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right).$$

Thus,

$$\begin{aligned} x_i(\mathbf{q},\bar{\mathbf{q}}) &= \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + x_i\left(\bar{\mathbf{q}},\bar{\mathbf{q}}\right) \\ &= \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + x_i\left(\mathbf{q^0},\bar{\mathbf{q}}\right) - \frac{C\left(Q^0\right) - C\left(\bar{Q}\right)}{n} \\ &= \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + \frac{C\left(\bar{Q}\right)}{n} + u_i\left(q_i^0,\bar{q}_i\right) - \frac{1}{n}\sum_{j\in N}u_j\left(q_j^0,\bar{q}_j\right). \end{aligned}$$

A.3 Proof of Proposition 1

Let $\left(\mathbf{q}^0,\bar{\mathbf{q}}\right) \in \mathcal{P}$ and $\left(\mathbf{q}^1,\bar{\mathbf{q}}\right) \in \mathcal{P}$ be two profiles associated respectively with the uniform responsibility levels r^0 and $r^1 \neq r^0$. Suppose that $x\left(\mathbf{q},\bar{\mathbf{q}}\right)$ satisfies **UWUR** so that it satisfies in particular **UWRR** for the reference responsibility level r^0. If it does also satisfy **SR,** it must be written as

$$x_i\left(\mathbf{q},\bar{\mathbf{q}}\right) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + u_i\left(q_i^0,\bar{q}_i\right) - \frac{1}{n}\sum_{j\in N}u_j\left(q_j^0,\bar{q}_j\right), \qquad \text{for all } i \in N.$$

This says in particular that when $\mathbf{q} = \mathbf{q^1}$, we have:

$$x_i\left(\mathbf{q^1}, \bar{\mathbf{q}}\right) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i\left(\mathbf{r}^1, C - C(\bar{Q})\right) + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right), \qquad \text{for all } i \in N.$$

By symmetry of ξ, we have $\xi_i\left(\mathbf{r}^1, C - C\left(\bar{Q}\right)\right) = \left[C\left(Q^1\right) - C\left(\bar{Q}\right)\right]/n$, for all $i \in N$ so that

$$x_i\left(\mathbf{q^1}, \bar{\mathbf{q}}\right) = \frac{C\left(Q^1\right)}{n} + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right), \qquad \text{for all } i \in N.$$

If $x\left(\mathbf{q}, \bar{\mathbf{q}}\right)$ satisfies **UWRR** for the reference responsibility level r^1 (to which $\mathbf{q^1}$ is associated), it must be the case that

$$u_i\left(q_i^1, \bar{q}_i\right) - x_i\left(\mathbf{q^1}, \bar{\mathbf{q}}\right) = u_j\left(q_j^1, \bar{q}_j\right) - x_j\left(\mathbf{q^1}, \bar{\mathbf{q}}\right), \qquad \text{for all } i, j \in N.$$

From the expression of the $x_i\left(\mathbf{q^1}, \bar{\mathbf{q}}\right)$ established above, we must have

$$u_i\left(q_i^1, \bar{q}_i\right) - u_i\left(q_i^0, \bar{q}_i\right) = u_j\left(q_j^1, \bar{q}_j\right) - u_j\left(q_j^0, \bar{q}_j\right), \qquad \text{for all } i, j \in N.$$

This implies in turn that

$$u_i\left(q_i^1, \bar{q}_i\right) - u_i\left(q_i^0, \bar{q}_i\right) = \frac{1}{n}\sum_{j\in N}\left[u_j\left(q_j^1, \bar{q}_j\right) - u_j\left(q_j^0, \bar{q}_j\right)\right], \qquad \text{for all } i \in N.$$

This must be true for any responsibility level r^0 and r^1 and the associated profiles $\left(\mathbf{q}^0, \bar{\mathbf{q}}\right) \in \mathcal{P}$ and $\left(\mathbf{q}^1, \bar{\mathbf{q}}\right) \in \mathcal{P}$. Thus, by setting $r^1 = 0$ and considering the associated profile $\left(\bar{\mathbf{q}}, \bar{\mathbf{q}}\right) \in \mathcal{P}$, we obtain that, for **SR** and **UWUR** to be compatible, the utility function must be such that

$$u_i\left(q_i^0, \bar{q}_i\right) = \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right) \tag{5}$$

for all $i \in N$ and for all profiles $\left(\mathbf{q}^0, \bar{\mathbf{q}}\right) \in \mathcal{P}$ such that

$$r\left(q_i^0, \bar{q}_i\right) = r^0, \qquad \text{for all } i \in N.$$

Fix r^0 and $\bar{\mathbf{q}}$ and define, for all $i \in N$, $q\left(r^0, \bar{q}_i\right) = \left\{q \in \mathbb{R}_+ | r\left(q, \bar{q}_i\right) = r^0\right\}$. By continuity and strict monotonicity of r, $q\left(r^0, \bar{q}_i\right)$ is a singleton and $\left(r^0, \bar{q}_i\right) \mapsto q\left(r^0, \bar{q}_i\right)$ defines a continuous function that is increasing in its first argument.

Also, define $u^0 = u_1\left(q\left(r^0, \bar{q}_1\right), \bar{q}_1\right)$. It follows from Expression (5) that we must have $u_i\left(q_i^0, \bar{q}_i\right) = u^0$ for any i and any $\left(q_i^0, \bar{q}_i\right)$ such that $r\left(q_i^0, \bar{q}_i\right) = r^0$ or, equivalently, that $u_i\left(q\left(r^0, \bar{q}_i\right), \bar{q}_i\right) = u^0$ for all i and all $\bar{q}_i$. Because u^0 depends neither upon i, nor upon$\bar{q}_i$, it must be that $\left(\bar{q}_i, r^0\right) \mapsto u_i\left(q\left(r^0, \bar{q}_i\right), \bar{q}_i\right)$ is a function of r^0 only. Therefore, for all r^0, all i and all $\bar{q}_i$,

$$u_i\left(q\left(r^0, \bar{q}_i\right), \bar{q}_i\right) = v\left(r^0\right)$$

for some function v on $\mathbb{R}$. Because u_i and q are both continuous and increasing in their first argument, v is also a continuous increasing function.

Finally, let $(q_i, \bar{q}_i) \in \mathbb{D}$, evaluating the above expression at $r^0 = r(q_i, \bar{q}_i)$, and noticing that

$$q\left(r\left(q_i, \bar{q}_i\right), \bar{q}_i\right) = q_i$$

yields:

$$u_i\left(q_i, \bar{q}_i\right) = v\left(r\left(q_i, \bar{q}_i\right)\right).$$

This in turn implies that the utility must be a transformation of the responsibility function:

$$u_i = u \equiv v \circ r.$$

Because v is a continuous and increasing function of $\mathbb{R}$, we can write:

$$r = \rho \circ u,$$

with $\rho = v^{-1}$, so that r is a transformation of the common utility function$u$.

A.4 Proof of Theorem 3

Only if. Let x satisfy **EWER** and **SR**. Because **EWER** is more demanding than **UWUR**, x must also satisfy **UWUR** . By Proposition 1, this can only occur if $u_i = u$ for some utility function u and $r = \rho \circ u$ for some continuous and increasing function ρ. Because **UWUR** is more demanding than **UWRR**, x must also satisfy **UWRR**. By Theorem 2, x must be a Conditional Equivalent solution:

$$x_i^{CE}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + u\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j \in N} u\left(q_j^0, \bar{q}_j\right),$$

where u is the common utility function and $\mathbf{q^0}$ is such that, for all $i \in N$, $r\left(q_i^0, \bar{q}_i\right) = r^0$ for some reference responsibility level, r^0. Moreover, it follows from $r = \rho \circ u$ that $u\left(q_i^0, \bar{q}_i\right) = \rho^{-1}\left(r^0\right)$ for all $i \in N$. Hence,

$$x_i^{CE}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right), \qquad \text{for all } i \in N.$$

If. Let x be defined as in the statement of the Theorem. Let $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$ such that $r\left(q_i, \bar{q}_i\right) = r\left(q_j, \bar{q}_j\right)$for some $i, j \in N$. It follows from the symmetry of ξ that

$$\xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) = \xi_j\left(\mathbf{r}, C - C(\bar{Q})\right).$$

As a result,

$$x_i^{CE}(\mathbf{q}, \bar{\mathbf{q}}) = x_j^{CE}(\mathbf{q}, \bar{\mathbf{q}}).$$

Moreover, because $r = \rho \circ u$ for some continuous and increasing function ρ, we can write $u = \rho^{-1} \circ r$. Thus,

$$r\left(q_i, \bar{q}_i\right) = r\left(q_j, \bar{q}_j\right) \implies u\left(q_i, \bar{q}_i\right) = u\left(q_j, \bar{q}_j\right),$$

and $u_i = u_j = u$ yields

$$u_i\left(q_i, \bar{q}_i\right) - x_i^{CE}(\mathbf{q}, \bar{\mathbf{q}}) = u_j\left(q_j, \bar{q}_j\right) - x_j^{CE}(\mathbf{q}, \bar{\mathbf{q}}).$$

A.5 Proof of Theorem 4

Let $\bar{q}_0 \in \mathbb{R}_+$ be a reference level of needs and denote by $\bar{\mathbf{q}}_0 = (\bar{q}_0, \bar{q}_0, ..., \bar{q}_0) \in \mathbb{R}_+^n$ the associated reference vector. Consider the profile $(\mathbf{q}; \bar{\mathbf{q}}_0)$. By budget balance and equal treatment of equals

$$x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) = \frac{C(n\bar{q}_0)}{n}.$$

By **SRRN**,

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}_0\right) - x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) = \xi_i\left(\mathbf{r}_0, C - C\left(n\bar{q}_0\right)\right) \qquad \text{for all } i \in N,$$

where $r_{0,i} = r\left(q_i, \bar{q}_0\right)$ for all i.

Define $\bar{\mathbf{q}}_0^1 = (\bar{q}_1, \bar{q}_0, ..., \bar{q}_0)$. Applying **GS** between $(\mathbf{q}, \bar{\mathbf{q}}_0)$ and $\left(\mathbf{q}, \bar{\mathbf{q}}_0^1\right)$ yields, for all $j \neq 1$:

$$\begin{aligned} & u_1(q_1, \bar{q}_1) - x_1^1 - u_1(q_1, \bar{q}_0) + x_1^0 \\ = \; & u_j(q_j, \bar{q}_0) - x_j^1 - u_j(q_j, \bar{q}_0) + x_j^0 \end{aligned}$$

where $x_j^0 = x_j(\mathbf{q}, \bar{\mathbf{q}}_0)$ and $x_j^1 = x_j\left(\mathbf{q}, \bar{\mathbf{q}}_0^1\right)$ for all $j \in N$. This yields

$$x_j^0 - x_j^1 = u_1(q_1, \bar{q}_1) - u_1(q_1, \bar{q}_0) + x_1^0 - x_1^1,$$

hence, by budget balance

$$\begin{aligned} x_1^1 - x_1^0 &= \frac{n-1}{n}\left[u_1(q_1, \bar{q}_1) - u_1(q_1, \bar{q}_0)\right] \\ x_j^1 - x_j^0 &= -\frac{1}{n}\left[u_1(q_1, \bar{q}_1) - u_1(q_1, \bar{q}_0)\right] \end{aligned}$$

all $j \neq 1$. Applying **GS** to profiles $\left(\mathbf{q}, \bar{\mathbf{q}}_0^k\right)$ where $\bar{\mathbf{q}}_0^k = (\bar{q}_1, \bar{q}_2, ..., \bar{q}_k, \bar{q}_0, ..., \bar{q}_0)$, successively leads to the following expression, for all iterations, $k = 1, ..., n$, and all agents $1 \leq i \leq k \leq j \leq n$:

$$\begin{aligned} & u_i(q_i, \bar{q}_i) - x_i^k - u_i(q_i, \bar{q}_i) + x_i^{k-1} \\ = \; & u_k(q_k, \bar{q}_k) - x_k^k - u_k(q_k, \bar{q}_0) + x_k^{k-1} \\ = \; & u_j(q_j, \bar{q}_0) - x_j^k - u_j(q_j, \bar{q}_0) + x_j^{k-1} \end{aligned}$$

Hence, for all $k = 1, ..., n$, and all agents $1 \leq i \leq k \leq j \leq n$:

$$\begin{aligned} & x_i^{k-1} - x_i^k \\ = \; & u_k(q_k; \bar{q}_k) - u_k(q_k; \bar{q}_0) + x_k^{k-1} - x_k^k \\ = \; & x_j^{k-1} - x_j^k \end{aligned}$$

By budget balance, $\sum_j x_j^k - x_j^{k-1} = 0$, yielding

$$\begin{aligned} x_k^k - x_k^{k-1} &= \frac{n-1}{n}\left[u_k(q_k; \bar{q}_k) - u_k(q_k; \bar{q}_0)\right] \\ x_j^k - x_j^{k-1} &= -\frac{1}{n}\left[u_k(q_k; \bar{q}_k) - u_k(q_k; \bar{q}_0)\right] \qquad \text{for all } j \neq k \end{aligned}$$

Summing up over all iterations k yields the following:

$$\begin{aligned}
x_1^n - x_1^0 &= \sum_{k>1}^{n} \left(x_1^k - x_1^{k-1} \right) + x_1^1 - x_1^0 \\
&= -\frac{1}{n} \sum_{k>1}^{n} \left[u_k \left(q_k ; \bar{q}_k \right) - u_k \left(q_k ; \bar{q}_0 \right) \right] + \left(1 - \frac{1}{n} \right) \left[u_1 \left(q_1 ; \bar{q}_1 \right) - u_1 \left(q_1 ; \bar{q}_0 \right) \right] \\
&= \left[u_1 \left(q_1 ; \bar{q}_1 \right) - u_1 \left(q_1 ; \bar{q}_0 \right) \right] - \frac{1}{n} \sum_{k=1}^{n} \left[u_k \left(q_k ; \bar{q}_k \right) - u_k \left(q_k ; \bar{q}_0 \right) \right]
\end{aligned}$$

Likewise, for all $i \in N$:

$$x_i^n - x_i^0 = \left[u_i \left(q_i ; \bar{q}_i \right) - u_i \left(q_i ; \bar{q}_0 \right) \right] - \frac{1}{n} \sum_{k=1}^{n} \left[u_k \left(q_k ; \bar{q}_k \right) - u_k \left(q_k ; \bar{q}_0 \right) \right]$$

Finally, upon noticing that $x_i^n = x\left(\mathbf{q}, \bar{\mathbf{q}}\right)$,

$$\begin{aligned}
x_i \left(\mathbf{q} ; \bar{\mathbf{q}} \right) &= \xi_i \left(\mathbf{r}_0, C - C \left(n\bar{q}_0 \right) \right) + x_i \left(\mathbf{q}, \bar{\mathbf{q}}_0 \right) \\
&\quad + \left[u_i \left(q_i ; \bar{q}_i \right) - u_i \left(q_i ; \bar{q}_0 \right) \right] - \frac{1}{n} \sum_{k=1}^{n} \left[u_k \left(q_k ; \bar{q}_k \right) - u_k \left(q_k ; \bar{q}_0 \right) \right].
\end{aligned}$$

A.6 Proof of tightness of the characterization of EE by SRRN and GS

Consider a profile $\left(\mathbf{q}, \bar{\mathbf{q}}_{\mathbf{1}}\right)$ such that $\bar{\mathbf{q}}_{\mathbf{1}} = \left(\bar{q}_1, \bar{q}_1, ..., \bar{q}_1\right)$ with $\bar{q}_1 \neq \bar{q}_0$ then:

$$\begin{aligned}
x_i^{EE} \left(\mathbf{q}, \bar{\mathbf{q}}_{\mathbf{1}} \right) - x_i^{EE} \left(\bar{\mathbf{q}}_{\mathbf{1}}, \bar{\mathbf{q}}_{\mathbf{1}} \right) &= \frac{C(n\bar{q}_0)}{n} + \xi_i \left(\mathbf{r}_0, C - C \left(n\bar{q}_0 \right) \right) \\
&\quad + \left[u_i \left(q_i, \bar{q}_1 \right) - u_i \left(q_i ; \bar{q}_0 \right) \right] - \frac{1}{n} \sum_{k=1}^{n} \left[u_k \left(q_k, \bar{q}_1 \right) - u_k \left(q_k, \bar{q}_0 \right) \right] \\
&\quad - \left(\frac{C(n\bar{q}_0)}{n} + \xi_i \left(\bar{\mathbf{r}}_0, C - C \left(n\bar{q}_0 \right) \right) \right. \\
&\quad \left. + \left[u_i \left(\bar{q}_1, \bar{q}_1 \right) - u_i \left(\bar{q}_1, \bar{q}_0 \right) \right] - \frac{1}{n} \sum_{k=1}^{n} \left[u_k \left(\bar{q}_1, \bar{q}_1 \right) - u_k \left(\bar{q}_1, \bar{q}_0 \right) \right] \right)
\end{aligned}$$

where $\bar{\mathbf{r}}_0 \equiv r(\bar{q}_1, \bar{q}_0)$. Hence,

$$\begin{aligned} x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}_1) - x_i^{EE}(\bar{\mathbf{q}}_1, \bar{\mathbf{q}}_1) &= \xi_i(\mathbf{r}_0, C - C(n\bar{q}_0)) + [u_i(q_i, \bar{q}_1) - u_i(q_i, \bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n}[u_k(q_k, \bar{q}_1) - u_k(q_k, \\ &\quad - \left(\xi_i(\bar{\mathbf{r}}_0, C - C(n\bar{q}_0)) + [u_i(\bar{q}_1, \bar{q}_1) - u_i(\bar{q}_1, \bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n}[u_k(\bar{q}_1, \bar{q}_1) - u_k \right. \end{aligned}$$

which simplifies into:

$$\begin{aligned} x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}_1) - x_i^{EE}(\bar{\mathbf{q}}_1, \bar{\mathbf{q}}_1) &= \xi_i(\mathbf{r}_0, C - C(n\bar{q}_0)) - \frac{1}{n}(C(n\bar{q}_1) - C(n\bar{q}_0)) \\ &\quad + (u_i(q_i, \bar{q}_1) - u_i(\bar{q}_1, \bar{q}_1) - [u_i(q_i, \bar{q}_0) - u_i(\bar{q}_1, \bar{q}_0)]) \\ &\quad - \frac{1}{n}\sum_{k=1}^{n}(u_k(q_k, \bar{q}_1) - u_k(\bar{q}_1, \bar{q}_1) - [u_k(q_k, \bar{q}_0) - u_k(\bar{q}_1, \bar{q}_0)]) \end{aligned}$$

The above expression reveals that $x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}_1) - x_i^{EE}(\bar{\mathbf{q}}_1, \bar{\mathbf{q}}_1)$ depends on u_i, hence cannot be driven only by the cost sharing function ξ. In other words, it cannot be the case that:

$$x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}_1) - x_i^{EE}(\bar{\mathbf{q}}_1, \bar{\mathbf{q}}_1) = \xi_i(\mathbf{r}_1, C - C(n\bar{q}_1)),$$

as required by **SRUN**.

B Section 5 Proofs

B.1 Proof of Proposition 3

Let $\mathbf{q} = \bar{\mathbf{q}}$. By **UWMC**,

$$\begin{aligned} x_i(\bar{\mathbf{q}}, \bar{\mathbf{q}}) &= x_j(\bar{\mathbf{q}}, \bar{\mathbf{q}}) \quad \text{for all } i, j \in N \\ &\Longrightarrow x_i(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = \frac{C(\bar{Q})}{n} \quad \text{for all } i \in N \end{aligned}$$

Without any loss of generality, assume that $r_1 \leq r_2 \leq ... \leq r_n$. Let $f_i : w \mapsto r(w, \bar{q}_i)$ map consumption to individual responsibility for agent i. By construction, f_i is monotonic and strictly increasing. Its inverse, $g_i : v \mapsto f_i^{-1}(v)$ is well defined and is also monotonic and strictly increasing. Note that $g_i(r_i) = q_i$ for all $i \in N$.

Define the following profile:

$$\mathbf{q}^1 = \left(q_1, g_2\left(r_1\right), ..., g_i\left(r_1\right), ..., g_n\left(r_1\right)\right).$$

Note that, by construction $\left(\mathbf{q}^1, \bar{\mathbf{q}}\right)$ is such that

$$r_i^1 = r_1,$$

for all $i \in N$.

Applying **SR** with profile $\left(\mathbf{q}^1; \bar{\mathbf{q}}\right)$ yields:

$$x_i(\mathbf{q}^1; \bar{\mathbf{q}}) - x_i(\bar{\mathbf{q}}; \bar{\mathbf{q}}) = \xi_i(\mathbf{r}^1, C - C(\bar{Q})),$$

where $\xi_i(\mathbf{r}, C - C(\bar{Q}))$ is symmetric in $\mathbf{r}$. Since all r_i^1 are identical, we have

$$\xi_i(\mathbf{r}^1, C - C(\bar{Q})) = \frac{1}{n}\left[C\left(Q^1\right) - C(\bar{Q})\right],$$

where

$$Q^1 = \sum_{i=1}^{n} q_i^1 = \sum_{i=1}^{n} g_i\left(r_1\right).$$

Similarly, let

$$\mathbf{q}^2 = \left(q_1, q_2, g_3\left(r_2\right), ..., g_i\left(r_2\right), ..., g_n\left(r_2\right)\right).$$

Again by construction $\left(\mathbf{q}^2, \bar{\mathbf{q}}\right)$ is such that

$$r_i^2 = r_2,$$

for all $i = 2, ..., n$.

Applying now **SR** with profile $\left(\mathbf{q}^2; \bar{\mathbf{q}}\right)$ yields:

$$x_i(\mathbf{q}^2, \bar{\mathbf{q}}) - x_i(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = \xi_i(\mathbf{r}^2, C - C(\bar{Q})),$$

where $\xi_i(\mathbf{r}, C - C(\bar{Q}))$ is symmetric in $\mathbf{r}$, therefore

$$\xi_i(\mathbf{r}^2, C - C(\bar{Q})) = \xi_j(\mathbf{r}^2, C - C(\bar{Q}))$$

for all $i, j \geq 2$. In words, agents $2, ..., n$ are assigned the same cost share.

Moreover, by assumption $r_1 \leq r_2$. Applying **IHR** between profiles $\left(\mathbf{q}^1, \bar{\mathbf{q}}\right)$ and

$\left(\mathbf{q}^2, \bar{\mathbf{q}}\right)$ yields that agent 1's contribution is the same under both profiles:

$$\xi_1(\mathbf{r}^1; C - C(\bar{Q})) = \xi_1(\mathbf{r}^2; C - C(\bar{Q})) = \frac{1}{n}\left[C\left(Q^1\right) - C(\bar{Q})\right],$$

Thus, agents $2, ..., n$ share the remaining cost equally:

$$\begin{aligned} \xi_i(\mathbf{r}^2; C - C(\bar{Q})) &= \frac{1}{n-1}\left[C\left(Q^2\right) - C\left(\bar{Q}\right) - \frac{1}{n}\left[C\left(Q^1\right) - C(\bar{Q})\right]\right] \\ &= \frac{1}{n-1}\left[C\left(Q^2\right) - C\left(Q^1\right)\right] + \frac{1}{n}\left[C\left(Q^1\right) - C(\bar{Q})\right] \end{aligned}$$

for all $i \geq 2$, where

$$Q^2 = \sum_{i=1}^{n} q_i^2 = q_1 + \sum_{i=2}^{n} g_i\left(r_2\right) \geq Q^1.$$

Alternatively,

$$\xi_i(\mathbf{r}^2; C - C(\bar{Q})) - \xi_i(\mathbf{r}^1; C - C(\bar{Q})) = \frac{1}{n-1}\left[C\left(Q^2\right) - C\left(Q^1\right)\right]$$

all $i \geq 2$.

Proof. Similarly, for all $k \geq 2$, we define

$$\mathbf{q}^k = \left(q_1, q_2, ..., q_k, g_{k+1}\left(r_k\right), ..., g_n\left(r_k\right)\right)$$

and obtain by **SR** that

$$x_i(\mathbf{q}^k; \bar{\mathbf{q}}) - x_i(\bar{\mathbf{q}}; \bar{\mathbf{q}}) = \xi_i(\mathbf{r}^k, C - C(\bar{Q})),$$

for all $i \in N$. It follows that

$$\begin{aligned} \xi_i(\mathbf{r}^k; C - C(\bar{Q})) - \xi_i(\mathbf{r}^{k-1}; C - C(\bar{Q})) &= 0 \qquad \text{for all } i < k, \text{ and} \\ \xi_i(\mathbf{r}^k; C - C(\bar{Q})) - \xi_i(\mathbf{r}^{k-1}; C - C(\bar{Q})) &= \frac{1}{n-k+1}\left[C\left(Q^k\right) - C\left(Q^{k-1}\right)\right] \qquad \text{for all } i \geq k, \end{aligned}$$

with

$$Q^k = \sum_{i=1}^{n} q_i^k = \sum_{i=1}^{k-1} q_i + \sum_{i=k}^{n} g_i\left(r_k\right).$$

Observe that

$$Q^{k+1} - Q^k = \sum_{i=k+1}^{n} \left[g_i \left(r_{k+1} \right) - g_i \left(r_k \right) \right] \geq 0$$

by monotonicity of the g_i's. It follows that $x_{i+1}\left(\mathbf{q}^k, \bar{\mathbf{q}}\right) \geq x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}\right)$, all $i \in N$. It follows from our initial ordering of the agents that agents with a higher r_i pay a higher bill for all k.

To sum up, upon observing that $\mathbf{r}^n = \mathbf{r}$ (as associated to $(\mathbf{q}, \bar{\mathbf{q}})$), we obtain

$$x_k\left(\mathbf{q}, \bar{\mathbf{q}}\right) - x_k\left(\bar{\mathbf{q}}, \bar{\mathbf{q}}\right) = \sum_{i=1}^{k} \frac{1}{n-i+1} \left[C\left(Q^i\right) - C\left(Q^{i-1}\right) \right]$$

where $Q^0 = \bar{Q}$. Finally,

$$\begin{aligned}
x_k\left(\mathbf{q}, \bar{\mathbf{q}}\right) &= \frac{1}{n} C\left(\bar{Q}\right) + \sum_{i=1}^{k} \frac{1}{n-i+1} \left[C\left(Q^i\right) - C\left(Q^{i-1}\right) \right] \\
&= \left[\frac{1}{n} - \frac{1}{n} \right] C\left(Q^0\right) + \left[\frac{1}{n} - \frac{1}{n-1} \right] C\left(Q^1\right) + \left[\frac{1}{n-1} - \frac{1}{n-2} \right] C\left(Q^2\right) \\
&\quad + ... + \left[\frac{1}{n-i+1} - \frac{1}{n-i} \right] C\left(Q^i\right) + ... + \frac{1}{n-k+1} C\left(Q^k\right) \\
x_k\left(\mathbf{q}, \bar{\mathbf{q}}\right) &= \frac{C\left(Q^k\right)}{(n-k+1)} - \sum_{i=1}^{k-1} \frac{C\left(Q^i\right)}{(n-i)(n-i+1)} = \frac{C\left(Q^k\right)}{n-k} - \sum_{i=1}^{k} \frac{C\left(Q^i\right)}{(n-i)(n-i+1)}
\end{aligned}$$

with

$$Q^k = \sum_{i=1}^{k-1} q_i + \sum_{i=k}^{n} g_i\left(r_k\right).$$

□

B.2 Proof of Proposition 3

Let $\bar{q}_0 \in \mathbb{R}_+$ be a reference level of needs and denote by $\bar{\mathbf{q}}_0 = (\bar{q}_0, \bar{q}_0, ..., \bar{q}_0) \in \mathbb{R}^n_+$ the associated reference vector. Consider the profile $(\mathbf{q}, \bar{\mathbf{q}}_0)$. By budget balance and Equal Treatment of Equals,

$$x_i\left(\bar{\mathbf{q}}_0; \bar{\mathbf{q}}_0\right) = \frac{C(n\bar{q}_0)}{n}.$$

Without loss of generality, assume that $q_1 \leq q_2 \leq ... \leq q_n$, so that $r_{0,1} \leq r_{0,2} \leq ... \leq r_{0,n}$, where $r_{0,i} = r\left(q_i, \bar{q}_0\right)$ for all $i \in N$.

Define

$$\mathbf{q}^k = (q_1, q_2, ..., q_{k-1}, q_k, ..., q_k)$$

for all $k = 1...n$.
Notice that $\mathbf{q}^1 = (q_1, q_1, ..., q_1)$; hence by Equal Treatment of Equals, $x_i\left(\mathbf{q}^1; \bar{\mathbf{q}}_0\right) = C\left(nq_1\right)/n$ so that

$$x_i\left(\mathbf{q}^1; \bar{\mathbf{q}}_0\right) - x_i\left(\bar{\mathbf{q}}_0; \bar{\mathbf{q}}_0\right) = \frac{1}{n}\left[C\left(nq_1\right) - C\left(n\bar{q}_0\right)\right]$$

for all $i \in N$, so that

$$x_i\left(\mathbf{q}^1; \bar{\mathbf{q}}_0\right) = \frac{C\left(nq_1\right)}{n}$$

Similarly, for $k \geq 2$, **ERRN** yields

$$x_i\left(\mathbf{q}^k; \bar{\mathbf{q}}_0\right) - x_i\left(\bar{\mathbf{q}}_0; \bar{\mathbf{q}}_0\right) = \xi_i\left(\mathbf{r}_0^k, C - C\left(n\bar{q}_0\right)\right)$$

and

$$x_i\left(\mathbf{q}^{k-1}; \bar{\mathbf{q}}_0\right) - x_i\left(\bar{\mathbf{q}}_0; \bar{\mathbf{q}}_0\right) = \xi_i\left(\mathbf{r}_0^{k-1}, C - C\left(n\bar{q}_0\right)\right)$$

with $r_{0,i}^k = r\left(q_i^k, \bar{q}_0\right)$ and $r_{0,i}^{k-1} = r\left(q_i^{k-1}, \bar{q}_0\right)$. Therefore, by subtraction,

$$x_i\left(\mathbf{q}^k; \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}; \bar{\mathbf{q}}_0\right) = \xi_i\left(\mathbf{r}_0^k, C - C\left(n\bar{q}_0\right)\right) - \xi_i\left(\mathbf{r}_0^{k-1}, C - C\left(n\bar{q}_0\right)\right)$$

for all $i \in N$ and summing up over all agents, we find

$$\sum_{i=1}^{n}\left[x_i\left(\mathbf{q}^k; \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}; \bar{\mathbf{q}}_0\right)\right] = C\left(Q^k\right) - C\left(Q^{k-1}\right),$$

where $Q^{k-1} = \sum_{l=1}^{n} q_l^{k-1} = \sum_{l=1}^{k-1} q_l + (n-k+1)\, q_{k-1}$ and $Q^k = \sum_{l=1}^{n} q_l^k = \sum_{l=1}^{k} q_l + (n-k)\, q_k$.
Observe that if $i < j$ then $r_{0,i}^{k-1} \leq r_{0,j}^{k-1}$ and $r_{0,i}^k \leq r_{0,j}^k$. Moreover for all $1 \leq i \leq k-1$, $\mathbf{q}_i^{k-1} = \mathbf{q}_i^k = q_i$, and $\mathbf{r}_{0,i}^{k-1} = \mathbf{r}_{0,i}^k = r\left(q_i, \bar{q}_0\right)$. Therefore, by **IHR**,

$$x_i\left(\mathbf{q}^k; \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}; \bar{\mathbf{q}}_0\right) = 0,$$

for all $1 \leq i \leq k-1$. It follows that the previous summation can truncated from below:

$$\sum_{i=k}^{n}\left[x_i\left(\mathbf{q}^k; \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}; \bar{\mathbf{q}}_0\right)\right] = C\left(Q^k\right) - C\left(Q^{k-1}\right).$$

Moreover, for all $i,j \geq k$, we have $q_i^{k-1} = q_j^{k-1} = q_{k-1}$ and $q_i^k = q_j^k = q_k$. Therefore, by Equal Treatment of Equals,

$$x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) = x_j\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right)$$

all $i,j \geq k$, and

$$x_i\left(\mathbf{q}^{k}, \bar{\mathbf{q}}_0\right) = x_j\left(\mathbf{q}^{k}, \bar{\mathbf{q}}_0\right).$$

Hence,

$$x_i\left(\mathbf{q}^{k}, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) = \frac{1}{n-k+1}\left[C\left(Q^k\right) - C\left(Q^{k-1}\right)\right]$$

all $i \geq k$, with the convention that $Q^0 = n\bar{q}_0$.

Finally, upon observing that $\mathbf{q}^n = \mathbf{q}$, it follows by summation that

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}_0\right) - x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) = \sum_{k=1}^{i} \frac{1}{n-k+1}\left[C\left(Q^k\right) - C\left(Q^{k-1}\right)\right].$$

Define $\bar{\mathbf{q}}_0^1 = (\bar{q}_1, \bar{q}_0, ..., \bar{q}_0)$. Applying **GS** between $(\mathbf{q}, \bar{\mathbf{q}}_0)$ and $\left(\mathbf{q}, \bar{\mathbf{q}}_0^1\right)$ yields, for all $j \neq 1$:

$$\begin{aligned} & u\left(q_1, \bar{q}_1\right) - x_1^1 - u\left(q_1, \bar{q}_0\right) + x_1^0 \\ = \quad & u\left(q_j, \bar{q}_0\right) - x_j^1 - u\left(q_j, \bar{q}_0\right) + x_j^0 \end{aligned}$$

where $x_j^0 = x_j\left(\mathbf{q}, \bar{\mathbf{q}}_0\right)$ and $x_j^1 = x_j\left(\mathbf{q}, \bar{\mathbf{q}}_0^1\right)$ for all $j \in N$. This yields

$$x_j^0 - x_j^1 = u\left(q_1, \bar{q}_1\right) - u\left(q_1, \bar{q}_0\right) + x_1^0 - x_1^1.$$

Since total consumption is unchanged, we have, by budget balance

$$\begin{aligned} x_1^1 - x_1^0 &= \frac{n-1}{n}\left[u\left(q_1, \bar{q}_1\right) - u\left(q_1, \bar{q}_0\right)\right] \\ x_j^1 - x_j^0 &= -\frac{1}{n}\left[u\left(q_1, \bar{q}_1\right) - u\left(q_1, \bar{q}_0\right)\right] \end{aligned}$$

all $j \neq 1$.

Iterating and applying **GS** to profiles $\left(\mathbf{q}, \bar{\mathbf{q}}_0^k\right)$ where $\bar{\mathbf{q}}_0^k = (\bar{q}_1, \bar{q}_2, ..., \bar{q}_k, \bar{q}_0, ..., \bar{q}_0)$, successively leads to the following expression, for all iterations, $k = 1, ..., n$, and all agents $1 \leq i \leq k \leq j \leq n$:

$$\begin{aligned} & u(q_i, \bar{q}_i) - x_i^k - u(q_i, \bar{q}_i) + x_i^{k-1} \\ = \; & u(q_k, \bar{q}_k) - x_k^k - u(q_k, \bar{q}_0) + x_k^{k-1} \\ = \; & u(q_j, \bar{q}_0) - x_j^k - u(q_j, \bar{q}_0) + x_j^{k-1} \end{aligned}$$

Hence, for all $k = 1, ..., n$, and all agents $1 \leq i \leq k \leq j \leq n$:

$$\begin{aligned} & x_i^{k-1} - x_i^k \\ = \; & u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0) + x_k^{k-1} - x_k^k \\ = \; & x_j^{k-1} - x_j^k \end{aligned}$$

Since total consumption does not change from $\left(\mathbf{q}, \bar{\mathbf{q}}_0^{k-1}\right)$ to $\left(\mathbf{q}, \bar{\mathbf{q}}_0^{k}\right)$, but only needs, budget balance implies $\sum_j \left(x_j^k - x_j^{k-1}\right) = 0$. Therefore,

$$\begin{aligned} x_j^k - x_j^{k-1} &= -\frac{1}{n} \left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)\right] \qquad \text{for all } j \neq k \\ x_k^k - x_k^{k-1} &= \frac{n-1}{n} \left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)\right] \end{aligned}$$

Summing up over all iterations $k = 1, ..., n$ yields the following for agent 1:

$$\begin{aligned} x_1^n - x_1^0 &= \sum_{k>1}^{n} \left(x_1^k - x_1^{k-1}\right) + x_1^1 - x_1^0 \\ &= -\frac{1}{n} \sum_{k>1}^{n} \left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)\right] + \frac{n-1}{n} \left[u(q_1, \bar{q}_1) - u(q_1, \bar{q}_0)\right] \\ &= \left[u(q_1, \bar{q}_1) - u(q_1, \bar{q}_0)\right] - \frac{1}{n} \sum_{k=1}^{n} \left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)\right] \end{aligned}$$

Similarly, for all $i > 1$:

$$x_i^n - x_i^0 = \left[u(q_i, \bar{q}_i) - u(q_i, \bar{q}_0)\right] - \frac{1}{n} \sum_{k=1}^{n} \left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)\right]$$

Finally, observing that $\bar{\mathbf{q}}_0^n = \bar{\mathbf{q}}$ yields the following:

$$\begin{aligned} x_i(\mathbf{q}, \bar{\mathbf{q}}) &= x_i(\mathbf{q}, \bar{\mathbf{q}}_0^n) = \frac{C(n\bar{q}_0)}{n} + \sum_{k=1}^{i} \frac{1}{n-k+1}\left[C\left(Q^k\right) - C\left(Q^{k-1}\right)\right] \\ &\quad + \left[u(q_i, \bar{q}_i) - u(q_i, \bar{q}_0)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)\right] \\ &= \frac{C(n \inf q_j)}{n} + \sum_{k=1}^{i-1} \frac{1}{n-k}\left[C\left(Q^{k+1}\right) - C\left(Q^k\right)\right] \\ &\quad + \left[u(q_i, \bar{q}_i) - u(q_i, \bar{q}_0)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)\right] \end{aligned}$$

Alternatively,

$$\begin{aligned} x_i(\mathbf{q}, \bar{\mathbf{q}}) &= \frac{C\left(Q^i\right)}{n-i+1} - \sum_{k=1}^{i-1} \frac{C\left(Q^k\right)}{(n-k)(n-k+1)} \\ &\quad + \left[u(q_i, \bar{q}_i) - u(q_i, \bar{q}_0)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)\right] \end{aligned}$$

where $Q^k = \sum_{l=1}^{k} q_l + (n-k)\, q_k$ for all $k = 1, ..., n$.

B.3 Proof of Proposition 4

Let $\mathbf{q} = \bar{\mathbf{q}}$, By **UWMC**,

$$\begin{aligned} x_i(\bar{\mathbf{q}}; \bar{\mathbf{q}}) &= x_j(\bar{\mathbf{q}}; \bar{\mathbf{q}}) \qquad \text{for all } i, j \in N \\ &\Longrightarrow \quad x_i(\bar{\mathbf{q}}; \bar{\mathbf{q}}) = \frac{C\left(\bar{Q}\right)}{n} \end{aligned}$$

Without any loss of generality, assume that $r_1 \leq r_2 \leq ... \leq r_n$. Let $f_i : w \mapsto r(w, \bar{q}_i)$ map consumption to individual responsibility for agent i. By construction, f_i is monotonic and strictly increasing. Its inverse, $g_i : v \mapsto f_i^{-1}(v)$ is well defined and is also monotonic and strictly increasing. Note that $g_i(r_i) = q_i$ for all $i \in N$.

Define the following profile:

$$\mathbf{q}^n = (g_1(r_n), ..., g_i(r_n), ..., g_{n-1}(r_n), q_n).$$

Note that, by construction $(\mathbf{q}^n, \bar{\mathbf{q}})$ is such that

$$r_i^n = r_n,$$

for all $i \in N$.

Applying **SR** with profile $(\mathbf{q}^n; \bar{\mathbf{q}})$ yields:

$$x_i(\mathbf{q}^n; \bar{\mathbf{q}}) - x_i(\bar{\mathbf{q}}; \bar{\mathbf{q}}) = \xi_i(\mathbf{r}^n, C - C(\bar{Q})),$$

where $\xi_i(\mathbf{r}, C - C(\bar{Q}))$ is symmetric in $\mathbf{r}$. Since all r_i^n are identical, we have

$$\xi_i(\mathbf{r}^n, C - C(\bar{Q})) = \frac{1}{n}\left[C\left(Q^n\right) - C(\bar{Q})\right],$$

where

$$Q^n = \sum_{i=1}^{n} q_i^n = \sum_{i=1}^{n} g_i\left(r_n\right).$$

This gives

$$x_i(\mathbf{q}^n; \bar{\mathbf{q}}) = \frac{1}{n} C\left(Q^n\right)$$

Similarly, let

$$\mathbf{q}^{n-1} = \left(g_1\left(r_{n-1}\right), ..., g_i\left(r_{n-1}\right), ..., g_{n-2}\left(r_{n-1}\right), q_{n-1}, q_n\right).$$

Again by construction $\left(\mathbf{q}^{n-1}, \bar{\mathbf{q}}\right)$ is such that

$$r_i^{n-1} = r_{n-1},$$

for all $i = 1, ..., n-1$.

Applying now **SR** with profile $\left(\mathbf{q}^{n-1}; \bar{\mathbf{q}}\right)$ yields:

$$x_i(\mathbf{q}^{n-1}, \bar{\mathbf{q}}) - x_i(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = \xi_i(\mathbf{r}^{n-1}, C - C(\bar{Q})),$$

where $\xi_i(\mathbf{r}, C - C(\bar{Q}))$ is symmetric in $\mathbf{r}$, therefore

$$\xi_i(r^{n-1}, C - C(\bar{Q})) = \xi_j(\mathbf{r}^{n-1}, C - C(\bar{Q}))$$

for all $i, j \leq n-1$. In words, agents $1, ..., n-1$ are assigned the same cost share. Moreover, by assumption $r_n \geq r_{n-1}$. Applying **ILR** between profiles $(\mathbf{q}^n, \bar{\mathbf{q}})$ and

$\left(\mathbf{q}^{n-1},\bar{\mathbf{q}}\right)$ yields that agent n's contribution is the same under both profiles:

$$\xi_n(\mathbf{r}^{n-1};C-C(\bar{Q}))=\xi_n(\mathbf{r}^{n};C-C(\bar{Q}))=\frac{1}{n}\left[C\left(Q^n\right)-C(\bar{Q})\right],$$

Thus, agents $1,...,n-1$ share the remaining cost equally:

$$\begin{aligned}\xi_i(\mathbf{r}^{n-1};C-C(\bar{Q})) &= \frac{1}{n-1}\left[C\left(Q^{n-1}\right)-C\left(\bar{Q}\right)-\frac{1}{n}\left[C\left(Q^n\right)-C(\bar{Q})\right]\right]\\ &= \frac{1}{n}\left[C\left(Q^n\right)-C(\bar{Q})\right]-\frac{1}{n-1}\left[C\left(Q^n\right)-C\left(Q^{n-1}\right)\right]\end{aligned}$$

for all $i\leq n-1$, where

$$Q^{n-1}=\sum_{i=1}^{n}q_i^{n-1}=\sum_{i=1}^{n-1}g_i\left(r_{n-1}\right)+q_n\leq Q^n.$$

Alternatively,

$$\xi_i(\mathbf{r}^{n-1};C-C(\bar{Q}))-\xi_i(\mathbf{r}^{n};C-C(\bar{Q}))=-\frac{1}{n-1}\left[C\left(Q^n\right)-C\left(Q^{n-1}\right)\right]$$

all $i\leq n-1$.

Similarly, for all $k\leq n-1$, we define

$$\mathbf{q}^k=\left(g_1\left(r_k\right),...,g_{k-1}\left(r_k\right),q_k,...,q_{n-1},q_n\right)$$

and obtain by **SR** that

$$x_i(\mathbf{q}^k;\bar{\mathbf{q}})-x_i(\bar{\mathbf{q}};\bar{\mathbf{q}})=\xi_i(\mathbf{r}^k,C-C(\bar{Q})),$$

for all $i\in N$. It follows that

$$\begin{aligned}\xi_i(\mathbf{r}^{k};C-C(\bar{Q}))-\xi_i(\mathbf{r}^{k+1};C-C(\bar{Q})) &= 0 \qquad \text{for all } i>k\text{, and}\\ \xi_i(\mathbf{r}^{k};C-C(\bar{Q}))-\xi_i(\mathbf{r}^{k+1};C-C(\bar{Q})) &= -\frac{1}{k}\left[C\left(Q^{k+1}\right)-C\left(Q^k\right)\right] \qquad \text{for all } i\leq k,\end{aligned}$$

with

$$Q^k=\sum_{i=1}^{n}q_i^k=\sum_{i=1}^{k}g_i\left(r_k\right)+\sum_{i=k+1}^{n}q_i.$$

Observe that

$$Q^{k+1} - Q^k = \sum_{i=1}^{k} \left[g_i \left(r_{k+1} \right) - g_i \left(r_k \right) \right] \geq 0$$

by monotonicity of the g_i's. It follows that $x_{i+1} \left(\mathbf{q}^k, \bar{\mathbf{q}} \right) \geq x_i \left(\mathbf{q}^k, \bar{\mathbf{q}} \right)$, all $i \in N$. It follows from our initial ordering of the agents that agents with a higher r_i pay a higher bill for all k.

To sum up, upon observing that $\mathbf{r}^1 = \mathbf{r}$ (as associated to $(\mathbf{q}, \bar{\mathbf{q}})$), we obtain

$$x_k \left(\mathbf{q}, \bar{\mathbf{q}} \right) - x_k \left(\bar{\mathbf{q}}, \bar{\mathbf{q}} \right) = \frac{1}{n} \left[C \left(Q^n \right) - C(\bar{Q}) \right] - \sum_{i=k}^{n-1} \frac{1}{i} \left[C \left(Q^{i+1} \right) - C \left(Q^i \right) \right]$$

where $Q^1 = Q$. Finally,

$$\begin{aligned} x_k \left(\mathbf{q}, \bar{\mathbf{q}} \right) &= \frac{1}{n} C \left(Q^n \right) - \sum_{i=k}^{n-1} \frac{1}{i} \left[C \left(Q^{i+1} \right) - C \left(Q^i \right) \right] \\ &= \left[\frac{1}{n} - \frac{1}{n-1} \right] C \left(Q^n \right) + \left[\frac{1}{n-1} - \frac{1}{n-2} \right] C \left(Q^{n-1} \right) + \left[\frac{1}{n-2} - \frac{1}{n-3} \right] C \left(Q^{n-2} \right) \\ &\quad + ... + \left[\frac{1}{i} - \frac{1}{i-1} \right] C \left(Q^i \right) + ... + \frac{1}{k} C \left(Q^k \right) \\ x_k \left(\mathbf{q}, \bar{\mathbf{q}} \right) &= \frac{C \left(Q^k \right)}{k-1} - \sum_{i=k}^{n} \frac{C \left(Q^i \right)}{i \left(i-1 \right)} = \frac{C \left(Q^k \right)}{k} - \sum_{i=k+1}^{n} \frac{C \left(Q^i \right)}{i \left(i-1 \right)} \end{aligned}$$

with

$$Q^k = \sum_{i=1}^{k} g_i \left(r_k \right) + \sum_{i=k+1}^{n} q_i.$$

B.4 Proof of Proposition 5

Let $\bar{q}_0 \in \mathbb{R}_+$ be a reference level of needs and denote by $\bar{\mathbf{q}}_0 = (\bar{q}_0, \bar{q}_0, ..., \bar{q}_0) \in \mathbb{R}^n_+$ the associated reference vector. Consider the profile $(\mathbf{q}, \bar{\mathbf{q}}_0)$. By budget balance and Equal Treatment of Equals,

$$x_i \left(\bar{\mathbf{q}}_0 ; \bar{\mathbf{q}}_0 \right) = \frac{C(n\bar{q}_0)}{n}.$$

Without loss of generality, assume that $q_1 \leq q_2 \leq ... \leq q_n$, so that $r_{0,1} \leq r_{0,2} \leq ... \leq r_{0,n}$, where $r_{0,i} = r \left(q_i, \bar{q}_0 \right)$ for all $i \in N$.
Define

$$\mathbf{q}^k = \left(q_k, ..., q_k, q_{k+1}, ..., q_{n-1}, q_n \right)$$

for all $k = 1...n$.

Notice that $\mathbf{q}^n = (q_n, q_n, ..., q_n)$; hence by Equal Treatment of Equals, $x_i(\mathbf{q}^n; \bar{\mathbf{q}}_0) = C(nq_n)/n$ so that

$$x_i(\mathbf{q}^n; \bar{\mathbf{q}}_0) - x_i(\bar{\mathbf{q}}_0; \bar{\mathbf{q}}_0) = \frac{1}{n}[C(nq_n) - C(n\bar{q}_0)]$$

for all $i \in N$.

Similarly, for $k \leq n-1$, **ERRN** yields

$$x_i\left(\mathbf{q}^{k+1}; \bar{\mathbf{q}}_0\right) - x_i(\bar{\mathbf{q}}_0; \bar{\mathbf{q}}_0) = \xi_i\left(\mathbf{r}_0^{k+1}, C - C(n\bar{q}_0)\right)$$

and

$$x_i\left(\mathbf{q}^k; \bar{\mathbf{q}}_0\right) - x_i(\bar{\mathbf{q}}_0; \bar{\mathbf{q}}_0) = \xi_i\left(\mathbf{r}_0^k, C - C(n\bar{q}_0)\right)$$

with $r_{0,i}^{k+1} = r\left(q_i^{k+1}, \bar{q}_0\right)$ and $r_{0,i}^k = r\left(q_i^k, \bar{q}_0\right)$. Therefore, by subtraction,

$$x_i\left(\mathbf{q}^k; \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k+1}; \bar{\mathbf{q}}_0\right) = \xi_i\left(\mathbf{r}_0^k, C - C(n\bar{q}_0)\right) - \xi_i\left(\mathbf{r}_0^{k+1}, C - C(n\bar{q}_0)\right)$$

for all $i \in N$ and summing up over all agents, we find

$$\sum_{i=1}^{n}\left[x_i\left(\mathbf{q}^k; \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k+1}; \bar{\mathbf{q}}_0\right)\right] = -\left[C\left(Q^{k+1}\right) - C\left(Q^k\right)\right],$$

where $Q^k = \sum_{l=1}^n q_l^k = kq_k + \sum_{l=k+1}^n q_l$ and $Q^{k+1} = \sum_{l=1}^n q_l^{k+1} = (k-1)\,q_{k+1} + \sum_{l=k+1}^n q_l$.

Observe that if $i < j$ then $r_{0,i}^k \leq r_{0,j}^k$ and $r_{0,i}^{k+1} \leq r_{0,j}^{k+1}$. Moreover for all $k+1 \leq i \leq n$, $\mathbf{q}_i^k = \mathbf{q}_i^{k+1} = q_i$, and $\mathbf{r}_{0,i}^k = \mathbf{r}_{0,i}^{k+1} = r(q_i, \bar{q}_0)$. Therefore, by **ILR**,

$$x_i\left(\mathbf{q}^k; \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k+1}; \bar{\mathbf{q}}_0\right) = 0,$$

for all $k+1 \leq i \leq n$. It follows that the previous summation can truncated from above:

$$\sum_{i=1}^{k}\left[x_i\left(\mathbf{q}^k; \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k+1}; \bar{\mathbf{q}}_0\right)\right] = -\left[C\left(Q^{k+1}\right) - C\left(Q^k\right)\right],$$

where $1 \leq k \leq n-1$.

Moreover, for all $i, j \leq k$, we have $q_i^{k+1} = q_j^{k+1} = q_{k+1}$ and $q_i^k = q_j^k = q_k$.

Therefore, by Equal Treatment of Equals,

$$x_i\left(\mathbf{q}^{k+1},\bar{\mathbf{q}}_0\right)=x_j\left(\mathbf{q}^{k+1},\bar{\mathbf{q}}_0\right)$$

all $i,j\leq k$, and

$$x_i\left(\mathbf{q}^{k},\bar{\mathbf{q}}_0\right)=x_j\left(\mathbf{q}^{k},\bar{\mathbf{q}}_0\right).$$

Hence,

$$x_i\left(\mathbf{q}^{k},\bar{\mathbf{q}}_0\right)-x_i\left(\mathbf{q}^{k-1},\bar{\mathbf{q}}_0\right)=-\frac{1}{k}\left[C\left(Q^{k+1}\right)-C\left(Q^{k}\right)\right]$$

all $i\leq k$.

Finally, upon observing that $\mathbf{q}^1=\mathbf{q}$, it follows by summation that

$$x_i\left(\mathbf{q},\bar{\mathbf{q}}_0\right)-x_i\left(\bar{\mathbf{q}}_0,\bar{\mathbf{q}}_0\right)=\frac{1}{n}\left[C\left(nq_n\right)-C\left(n\bar{q}_0\right)\right]-\sum_{k=i}^{n-1}\frac{1}{k}\left[C\left(Q^{k+1}\right)-C\left(Q^{k}\right)\right],$$

so that

$$x_i\left(\mathbf{q},\bar{\mathbf{q}}_0\right)=\frac{1}{n}C\left(nq_n\right)-\sum_{k=i}^{n-1}\frac{1}{k}\left[C\left(Q^{k+1}\right)-C\left(Q^{k}\right)\right].$$

Define $\bar{\mathbf{q}}_0^1=\left(\bar{q}_1,\bar{q}_0,...,\bar{q}_0\right)$. Applying **GS** between $\left(\mathbf{q},\bar{\mathbf{q}}_0\right)$ and $\left(\mathbf{q},\bar{\mathbf{q}}_0^1\right)$ yields, for all $j\neq 1$:

$$\begin{aligned}&u\left(q_1,\bar{q}_1\right)-x_1^1-u\left(q_1,\bar{q}_0\right)+x_1^0\\=\;&u\left(q_j,\bar{q}_0\right)-x_j^1-u\left(q_j,\bar{q}_0\right)+x_j^0\end{aligned}$$

where $x_j^0=x_j\left(\mathbf{q},\bar{\mathbf{q}}_0\right)$ and $x_j^1=x_j\left(\mathbf{q},\bar{\mathbf{q}}_0^1\right)$ for all $j\in N$. This yields

$$x_j^0-x_j^1=u\left(q_1,\bar{q}_1\right)-u\left(q_1,\bar{q}_0\right)+x_1^0-x_1^1.$$

Since total consumption is unchanged, we have, by budget balance

$$\begin{aligned}x_1^1-x_1^0&=\frac{n-1}{n}\left[u\left(q_1,\bar{q}_1\right)-u\left(q_1,\bar{q}_0\right)\right]\\x_j^1-x_j^0&=-\frac{1}{n}\left[u\left(q_1,\bar{q}_1\right)-u\left(q_1,\bar{q}_0\right)\right]\end{aligned}$$

all $j\neq 1$.

Iterating and applying **GS** to profiles $\left(\mathbf{q},\bar{\mathbf{q}}_0^k\right)$ where $\bar{\mathbf{q}}_0^k=\left(\bar{q}_1,\bar{q}_2,...,\bar{q}_k,\bar{q}_0,...,\bar{q}_0\right)$, successively leads to the following expression, for all iterations, $k=1,...,n$, and all agents $1 \leq i \leq k \leq j \leq n$:

$$\begin{aligned} & u(q_i, \bar{q}_i) - x_i^k - u(q_i, \bar{q}_i) + x_i^{k-1} \\ = \; & u(q_k, \bar{q}_k) - x_k^k - u(q_k, \bar{q}_0) + x_k^{k-1} \\ = \; & u(q_j, \bar{q}_0) - x_j^k - u(q_j, \bar{q}_0) + x_j^{k-1} \end{aligned}$$

Hence, for all $k = 1, ..., n$, and all agents $1 \leq i \leq k \leq j \leq n$:

$$\begin{aligned} & x_i^{k-1} - x_i^k \\ = \; & u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0) + x_k^{k-1} - x_k^k \\ = \; & x_j^{k-1} - x_j^k \end{aligned}$$

Since total consumption does not change from $(\mathbf{q}, \bar{\mathbf{q}}_0^{k-1})$ to $(\mathbf{q}, \bar{\mathbf{q}}_0^k)$, but only needs, budget balance implies $\sum_j (x_j^k - x_j^{k-1}) = 0$. Therefore,

$$\begin{aligned} x_j^k - x_j^{k-1} &= -\frac{1}{n}[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)] \qquad \text{for all } j \neq k \\ x_k^k - x_k^{k-1} &= \frac{n-1}{n}[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)] \end{aligned}$$

Summing up over all iterations $k = 1, ..., n$ yields the following for agent 1:

$$\begin{aligned} x_1^n - x_1^0 &= \sum_{k>1}^{n} (x_1^k - x_1^{k-1}) + x_1^1 - x_1^0 \\ &= -\frac{1}{n}\sum_{k>1}^{n}[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)] + \frac{n-1}{n}[u(q_1, \bar{q}_1) - u(q_1, \bar{q}_0)] \\ &= [u(q_1, \bar{q}_1) - u(q_1, \bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n}[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)] \end{aligned}$$

Similarly, for all $i > 1$:

$$x_i^n - x_i^0 = [u(q_i, \bar{q}_i) - u(q_i, \bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n}[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)]$$

Finally, observing that $\bar{\mathbf{q}}_0^n = \bar{\mathbf{q}}$ yields the following:

$$\begin{aligned} x_i(\mathbf{q}, \bar{\mathbf{q}}) &= x_i(\mathbf{q}, \bar{\mathbf{q}}_0^n) = \frac{1}{n} C(Q^n) - \sum_{k=i}^{n-1} \frac{1}{k} \left[C\left(Q^{k+1}\right) - C\left(Q^k\right) \right] \\ &\quad + \left[u(q_i, \bar{q}_i) - u(q_i, \bar{q}_0) \right] - \frac{1}{n} \sum_{k=1}^{n} \left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0) \right] \end{aligned}$$

Alternatively,

$$\begin{aligned} x_i(\mathbf{q}, \bar{\mathbf{q}}) &= \frac{C\left(Q^i\right)}{i} - \sum_{k=i+1}^{n-1} \frac{C\left(Q^k\right)}{k(k-1)} \\ &\quad + \left[u(q_i, \bar{q}_i) - u(q_i, \bar{q}_0) \right] - \frac{1}{n} \sum_{k=1}^{n} \left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0) \right] \end{aligned}$$

where $Q^k = kq_k + \sum_{l=k+1}^{n} q_l$ for all $k = 1, ..., n$.

C Supplementary material: Calculations not intended for publication

C.1 Decreasing Returns to Scale: Quadratic Costs

SCE0 with absolute responsibility

Recall that

$$x^{SCE}(\rho) = \frac{C(\bar{Q})}{N} + \int_{z=0}^{\rho} \frac{1}{N - N^r(z)} C'\left(\hat{Q}(z)\right) \frac{d\hat{Q}(z)}{d\rho} dz,$$

where

$$\hat{Q}(\rho) = \sum_{s \in S} \left[\int_0^{+\infty} \inf\{g_s(z), g_s(\rho)\} n_s^r(z)\, dz \right].$$

Under the absolute responsibility view,

$$\frac{d\hat{Q}(\rho)}{d\rho} = \sum_{s \in S} (N_s - N_s^r(\rho))\, g_s'(\rho) = N - N^r(\rho),$$

with the second equality following from the fact that $g_s(r) = \bar{q}_s + \rho$. Hence,

$$x^{SCE}(\rho) = \frac{C(\bar{Q})}{N} + \int_{z=0}^{\rho} C'\left(\hat{Q}(z)\right) dz,$$

with

$$\begin{aligned} \hat{Q}_s(\rho) &= \int_0^{\rho} (\bar{q}_s + z)\, n_s^r(z)\, dz + (N_s - N_s^r(\rho))(\bar{q}_s + \rho) \\ &= \bar{q}_s N_s + \int_0^{\rho} z n_s^r(z)\, dz + (N_s - N_s^r(\rho))\, \rho \\ &= \bar{q}_s N_s + \int_0^{+\infty} \min\{z, \rho\} n_s^r(z)\, dz \end{aligned}$$

so that

$$\begin{aligned} \hat{Q}(\rho) &= \bar{Q} + \int_0^{r} z n^r(z)\, dz + (N - N^r(\rho))\, \rho \\ &= \bar{Q} + \int_0^{+\infty} \min\{z, \rho\} n^r(z)\, dz. \end{aligned}$$

Consider the case where $C(Q) = \frac{c}{2}Q^2$. It follows that $C'(Q) = cQ$, so that

$$\begin{aligned}
x^{SCE0}(\rho) &= \frac{c\bar{Q}^2}{2N} + c\int_{z=0}^{\rho} \hat{Q}(z)\,dz \\
&= \frac{c\bar{Q}^2}{2N} + c\int_{z=0}^{\rho} \left[\bar{Q} + \int_{y=0}^{+\infty} \min\{y,z\} n^r(y)\,dy\right] dz \\
&= \frac{c\bar{Q}^2}{2N} + c\bar{Q}\rho + c\int_{y=0}^{+\infty} n^r(y) \int_{z=0}^{\rho} \min\{y,z\} dz dy \\
&= \frac{c\bar{Q}^2}{2N} + c\bar{Q}\rho + c\int_{y=0}^{+\infty} n^r(y) \left[\int_{z=0}^{y} z dz + \int_{z=y}^{\rho} y dz\right] dy \\
&= \frac{c\bar{Q}^2}{2N} + c\bar{Q}\rho + c\int_{y=0}^{+\infty} n^r(y) \left[\frac{y^2}{2} + y(\rho - y)\right] dy, \\
&= \frac{c\bar{Q}^2}{2N} + c\bar{Q}\rho + c\int_{y=0}^{+\infty} n^r(y) \left[y\rho - \frac{y^2}{2}\right] dy.
\end{aligned}$$

Upon noticing that $\bar{Q} + \int_{y=0}^{+\infty} n^r(y)\,y dy = Q$ under absolute responsibility, the above expression rewrites as follows:

$$x^{SCE0}(\rho) = \frac{c\bar{Q}^2}{2N} - c\int_{y=0}^{+\infty} n^r(y) \frac{y^2}{2} dy + cQ\rho.$$

By budget balance,

$$\begin{aligned}
c\frac{Q^2}{2} &= \int_{z=0}^{+\infty} x^{SCE0}(z)\, n^r(z)\,dz \\
&= N\left[\frac{c\bar{Q}^2}{2N} - c\int_{y=0}^{+\infty} n^r(y) \frac{y^2}{2} dy\right] + cQ\int_{z=0}^{+\infty} z n^r(z) dz \\
&= N\left[\frac{c\bar{Q}^2}{2N} - c\int_{y=0}^{+\infty} n^r(y) \frac{y^2}{2} dy\right] + cQ\left(Q - \bar{Q}\right).
\end{aligned}$$

Thus,

$$\frac{c\bar{Q}^2}{2N} - c\int_{y=0}^{+\infty} n^r(y) \frac{y^2}{2} dy = \frac{1}{N}\left(\frac{cQ^2}{2} - cQ\left(Q - \bar{Q}\right)\right).$$

Finally, it follows that

$$\begin{aligned}
x^{SCE0}(\rho) &= \frac{1}{N}\left[cQ\left(\bar{Q} - \frac{Q}{2}\right)\right] + cQ\rho \\
&= \frac{1}{N}\frac{cQ^2}{2} + cQ\left(\rho - \frac{Q - \bar{Q}}{N}\right)
\end{aligned}$$

Upon recalling that $\rho = q - \bar{q}_s$ under absolute responsibility, individual contributions are a function of the sole four variables $q, \bar{q}_s, Q, \bar{Q}$:

$$x^{SCE0}\left(q, \bar{q}_s, Q, \bar{Q}\right) = \frac{1}{N}\frac{cQ^2}{2} + cQ\left(q - \bar{q}_s - \frac{Q - \bar{Q}}{N}\right).$$

SCE0 with relative responsibility

Recall that

$$x^{SCE}\left(\rho\right) = \frac{C\left(\bar{Q}\right)}{N} + \int_{z=0}^{\rho} \frac{1}{N - N^r\left(z\right)} C'\left(\hat{Q}\left(z\right)\right)\frac{d\hat{Q}\left(z\right)}{dz}dz,$$

where

$$\begin{aligned}\hat{Q}\left(\rho\right) &= \sum_{s\in S}\left[\int_0^{+\infty} \inf\{g_s\left(z\right), g_s\left(\rho\right)\} n_s^r\left(z\right)dz\right] \\ &= \sum_{s\in S}\left[\int_0^{\rho} g_s\left(z\right) n_s^r\left(z\right)dz + \left(N_s - N_s^r\left(\rho\right)\right) g_s\left(\rho\right)\right]\end{aligned}$$

Under relative responsibility, $\rho = \left(q - \bar{q}_s\right)/\bar{q}_s$ so that $g_s\left(\rho\right) = \bar{q}_s\left(1 + \rho\right)$. It follows that $g_s'\left(\rho\right) = \bar{q}_s$ and

$$\frac{d\hat{Q}_s\left(\rho\right)}{d\rho} = \left(N_s - N_s^r\left(\rho\right)\right) g_s'\left(\rho\right) = \left(N_s - N_s^r\left(\rho\right)\right)\bar{q}_s.$$

We now make an additional assumption. Namely, we posit that responsibility is evenly spread across types, so that its distribution is independent of needs, $\bar{q}_s$:

$$N_s^r\left(\rho\right) = \alpha\left(\rho\right) N_s \qquad \forall s \in S,$$

for some increasing function $\alpha : \mathbb{R}_+ \to [0, 1]$ which we take to be differentiable. This yields:

$$\frac{d\hat{Q}\left(\rho\right)}{d\rho} = \left(1 - \alpha\left(\rho\right)\right)\bar{Q}.$$

Also, because $N - N^r\left(r\right) = \left(1 - \alpha\left(\rho\right)\right) N$, we have

$$\frac{1}{N - N^r\left(z\right)}\frac{d\hat{Q}\left(\rho\right)}{d\rho} = \frac{\bar{Q}}{N}$$

and $x^{SCE0}(r)$ simplifies to

$$x^{SCE0}(\rho) = \frac{C(\bar{Q})}{N} + \frac{\bar{Q}}{N}\int_{z=0}^{\rho} C'\left(\hat{Q}(z)\right) dz.$$

Upon noticing that $n_s^r(\rho) = \alpha'(\rho) N_s$ we get

$$\begin{aligned}
\hat{Q}(\rho) &= \int_0^{+\infty} \sum_{s\in S} \inf\{g_s(z), g_s(\rho)\} N_s \alpha'(z)\, dz \\
&= \int_0^{+\infty} \inf\{\sum_{s\in S} N_s g_s(z), \sum_{s\in S} N_s g_s(\rho)\}\alpha'(z)\, dz
\end{aligned}$$

where the summation sign enters the min operator because, for any $s \in S$, $g_s(z) \leq g_s(\rho)$ if and only if $z \leq \rho$. Therefore,

$$\begin{aligned}
\hat{Q}(\rho) &= \int_0^{+\infty} \inf\{\sum_{s\in S} N_s \bar{q}_s(1+z), \sum_{s\in S} N_s \bar{q}_s(1+\rho)\}\alpha'(z)\, dz \\
&= \bar{Q}\left[1 + \int_0^{+\infty} \inf\{z,\rho\}\alpha'(z)\, dz\right]
\end{aligned}$$

Assuming $C(Q) = \frac{1}{2}cQ^2$, the expression for the SCE0 solution follows:

$$\begin{aligned}
x^{SCE}(\rho) &= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}c}{N}\int_{z=0}^{\rho} \hat{Q}(z)\, dz \\
&= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}c}{N}\int_{z=0}^{\rho} \bar{Q}\left[1 + \int_{y=0}^{+\infty} \inf\{y,z\}\alpha'(y)\, dy\right] dz \\
&= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}^2 c\rho}{N} + \frac{\bar{Q}^2 c}{N}\int_{y=0}^{+\infty}\int_{z=0}^{r} \inf\{y,z\}\alpha'(y)\, dydz \\
&= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}^2 c\rho}{N} + \frac{\bar{Q}^2 c}{N}\int_{y=0}^{+\infty} \alpha'(y)\left[\int_{z=0}^{y} zdz + y\int_{z=y}^{\rho} dz\right] dy \\
&= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}^2 c\rho}{N} + \frac{\bar{Q}^2 c}{N}\int_{y=0}^{+\infty} \frac{n^r(y)}{N}\left[\frac{y^2}{2} + y(\rho - y)\right] dy \\
&= \frac{c\bar{Q}^2}{2N} + \frac{c\bar{Q}^2}{N}\rho + \frac{c\bar{Q}^2}{N^2}\int_{y=0}^{+\infty}\left[\left(\rho - \frac{y}{2}\right) y n^r(y)\right] dy \\
&= \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2}\int_{y=0}^{+\infty} y^2 n^r(y)\, dy + \frac{c\bar{Q}^2}{N}\left[1 + \frac{1}{N}\int_{y=0}^{+\infty} y n^r(y)\, dy\right]\rho
\end{aligned}$$

For households of type s this writes

$$\begin{aligned} x^{SCE0}(\rho) &= \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2}\int_{y=0}^{+\infty} y^2 n^r(y)\,dy + \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} y n^r(y)\,dy\right]\left(\frac{q-\bar{q}_s}{\bar{q}_s}\right) \\ &= \left\{\frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2}\int_{y=0}^{+\infty} y^2 n^r(y)\,dy - \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} y n^r(y)\,dy\right]\right\} \\ &\quad + \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} y n^r(y)\,dy\right]\frac{q}{\bar{q}_s} \end{aligned}$$

Also, by budget balance,

$$\begin{aligned} \frac{cQ^2}{2} &= \sum_s \int_{z=0}^{+\infty} x^{SCE0}(z) n_s^r(z)dz \\ &= \sum_s \int_{z=0}^{+\infty}\left\{\frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2}\int_{y=0}^{+\infty} y^2 n^r(y)\,dy - \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} y n^r(y)\,dy\right]\right\} n_s^r(z)dz \\ &+ \sum_s \int_z \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} y n^r(y)\,dy\right]\frac{q}{\bar{q}_s} n_s^r(z)dz \\ &= \left\{\frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2}\int_{y=0}^{+\infty} y^2 n^r(y)\,dy - \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} y n^r(y)\,dy\right]\right\}\sum_s \int_{z=0}^{+\infty} n_s^r(z)dz \\ &+ \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} y n^r(y)\,dy\right]\sum_s \int_z (z+1)\, n_s^r(z)dz \\ &= \left\{\frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2}\int_{y=0}^{+\infty} y^2 n^r(y)\,dy - \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} y n^r(y)\,dy\right]\right\} N \\ &+ \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} y n^r(y)\,dy\right]\left[\sum_s \frac{Q_s}{\bar{q}_s}\right] \end{aligned}$$

because $z+1 = g_s(z)/\bar{q}_s$ and $\int_z (z+1)\, n_s^r(z)dz = \int_z [g_s(z)/\bar{q}_s]\, n_s^r(z)dz = \int_q (q/\bar{q}_s)\, n_s(q)dq = Q_s/\bar{q}_s$.

Therefore,

$$\begin{aligned} \left\{\frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2}\int_{y=0}^{+\infty} y^2 n^r(y)\,dy - \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} y n^r(y)\,dy\right]\right\} = \frac{1}{N}\left\{\frac{cQ^2}{2}\right. \\ \left. -\frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} y n^r(y)\,dy\right]\left[\sum_s \frac{Q_s}{\bar{q}_s}\right.\right. \end{aligned}$$

Finally,

$$x^{SCE0}(\rho) = \frac{1}{N}\left\{\frac{cQ^2}{2} - \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} yn^r(y)\,dy\right]\left[\sum_s \frac{Q_s}{\bar{q}_s}\right]\right\} + \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} yn^r(y)\,dy\right]\frac{q}{\bar{q}_s}$$

$$= \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\int_{y=0}^{+\infty} yn^r(y)\,dy\right]\left(\frac{q}{\bar{q}_s} - \frac{1}{N}\sum_s \frac{Q_s}{\bar{q}_s}\right).$$

Observing that $N_s(q) = N_s^r\left(\frac{q-\bar{q}_s}{\bar{q}_s}\right)$ implies $n_s(q)dq = \frac{1}{\bar{q}_s}n_s^r\left(\frac{q-\bar{q}_s}{\bar{q}_s}\right)dq = n_s^r(y)\,dy$. Hence,

$$x^{SCE0}(\rho) = \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\sum_s\int_{q=\bar{q}_s}^{+\infty} \frac{q-\bar{q}_s}{\bar{q}_s}n_s(q)\,dq\right]\left(\frac{q}{\bar{q}_s} - \frac{1}{N}\sum_s \frac{Q_s}{\bar{q}_s}\right)$$

$$= \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[1+\frac{1}{N}\sum_s\left(\frac{Q_s}{\bar{q}_s} - N_s\right)\right]\left(\frac{q}{\bar{q}_s} - \frac{1}{N}\sum_s \frac{Q_s}{\bar{q}_s}\right)$$

$$= \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[\frac{1}{N}\sum_s \frac{Q_s}{\bar{q}_s}\right]\left(\frac{q}{\bar{q}_s} - \frac{1}{N}\sum_s \frac{Q_s}{\bar{q}_s}\right).$$

Moreover, the distributional assumption that $N_s^r(r)/N_s = \alpha(\rho)$ for all s implies that:

$$Q_s = \int_{\bar{q}_s}^{+\infty} qn_s(q)\,dq$$

$$= \int_0^{+\infty} \bar{q}_s(1+y)\,n_s(y)\,dy$$

$$= \bar{q}_s\int_0^{+\infty}(1+y)\,\alpha'(y)\,N_s dy$$

$$= \bar{Q}_s\int_0^{+\infty}(1+y)\,\alpha'(y)\,dy$$

This says that $Q_s/\bar{Q}_s = \int_0^{+\infty}(1+y)\,\alpha'(y)\,dy$ is independent of s. Hence, for all s,

$$Q_s/\bar{Q}_s = Q/\bar{Q}.$$

Therefore,

$$
\begin{aligned}
x^{SCE0}(q,s) &= \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[\frac{1}{N}\frac{Q}{\bar{Q}}\sum_s \frac{\bar{Q}_s}{\bar{q}_s}\right]\left(\frac{q}{\bar{q}_s} - \frac{1}{N}\frac{Q}{\bar{Q}}\sum_s \frac{\bar{Q}_s}{\bar{q}_s}\right) \\
&= \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[\frac{Q}{\bar{Q}}\frac{1}{N}\sum_s N_s\right]\left(\frac{q}{\bar{q}_s} - \frac{Q}{\bar{Q}}\frac{1}{N}\sum_s N_s\right) \\
&= \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[\frac{Q}{\bar{Q}}\right]\left(\frac{q}{\bar{q}_s} - \frac{Q}{\bar{Q}}\right) \\
&= \frac{1}{N}\frac{cQ^2}{2} + cQ\frac{\bar{Q}}{N}\left(\frac{q}{\bar{q}_s} - \frac{Q}{\bar{Q}}\right).
\end{aligned}
$$

SEE

Recall that:

$$
\begin{aligned}
x^{SEE}(q,s) = \frac{C(N\bar{q}_0)}{N} &+ \int_{z=0}^{q}\frac{1}{N-N(z)}C'\left(\tilde{Q}(z)\right)\frac{d\tilde{Q}(z)}{dq}dz \\
&+ \left[u(q,\bar{q}_s) - u(q,\bar{q}_0)\right] - \frac{1}{N}\sum_{t\in S}\int_{z=0}^{\infty}\left[u(z,\bar{q}_t) - u(z,\bar{q}_0)\right]n_t(z)\,dz,
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{Q}(q) &= \int_0^q zn(z)\,dz + (N - N(q))\,q \\
&= \int_0^\infty \inf\{z,q\}n(z)\,dz.
\end{aligned}
$$

We have

$$
\begin{aligned}
\frac{d\tilde{Q}}{dq} &= qn(q) + (N - N(q)) - qn(q) \\
&= N - N(q),
\end{aligned}
$$

so that

$$
\begin{aligned}
\int_{z=0}^{q} \frac{1}{N-N(z)} C'\left(\tilde{Q}(z)\right) \frac{d\tilde{Q}(z)}{dq} dz &= \int_{z=0}^{q} C'\left(\tilde{Q}(z)\right) dz \\
&= \int_{z=0}^{q} c\tilde{Q}(z)\, dz \\
&= c\int_{z=0}^{q} \left(\int_{y=0}^{\infty} \inf\{y,z\}\, n(y)\, dy\right) dz \\
&= c\int_{y=0}^{\infty} \int_{z=0}^{q} \inf\{y,z\} dz n(y)\, dy \\
&= c\int_{y=0}^{\infty} \left(\int_{z=0}^{y} zdz + \int_{z=y}^{q} ydz\right) n(y)\, dy \\
&= c\int_{y=0}^{\infty} \left(\frac{y^2}{2} + y(q-y)\right) n(y)\, dy \\
&= c\int_{y=0}^{\infty} \left(yq - \frac{y^2}{2}\right) n(y)\, dy \\
&= cq\int_{y=0}^{\infty} yn(y)\, dy - c\int_{y=0}^{\infty} \frac{y^2}{2} n(y)\, dy \\
&= cQq - c\int_{y=0}^{\infty} \frac{y^2}{2} n(y)\, dy.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
x^{SEE}(q,s) &= \frac{C(N\bar{q}_0)}{N} - c\int_{y=0}^{\infty} \frac{y^2}{2} n(y)\, dy + cQq \\
&\quad + \left[u(q,\bar{q}_s) - u(q,\bar{q}_0)\right] - \frac{1}{N}\sum_{t\in S}\int_{z=0}^{\infty} \left[u(z,\bar{q}_t) - u(z,\bar{q}_0)\right] n_t(z)\, dz.
\end{aligned}
$$

By budget balance

$$
\sum_s \int_0^{\infty} x^{SEE}(z,s)\, n_s(z) dz = \frac{cQ^2}{2}
$$

so that

$$
N\left[\frac{C(N\bar{q}_0)}{N} - c\int_{y=0}^{\infty} \frac{y^2}{2} n(y)\, dy\right] + cQ^2 = \frac{cQ^2}{2};
$$

hence,

$$
\frac{C(N\bar{q}_0)}{N} - c\int_{y=0}^{\infty} \frac{y^2}{2} n(y)\, dy = -\frac{1}{N}\frac{cQ^2}{2}.
$$

Therefore, the cost component of x^{SEE} writes

$$\frac{C(N\bar{q}_0)}{N} - c\int_{y=0}^{\infty} \frac{y^2}{2} n\left(y\right) dy + cQq = -\frac{1}{N}\frac{cQ^2}{2} + cQq$$
$$= cQ\left(q - \frac{Q}{2N}\right).$$

To sum up,

$$x^{SEE}\left(q,s\right) = cQ\left(q - \frac{Q}{2N}\right)$$
$$+ \left[u\left(q,\bar{q}_s\right) - u\left(q,\bar{q}_0\right)\right] - \frac{1}{N}\sum_{t\in S}\int_{z=0}^{\infty}\left[u\left(z,\bar{q}_t\right) - u\left(z,\bar{q}_0\right)\right] n_t\left(z\right) dz.$$

C.2 Increasing Returns to Scale: Affine Costs

From the discrete to the distributional setting

Recall the expression for x^{DSCE0} :

$$x_i^{DSCE0}\left(\mathbf{q},\bar{\mathbf{q}}\right) = \frac{1}{n}C\left(\check{Q}^n\right) - \sum_{k=i}^{n-1}\frac{1}{k}\left[C\left(\check{Q}^{k+1}\right) - C\left(\check{Q}^k\right)\right] \qquad \text{for all } i \in N,$$

where, for all $k \in N$,

$$\check{Q}^k = \sum_{i=1}^{k} g_i\left(r_k\right) + \sum_{i=k+1}^{n} q_i.$$

where the set N is ordered so as to have $r_1 \leq r_2 \leq ... \leq r_n$.

In the distributional setting,

$$
\begin{aligned}
\check{Q}(\rho) &= \sum_{s\in S}\left[\int_{z=0}^{\rho} g_s(\rho)\, n_s^r(z)\, dz + \int_{z=\rho}^{\infty} g_s(z)\, n_s^r(z)\, dz\right] \\
&= \sum_{s\in S}\left[N_s^r(\rho)\, g_s(\rho) + \int_{z=\rho}^{\infty} g_s(z)\, n_s^r(z)\, dz\right] \\
&= \sum_{s\in S} N_s^r(\rho)\, g_s(\rho) + \int_{z=\rho}^{\infty} \sum_{s\in S} g_s(z)\, n_s^r(z)\, dz \\
&= \int_{z=0}^{\infty} \sum_{s\in S} \sup\{g_s(\rho), g_s(z)\}\, n_s^r(z)\, dz \\
&= \int_{z=0}^{\infty} \sum_{s\in S} g_s(\sup\{\rho, z\})\, n_s^r(z)\, dz.
\end{aligned}
$$

Define $\sup\vec{\rho}$ the largest responsibility level in the population and the associated virtual consumption level that brings all users to that same level of responsibility:

$$
\begin{aligned}
\check{Q}_{\sup} &\equiv \check{Q}(\sup\vec{\rho}) \\
&= \int_{z=0}^{\infty} \sum_{s\in S} g_s(\sup\{\sup\vec{\rho}, z\})\, n_s^r(z)\, dz \\
&= \sum_{s\in S} g_s(\sup\vec{\rho}) \int_{z=0}^{\infty} n_s^r(z)\, dz \\
&= \sum_{s\in S} N_s g_s(\sup\vec{\rho}).
\end{aligned}
$$

Likewise, the expression of x^{DSCE0} in the distributional setting becomes:

$$
x^{DSCE0}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{1}{N} C\left(\check{Q}_{\sup}\right) - \int_{z=\rho}^{\sup\vec{\rho}} \frac{1}{N^r(z)} C'\left(\check{Q}(z)\right) \frac{d\check{Q}(z)}{dz} dz.
$$

Moreover,

$$
\begin{aligned}
\frac{d\check{Q}(z)}{dz} &= \frac{d}{dz}\left[\sum_{s\in S} N_s^r(z)\, g_s(z) + \int_{y=z}^{\infty} \sum_{s\in S} g_s(y)\, n_s^r(y)\, dy\right] \\
&= \sum_{s\in S} n_s^r(z)\, g_s(z) + \sum_{s\in S} N_s^r(z)\, g_s'(z) - \sum_{s\in S} g_s(z)\, n_s^r(z) \\
&= \sum_{s\in S} N_s^r(z)\, g_s'(z). \qquad (6)
\end{aligned}
$$

DSCE0 with absolute responsibility

In the case of absolute responsibility, $\rho = q - \bar{q}_s$ so that $g_s(\rho) = \rho + \bar{q}_s$. Hence, $g_s'(\rho) = 1$ for all $s \in S$. It follows that:

$$\begin{aligned}
\check{Q}(\rho) &= \int_{z=0}^{\infty} \sum_{s \in S} g_s(\sup\{\rho, z\})\, n_s^r(z)\, dz \\
&= \int_{z=0}^{\infty} \sum_{s \in S} [\sup\{\rho, z\} + \bar{q}_s]\, n_s^r(z)\, dz \\
&= \int_{z=0}^{\infty} \sup\{\rho, z\}\, n^r(z)\, dz + \bar{Q}.
\end{aligned}$$

Moreover, Expression (6) becomes:

$$\frac{d\check{Q}(\rho)}{d\rho} = \sum_{s \in S} N_s^r(\rho) = N^r(\rho),$$

and,

$$\begin{aligned}
\check{Q}_{\sup} &= \sum_{s \in S} N_s g_s(\sup \vec{\rho}) \\
&= \sum_{s \in S} N_s [\bar{q}_s + \sup \vec{\rho}] \\
&= \bar{Q} + N \sup \vec{\rho}.
\end{aligned}$$

Hence,

$$x^{DSCE0}(q, s) = \frac{1}{N} C\left(\check{Q}_{\sup}\right) - \int_{z=\rho}^{\sup \vec{\rho}} C'\left(\check{Q}(z)\right) dz.$$

Suppose that $C(Q) = F + cQ$ with $F, c > 0$. We obtain:

$$\begin{aligned}
x^{DSCE0}(q, s) &= \frac{1}{N}\left(F + c\check{Q}_{\sup}\right) - c\left[\sup \vec{\rho} - \rho\right] \\
&= \frac{F + c\bar{Q}}{N} + c \sup \vec{\rho} - c \sup \vec{\rho} + c\rho \\
&= \frac{F + c\bar{Q}}{N} + c\rho \\
&= \frac{F + c\bar{Q}}{N} + c(q - \bar{q}_s)
\end{aligned}$$

DSCE0 with relative responsibility

When responsibility is measured by relative responsibility, $\rho = (q - \bar{q}_s)/\bar{q}_s$ so that $g_s(\rho) = \bar{q}_s(1+\rho)$. Hence, $g'_s(\rho) = \bar{q}_s$ for any $s \in S$. It follows that:

$$\begin{aligned}
\check{Q}(\rho) &= \int_{z=0}^{\infty} \sum_{s\in S} g_s(\sup\{\rho, z\})\, n_s^r(z)\, dz \\
&= \int_{z=0}^{\infty} \sum_{s\in S} [1+\sup\{\rho, z\}]\, \bar{q}_s n_s^r(z)\, dz \\
&= \bar{Q} + \int_{z=0}^{\infty} \sup\{\rho, z\} \sum_{s\in S} \bar{q}_s n_s^r(z)\, dz.
\end{aligned}$$

Moreover, Expression (6) becomes:

$$\frac{d\check{Q}(\rho)}{d\rho} = \sum_{s\in S} \bar{q}_s N_s^r(\rho),$$

and,

$$\begin{aligned}
\check{Q}_{\sup} &= \sum_{s\in S} N_s g_s(\sup \vec{\rho}) \\
&= \sum_{s\in S} N_s \bar{q}_s [1+\sup \vec{\rho}] \\
&= [1+\sup \vec{\rho}]\, \bar{Q}.
\end{aligned}$$

Therefore, taking $C(Q) = F + cQ$ yields:

$$\begin{aligned}
x^{DSCE0}(\mathbf{q}, \bar{\mathbf{q}}) &= \frac{1}{N} C\left(\check{Q}_{\sup}\right) - \int_{z=\rho}^{\sup \vec{\rho}} \frac{1}{N^r(z)} C'\left(\check{Q}(z)\right) \frac{d\check{Q}(z)}{dz} dz \\
&= \frac{1}{N}\left(F + c\check{Q}_{\sup}\right) - c\int_{z=\rho}^{\sup \vec{\rho}} \frac{1}{N^r(z)} \frac{d\check{Q}(z)}{dz} dz \\
&= \frac{F + c\bar{Q}}{N} + \frac{1}{N} c\bar{Q} \sup \vec{\rho} - c\int_{z=\rho}^{\sup \vec{\rho}} \frac{1}{N^r(z)} \sum_{s\in S} \bar{q}_s N_s^r(z)\, dz
\end{aligned}$$

Assuming that responsibility is evenly spread across types, the distribution of responsibility is independent of needs:

$$N_s^r(\rho) = \alpha(\rho)\, N_s,$$

for some increasing function $\alpha : \mathbb{R}_+ \to [0,1]$ which we take to be differentiable. This yields:

$$\sum_{s\in S} \bar{q}_s N_s^r(\rho) = \sum_{s\in S} \bar{q}_s \alpha(\rho) N_s = \alpha(\rho)\bar{Q},$$

and,

$$N^r(z) = \sum_{s\in S} N_s^r(z) = \alpha(\rho)\sum_{s\in S} N_s = \alpha(\rho) N.$$

Finally, it follows that:

$$\begin{aligned}
x^{DSCE0}(q,s) &= \frac{F+c\bar{Q}}{N} + c\frac{\bar{Q}}{N}\sup\vec{\rho} - c\int_{z=\rho}^{\sup\vec{\rho}} \frac{1}{\alpha(\rho) N}\alpha(\rho)\bar{Q}dz \\
&= \frac{F+c\bar{Q}}{N} + c\frac{\bar{Q}}{N}\sup\vec{\rho} - c\frac{\bar{Q}}{N}(\sup\vec{\rho} - \rho) \\
&= \frac{F+c\bar{Q}}{N} + c\frac{\bar{Q}}{N}\rho \\
&= \frac{F+c\bar{Q}}{N} + c\frac{\bar{Q}}{N}\frac{q-\bar{q}_s}{\bar{q}_s} \\
&= \frac{F}{N} + c\frac{1}{\bar{q}_s/\left(\bar{Q}/N\right)}q.
\end{aligned}$$

DSEE

$$\begin{aligned}
x_i(\mathbf{q},\bar{\mathbf{q}}) &= x_i(\mathbf{q},\bar{\mathbf{q}}_0^n) = \frac{1}{n}C\left(\breve{Q}^n\right) - \sum_{k=i}^{n-1}\frac{1}{k}\left[C\left(\breve{Q}^{k+1}\right) - C\left(\breve{Q}^k\right)\right] \\
&\quad + \left[u(q_i,\bar{q}_i) - u(q_i,\bar{q}_0)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u(q_k,\bar{q}_k) - u(q_k,\bar{q}_0)\right]
\end{aligned}$$

Alternatively,

$$\begin{aligned}
x_i(\mathbf{q},\bar{\mathbf{q}}) &= \frac{C\left(\breve{Q}^i\right)}{i} - \sum_{k=i+1}^{n-1}\frac{C\left(\breve{Q}^k\right)}{k(k-1)} \\
&\quad + \left[u(q_i,\bar{q}_i) - u(q_i,\bar{q}_0)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u(q_k,\bar{q}_k) - u(q_k,\bar{q}_0)\right]
\end{aligned}$$

where $\breve{Q}^k = kq_k + \sum_{l=k+1}^{n} q_l$ for all $k = 1,...,n$.

Let

$$\begin{aligned}\breve{Q}(q) &= N(q)\,q + \int_{z=q}^{\infty} zn(z)\,dz \\ &= \int_{z=0}^{\infty} \sup\{q,z\} n(z)\,dz\end{aligned}$$

Notice that

$$\frac{d\breve{Q}(q)}{dq} = N(q)$$

and define $\breve{Q}_{\sup} \equiv \breve{Q}(\sup \mathbf{q})$

$$\breve{Q}_{\sup} = N \sup \mathbf{q}$$

Therefore,

$$\begin{aligned}x^{DSEE}(q,s) &= \frac{1}{N} C\left(\breve{Q}_{\sup}\right) - \int_{z=q}^{\infty} \frac{1}{N(z)} C'\left(\breve{Q}(z)\right) \frac{d\breve{Q}(z)}{dz} dz \\ &\quad + \left[u(q,\bar{q}_s) - u(q,\bar{q}_0)\right] - \frac{1}{N}\sum_{t\in S}\int_{z=0}^{\infty}\left[u(z,\bar{q}_t) - u(z,\bar{q}_0)\right] n_t(z)\,dz \\ &= \frac{1}{N} C\left(\breve{Q}_{\sup}\right) - \int_{z=q}^{\sup \mathbf{q}} C'\left(\breve{Q}(z)\right) dz \\ &\quad + \left[u(q,\bar{q}_s) - u(q,\bar{q}_0)\right] - \frac{1}{N}\sum_{t\in S}\int_{z=0}^{\infty}\left[u(z,\bar{q}_t) - u(z,\bar{q}_0)\right] n_t(z)\,dz\end{aligned}$$

With $C(Q) = F + cQ$, the cost-sharing component becomes:

$$\begin{aligned}\frac{1}{N} C\left(\breve{Q}_{\sup}\right) - \int_{z=q}^{\sup \mathbf{q}} C'\left(\breve{Q}(z)\right) dz &= \frac{F}{N} + c \sup \mathbf{q} - c\int_{z=q}^{\sup \mathbf{q}} dz \\ &= \frac{F}{N} + cq.\end{aligned}$$

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