Well-Being Poverty and Labor Income Taxation: Theory and Application to Europe and the US*

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November 2015

Abstract

We study a model in which agents differ in their productive skills and their preferences over labor time/consumption bundles. We assume there is a poverty line, that is, there is a minimal level of consumption below which society finds it unacceptable to let people live. To avoid conflict with individual well-being, we capture the anti-poverty project be requiring redistribution to take place between agents on both sides of the poverty line *provided they have the same labor time*. We combine this requirement with efficiency and robustness requirements to derive social preferences. Maximizing these preferences under incentive compatibility constraints yields the following evaluation criterion: labor income tax schemes should minimize the labor time required to reach the poverty line. We apply this criterion to tax schemes of European countries and the US.

JEL Classification: D63, I32.

Keywords: well-being, poverty, labor income taxation.

*We thank Marc Fleurbaey for helpful comments and suggestions. The research leading to these results has received funding from the European Research Council under the European Union's Seventh Framework Programme (FP7/2007-2013) / ERC grant agreement n° 269831.

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1 Introduction

Eradicating poverty is a social objective that many embrace. In developed societies, this objective often requires that all incomes be above some threshold, the so-called poverty line. The US and all EU states, for instance, publish official poverty lines and official poverty rates, defined as the fraction of the population with incomes below the poverty line.[1] The EU Horizon 2020 objectives include a decrease in the number of people at risk-of-poverty, that is, having a disposable income below their national poverty line.

With an objective of poverty reduction stated in terms of poverty in income, tax and transfer systems are evaluated on the basis of the distribution of incomes they generate, and, in particular, on the distribution of incomes below the poverty line. This creates two difficulties.

The first difficulty is that focusing on income disregards the labor time it takes to agents to earn it, and labor time is of course a determinant of well-being. When increasing income from below to above the poverty line goes together with an increase in the labor time, anti-poverty policies may decrease the well-being of the income-poor.

This tension between fighting against income poverty and increasing well-being (of the poor) is reflected in the optimal income tax literature. Indeed, when it comes to design tax and transfer systems that would optimally alleviate poverty, it is typically assumed that the objective of the planner consists in leading agents to choose bundles that allow them to consume more than the poverty line, independently of the labor time of those bundles, and, consequently, independently of whether those agents gain or lose, in terms of well-being, from the fight against poverty. To phrase it differently, the tax and transfer systems which are proven optimal to alleviate poverty distort the opportunities offered to poor (that is, typically, low productivity) agents so as to incentivize them to earn more but the loss in opportunities may be

[1] Ideally, a poverty line should be defined in terms of the financial resources available to people for consumption. The official US poverty line was defined in pre-tax income and was criticized for that reason (and many others). See Citro and Michael (1995) and Iceland (2005) for accounts of those criticisms. The Supplemental Poverty Measure, published by the Census Bureau since 2011, is defined in a much closer way to disposable income. All EU official poverty lines are defined on terms of disposable incomes. Of course, there are many ways of defining disposable incomes, as a function of whether in kind or indirect financial help are considered as income, and whether a series of expenses are considered consumption or taxes. See Meyer and Sullivan 2012 for a detailed account of the many issues raised by the definition of the poverty line.

accompanied with a loss in well-being.[2]

The following simplified example illustrates this first difficulty. Let us assume that two tax and transfer systems are possible. The poverty line is at 11 and the minimal wage is equal to 10. In the first policy, the poorest agent works half-time, thereby earning a pre-tax income of 5, obtains a transfer of 2 and earns a disposable income of 7. In the second policy, that agent works full time, earning 10, and obtains a transfer of 1, so that her disposable income is equal to 11. Finally, let us assume that the agent strictly prefers to work half-time and get 7 over working full-time and getting 11. The first policy is the one that maximizes the well-being of the poor agent, whereas the second policy is the only one that allows the poor agent to reach the poverty line. This illustrates the tension between alleviating income poverty and increasing the well-being of the poor. Also note that the transfer needed to implement the first policy is higher than the transfer of the second policy, which means that the latter is less costly than the former in terms of well-being of the other, non-poor, agents. The cost of redistribution, which is at the heart of optimal income taxation theory, will also be at the heard of this paper.

That a decrease in income poverty can be accompanied with a decrease in the well-being of the poor has been noted since long (see, for instance, Kanbur, Keen and Tuomala, 1994a and b, and Wane, 2001). A similar difficulty also arises when poverty is defined in terms of commodity deprivation instead of lack of income (see Pirttila and Tuomala, 2004).[3] Kanbur, Keen and Tuomala (1994b) suggested a solution to that problem, that is, they suggested to compare the poverty line not with the actual income of an agent

[2]Those distortions lead to opportunity sets that typically fail to satisfy the properties derived by optimal tax theory. Kanbur, Keen and Tuomala (1994a), for instance, conclude that income tax rates should be negative on low incomes when all agents have a positive labor time (and there is no bunching at $y = 0$), in contradiction to the classical results (summarized, for instance, in Diamond, 1998). Similarly, Pirttila and Tuomala (2004) conclude that commodity taxation should not be uniform even if preferences are separable in leisure and goods, against the classical theorem of Atkinson and Stiglitz (1976).

[3]In Besley and Coate (1992), on the contrary, poor agents never suffer from the objective of the planner to decrease poverty. This comes from the fact that labor income taxation is not a tool in the hands of the planner. Instead, she has a fixed budget to allocate and she may require agents to work in exchange of a benefit. Compared with the results above on optimal taxation, this shows that assuming an exogenous budget and disregarding the possible distortions associated with collecting money through taxation assumes away some important incentive issues.

but with her equivalent income. The equivalent income, a concept introduced by Samuelson (1974) and Samuelson and Swamy (1974), is the income level that, given some fixed reference wage rate, would leave the agent indifferent with her actual situation. Kanbur, Keen and Tuomala do not give any result, however, and they limit themselves at mentioning the arbitrariness of the choice of the reference wage rate.

The second, and related, difficulty is that an agent's income is jointly determined by two ingredients. First, it depends on the opportunity set that is offered to that agent, which in turns depends on the tax and transfer system applying to that agent. Second, the income depends on the agent's choice, typically her labor time.

The evaluation of the ability of tax and transfer system to fight against poverty, being carried out on the basis of the statistical distribution of incomes, mixes the two ingredients and is, therefore, mistaken. Two systems offering the same opportunities to their agents should receive the same praise and blame, independently of the choices of these agents.

In this paper, we propose an anti-poverty requirement that does not conflict with individual well-being. Our strategy consists in dropping the classical objective of income poverty reduction and in constructing new social preferences. We do so under the assumption that agents may differ not only in their productivity, like in the papers cited above, but also in their preferences, so that the same opportunity set may lead different agents to different income levels. We also consider that different economies may have different profiles of preferences, so that the same tax and transfer system may yield different income distributions.

The key property we require from social preferences is that a transfer from an agent above the poverty line to another agent below the line be a social improvement only if both agents have the same labor time. This requirement turns out to be compatible with the property of Pareto efficiency. Our first result consists in proving that, together with an auxiliary robustness property, these two properties characterize unique social preferences.

Those social preferences are egalitarian in a specific well-being index representing the preferences of the agents. The well-being of an agent is computed as the labor time that, associated with a consumption level equal to the poverty line, leaves the agent indifferent to her actual labor time-consumption bundle. This well-being index does not belong to the family of equivalent incomes and does not require the arbitrary choice of any reference price. On the other hand, it depends on the value of the poverty line, which we take as

exogenous.

Next, we turn to the optimal tax exercise, and we study the consequence of maximizing the social preferences we have characterized under incentive compatibility constraints. These constraints are satisfied when agents choose their labor time given a tax function that determines consumption as a function of earnings. Our main result is the construction of a criterion that can be used to evaluate income tax functions. According to that criterion, income tax should minimize the pre-tax earning level that is required to reach an after-tax income equal to the poverty line.

This criterion allows us to evaluate tax schemes, independently of the distribution of incomes it generates. That is, only the opportunities offered to agents are considered. Of course, the opportunity sets we end up evaluating are two-dimensional budgets. Those budgets are not necessarily nested, so that set inclusion is insufficient to rank them. Our criterion plays the role of a one number summary of the opportunities associated with budgets. It may seem too simple to reduce budget sets to a number, but it it the well-known price to pay to obtain a complete ranking of them, and, as a consequence, a complete ranking of the tax systems.

We apply this criterion to evaluate the US and European labor income tax schemes. Those schemes are derived based on simulations using the OECD Tax-Benefit Calculator. The simplicity of our criterion allows us to identify the fiscal reforms that will increase social welfare in each of the considered countries. As a by-product of this analysis, we are able to compare countries and to identify those with better tax schemes according to our criterion. It turns out that the countries that offer better opportunities to the poor agents according to our criterion are not those that transfer the largest benefits to the agents who do not work at all. We also identify how each national tax scheme treats different households differently.

The paper is organized as follows. In Section 2, we present the model. In Section 3, we introduce our key property of poverty reduction that does not conflict with individual well-being, and we characterize social preferences. In Section 4, we move to a second-best context and we derive the evaluation criterion of tax functions. In Section 5, we use this criterion to evaluate US and European labor income tax schemes. In Section 6, we give some concluding comments.

2 The model

There are two goods, labor, denote ℓ, and consumption, denoted c. The population contains n agents. That means that there is a finite number of agents but we think of that number as a large one, so that the population is diverse in the relevant characteristics. A *bundle* for agent $i \in \{1, \ldots, n\}$ is a pair $z_i = (\ell_i, c_i)$, where ℓ_i is agent i's labor and c_i her good consumption. When we come to the second-best result and the application, the level of consumption will be the after-tax income of the agent. The agents' identical *consumption set* X is defined by the conditions $0 \leq \ell_i \leq 1$ and $c_i \geq 0$.

Agents have two characteristics, their preferences over the consumption set and their productivity. For any agent $i \in \{1, \ldots, n\}$, *preferences* are denoted R_i, and $z_i R_i z_i'$ (resp. $z_i P_i z_i'$, $z_i I_i z_i'$) means that bundle z_i is weakly preferred (resp. strictly preferred, indifferent) to bundle z_i'. We assume that individual preferences are continuous, convex and monotonic.[4] We further assume that consumption is necessary, in the sense that any bundle with a positive good consumption is always strictly preferred to any bundle with a zero consumption. This assumption will play a role during the construction of the social preferences. We let $\mathcal{R}$ denote this set of preferences.

The marginal productivity of labor is assumed to be fixed, as in a constant returns to scale technology. Agent i's earning ability is measured by her productivity or *wage rate*, denoted w_i, and is measured in consumption units, so that $w_i \geq 0$ is agent i's production when working $\ell_i = 1$ and, for any ℓ_i, $w_i \ell_i$ is the agent's pre-tax income (earnings). We assume that $w_i \in [w_{\min}, \infty)$, where $w_{\min}$ stands for the minimal wage rate. We also assume that there are agents in the economy whose wage equals the minimal-wage: $w_i = w_m$ for some $i \in \{1, \ldots, n\}$. Our notion of a minimal wage rate can either refer to a legal minimal wage or to the minimal statistical was rate. In the application below, we restrict our attention to the many countries having a legal minimal wage rate.

Fig. 1 displays the consumption set, with typical indifference curves, and earnings as a function of labor time. As illustrated on the figure, an agent's consumption c_i may differ from her earnings $w_i \ell_i$. This is a typical consequence of redistribution.

An allocation is a list $z = (z_1, ..., z_n)$. Social preferences will allow us to compare allocations in terms of fairness and efficiency. Social preferences

[4] Preferences are monotonic if $\ell_i \leq \ell_i'$ and $c_i > c_i'$ implies that $(\ell_i, c_i) P_i (\ell_i', c_i')$.

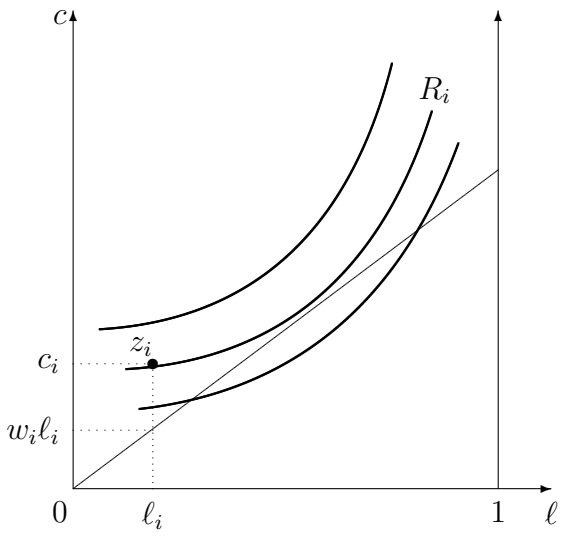


Figure 1: Illustration of the model

will be formalized as a complete ordering over all allocations in X^n, and will be denoted R, with asymmetric and symmetric components P and I, respectively. In other words, $z\ R\ z'$ (resp. $z\ P\ z'$, $z\ I\ z'$) means that z is at least as good as z', (resp. strictly better, equivalent).

Social preferences may depend on the population profile of characteristics $(R_1, ..., R_n)$ and $(w_1, ..., w_n)$. However, we may simplify the analysis and consider that the profile of wage rates is fixed. As a result, social preferences R are formally a *mapping* from the set of population profiles $\mathcal{R}^n$ to the set of complete orderings over allocations.

3 Social preferences

We assume that there is a poverty line, that is, a consumption level, p, with the property that society considers it unacceptable to let people live with less consumption than p. There are several ways of capturing the idea that society is ready to fight against poverty.

We now treat the discussion carried out in the introduction more formally. There are two pitfalls that the objective of poverty reduction should avoid. They are illustrated in Fig. 2. There are four bundles, z_1, z_2, z_3 and z_4 and

two agents, j and k having identical preferences $R_j = R_k$.

First, let us compare z_2 and z_3. We have $c_2 < p < c_3$, so that we could claim that assigning z_3 to j or k is socially preferable to assigning them z_2. We also have that $z_2 \; P_j \; z_3$ (and $z_2 \; P_k \; z_3$), though, so a social preference for z_3 over z_2 would immediately conflict with individual well-being and, therefore, the Pareto criterion.

Second, let us consider the allocation at which j consumes z_1 and k consumes z_4. Again, we might consider that j is poor, because $c_1 < p$, whereas k is non-poor, because $c_4 > p$. A transfer of income from k to j could then be thought of as a social improvement, in seeming line with the celebrated Pigou-Dalton progressive transfers in the literature on inequality measurement (see Pigou, 1912, and Dalton, 1920). Contrary to the first pitfall, such a transfer would not conflict with Pareto efficiency because one agent gains and one loses. The transfer is not desirable, though, because it amounts to exacerbate inequality rather than decrease it. Indeed, in well-being terms, agent k at z_4 is worst-off than agent j at z_1, in the sense that z_4 lies on a lower indifference curve than z_1 (this statement only makes sense because they both have the same preferences) and well-being inequality is larger at (z_2, z_3) than at (z_1, z_4).

To avoid these two pitfalls, we look for social preferences that satisfy both the Pareto criterion and the following *poverty reduction* property, requiring transfers to take place only under the proviso that the labor time of the two agents are the same. Note that this proviso guarantees that both agents agree that the bundle assigned to the agent whose consumption level is above the poverty line is better than the bundle assigned to the agent whose consumption level is below the poverty line.

This requirement is illustrated in Fig. 3. We state it formally.

Property 1 Poverty reduction
For all economy $(R_1, ..., R_n)$, for all pairs of allocations $z = (z_1, ..., z_n)$ and $z' = (z'_1, ..., z'_n)$, if, for two agents j and k and a positive quantity Δ

$$\ell_j = \ell'_j = \ell_k = \ell'_k, \tag{1}$$
$$c'_j = c_j + \Delta \leq p \leq c'_k = c_k - \Delta, \tag{2}$$

whereas $z_i = z'_i$ for all other agents, then z' is socially strictly preferred to z.

Here are three examples of social preferences that satisfy *poverty reduction*. The first example is the social preference relation based on the cele-

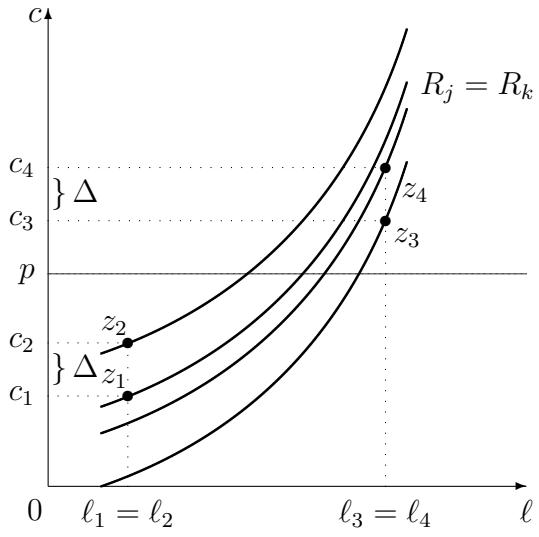


Figure 2: Individual well-being versus the objective of consumption poverty reduction

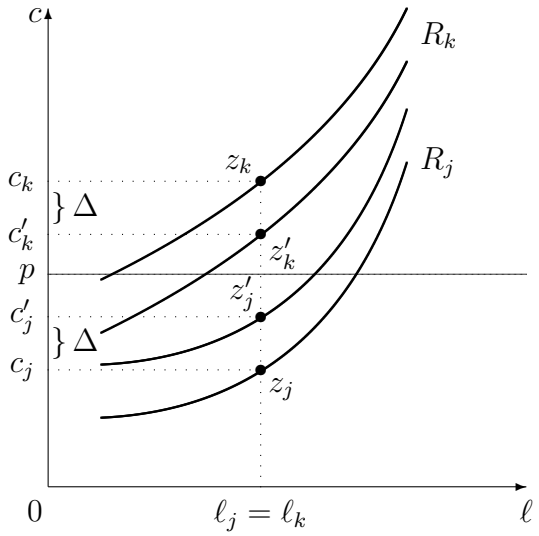


Figure 3: The property of *poverty reduction*: (z'_j, z'_k) is socially preferred to (z_j, z_k)

brated income poverty measurement introduced by Foster, Greer and Thorbecke (1984). We denote that social preference by R^{FGT}. It is formally defined as follows: For all economy $(R_1, ..., R_n)$, for all pairs of allocations $z = (z_1, ..., z_n)$ and $z' = (z'_1, ..., z'_n)$, z is socially weakly preferred to z' if and only if there is less poverty at z than at z', that is,

$$\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\max\{0, p - c_i\}}{p}\right)^{\alpha} \leq \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\max\{0, p - c'_i\}}{p}\right)^{\alpha}.$$

Social poverty is defined as the sum of individual poverty. Any agent consuming more than the poverty line has a poverty level equal to 0. An agent consuming nothing has a poverty level of 1. The α coefficient stands for the degree of inequality aversion among the poor, which amounts to the priority that is given to people at the very bottom of the consumption spectrum. If $\alpha = 0$, all poor agents contribute the same level to global poverty. It amounts to give priority to agents very close to the poverty line, as giving one euro to an agent so close to the poverty line that this agent quits poverty as a consequence of the transfer decreases poverty more than giving one euro to a very poor agent. This is the so-called headcount ratio. If $\alpha = 1$, transferring one euro to a poor agent decreases poverty the same way independently of the consumption level of this agent. This is the so-called poverty gap ratio, measuring the average share of p that needs to be transferred to poor agent to completely alleviate poverty. If $\alpha > 1$, transferring one euro to a poor agent decreases poverty more the poorer this agent. As soon as $\alpha > 0$, all social preferences R^{FGT} satisfy the property of *poverty reduction*.

Another example of social preferences that satisfy this property is the generalized utilitarian social welfare function that is often used in optimal taxation theory under the assumption that preferences are quasi-linear (see, for instance, Diamond 1998). We denote that social preference by R^U. It is formally defined as follows: for all economy $(R_1, ..., R_n)$ in which all agents have (possibly different) quasi-linear preferences represented by quasi-linear utility functions $(u_1, ..., u_n)$, for all pairs of allocations $z = (z_1, ..., z_n)$ and $z' = (z'_1, ..., z'_n)$, z is socially weakly preferred to z' if and only if

$$\sum_{i=1}^{n} g(u_i(z_i)) \geq \sum_{i=1}^{n} g(u_i(z'_i)),$$

where g is a strictly concave and strictly increasing real-valued function representing the inequality aversion of the planner. Let us observe that when

$\ell_j = \ell'_j = \ell_k = \ell'_k$ and $c'_j = c_j + \Delta < c'_k = c_k - \Delta$, quasi-linearity implies that $u_j(z'_j) - u_j(z_j) = \Delta = u_k(z_k) - u_k(z'_k)$. The fact that R^U satisfies *poverty reduction* comes then from the strict concavity of g.

A last example is a new social preference relation that we introduce in this paper. It works by applying the leximin aggregator to some particular well-being representation of individual preferences. We denote that social preference by R^{lex}. The leximin aggregator works by lexicographically applying the maximin aggregator: first maximize the well-being of the worst-off, in case of a tie, maximize the well-being of the second worst-off, and so on. The new representation of the preferences works as follows. It is a decreasing function of the labor time that leaves an agent indifferent between her current bundle and consuming the poverty line p at that labor time. Formally, the well-being of an agent having preferences R_i and consuming bundle $z_i = (\ell_i, c_i)$, denoted $W^p(z_i, R_i)$, is equal to $-w$ if this agent is indifferent between z_i and (w, p). It is illustrated in Fig. 4. Note that, given this construction, $w \in [0, 1]$, but the only relevant characteristic of that well-being measure is to be decreasing in labor time. How decreasing does not matter because we apply the leximin aggregator, which only uses the ordinal information on well-being levels.

There are two cases in which no w satisfying the indifference condition above exists. the first case is when $z_i \, P_i \, (0, p)$, that is, the agent strictly prefers her bundle over not working at all and consuming the poverty line. In this case, there is a consumption level c such that $z_i \, I_i \, (0, c)$. We fix the well-being at such a bundle equal to $c - p$.

The second case is when $(1, p) \, P_i \, z_i$, that is, the current bundle of this agent is so bad that she would prefer to work full-time and consume exactly the poverty line.In this case, there is a consumption level c such that $z_i \, I_i \, (1, c)$. We fix the well-being at such a bundle equal to $c - p - 1$.

We now check which social preferences satisfy the other properties we are interested in. Our first property is the classical Pareto property, which we now formally define. It guarantees that decreasing the preference satisfaction of an agent will never be a social improvement, even if this decrease goes together with an increase in income above the poverty line.

Property 2 PARETO
For all economy $(R_1, ..., R_n)$, for all pairs of allocations $z = (z_1, ..., z_n)$ and $z' = (z'_1, ..., z'_n)$, if all agents i weakly prefer z_i to z'_i, then z is socially weakly preferred to z'. If, moreover, z_j is strictly preferred to z'_j for one agent j,

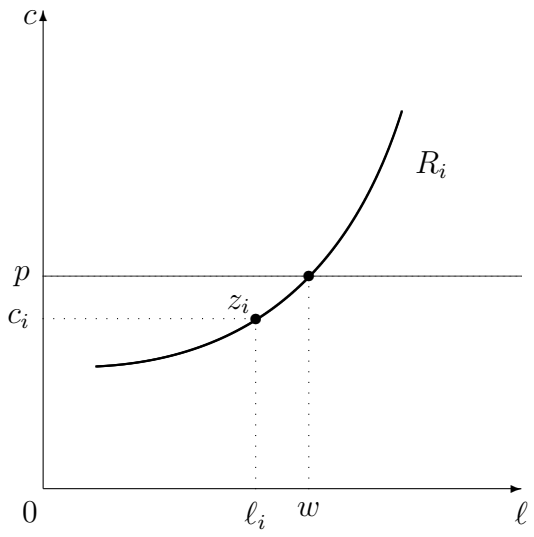


Figure 4: Illustration of the well-being measure W^p: $W^p(z_i, R_i) = -w$.

then z is socially strictly preferred to z'.

R^{FGT} does not satisfy Pareto. It should not be a surprise, because R^{FGT} only aggregates consumption levels and remains insensitive to increases in labor times.

On the other hand, R^U satisfies Pareto, which follows from the fact that these social preferences are directly defined as a function of the utility levels of the agents. These preferences R^U are defined for economies in which preferences are quasi-linear. We would like to be more general than that, and be able to define social preferences even when there are income effects. We need to generalize R^U to all preferences. Yet, we would like social preferences in non quasi-linear economies to be consistent with that in quasi-linear ones. We capture this requirement with the following property. It requires that social preferences be independent to changes in preferences that do not affect the indifference curves through the bundles we are contemplating. This is a cross economy robustness property, and it explains why we wanted to define the domain of economies as a function of all possible preference profiles in the economy. This is a way to make our conclusions independent of the precise profile of the preferences in the economy, for instance of the fact that all agents have quasi-linear preferences.

There are many ways in which *independence* can be justified. It is a weakening of Arrow's independence property, and a weakening that makes it compatible with fairness properties, as Samuelson (1977) and Pazner (1979) already mentioned. This *independence* property is also related to incentive compatibility. We postpone the discussion to the next section.

To define that property formally, we need the following terminology. For some preference relation $R_i \in \mathcal{R}$ and some bundle z_i, we let $I(z_i, R_i)$ denote the indifference curve at z_i, that is, the set of all bundles to which this agent is indifferent.

Property 3 Independence
For all pairs of economies $(R_1, ..., R_n)$ and $(R'_1, ..., R'_n)$, and all pairs of allocations $z = (z_1, ..., z_n)$ and $z' = (z'_1, ..., z'_n)$, if all agents have the same indifference curves at z_i and z'_i with preferences R_i and R'_i, that is, if $I(z_i, R_i) = I(z_i, R'_i)$ and $I(z'_i, R_i) = I(z'_i, R'_i)$ for all $i \in \{1, \ldots, n\}$, then the social preference towards z and z' is the same in both economies.

R^{FGT} satisfies this property, for all values of α. It immediately comes from the fact that R^{FGT} is defined without reference to preferences. It is therefore independent to all changes in preferences, including the ones that leave indifference curves unaffected. Unfortunately, no social preference can generalize R^U to non quasi-linear economies so as to satisfy that property. This will come as a corollary of our first result. The social preference relation that we have introduced above, R^{lex}, satisfies all three properties. Moreover, any social preference that satisfies all three properties needs to maximin the well-being measure W^p. That almost amounts to say that R^{lex} is the only social preference that satisfies the three properties.

Proposition 1 *Social preferences R^{lex} satisfy* poverty reduction, Pareto *and* independence. *Conversely, if social preferences R satisfy* poverty reduction, Pareto *and* independence, *then for all economy $(R_1, ..., R_n)$, for all pairs of allocations $z = (z_1, ..., z_n)$ and $z' = (z'_1, ..., z'_n)$, if*

$$\min_i W^p(z_i) > \min_i W^p(z'_i),$$

allocation z is socially strictly preferred to z'.

The formal proof of the proposition is similar to the proof of the main result

in Maniquet and Sprumont (2004) in a public good model.[5] We provide a complete proof in the Appendix.

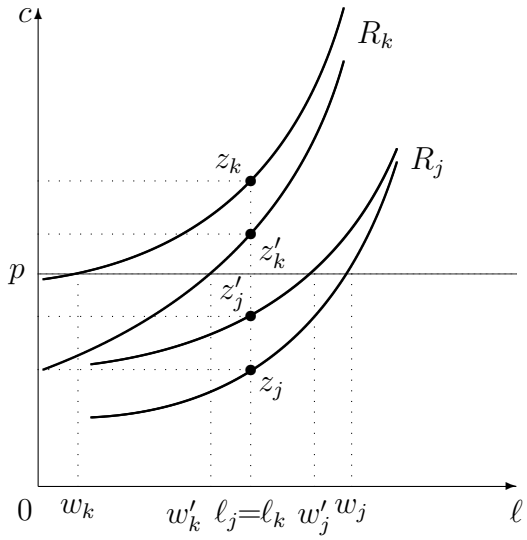


Figure 5: Illustration of why social preferences need to be maximin

Two aspects of Proposition 1 are unexpected. First, it tells us that we need to maximize the minimal well-being level, that is, we need to apply an infinite degree of inequality aversion, whereas the property of *poverty reduction* is nothing more than a version of the transfer principle, which is compatible with any degree of inequality aversion. The intuition of this result is not difficult to grasp, though. Let us look at Fig. 5, a variant of Fig. 3. The transfer from k to j needs to be declared a social improvement. In terms of well-being, however (that is, in terms of the labor time it takes to reach the poverty line along those indifference curves), we see that receiving $z'_j - z_j$ increases agent j's well-being by $w_j - w'_j$, whereas losing $z_k - z'_k$ decreases agent k's well-being by $w'_k - w_k$, a much larger amount. One can easily imagine that the well-being increase for agent j is arbitrarily small, whereas the corresponding decrease for agent k is arbitrarily large. An arbitrarily small increase of the poorer agent has to compensate an arbitrarily large decrease for the relatively richer one. This can only be achieved with a

[5]The main differences between the two models are that labor time is bounded above and consumption is bounded below in the model of this paper.

maximin objective.[6]

Second, Proposition 1 tells us to measure individual well-being according to W^p, which is basically expressed in labor time, whereas *poverty reduction* is a property of money transfer. Again, the intuition can be deduced from Fig. 5. In order to satisfy *poverty reduction* with egalitarian social preferences, one needs to be sure that agent j is considered poorer (that is, at a lower well-being level) at z_j than k at z_k, whatever their preferences R_j and R_k. As suggested in the picture, the indifference curve of these two agents may cross each other. Moreover, they may cross at the vertical of any point either left of $l_j = l_k$, or right. Consequently, the only certainty one can have about the indifference curves of agent j at z_j and of agent k at z_k is that the former crosses the poverty line at a larger labor time than $\ell_j = \ell_k$, whereas the latter crosses it at a smaller labor time. This is why well-being is measured in labor time along the poverty line, and agent k is claimed to have a larger well-being at z_k than agent j at z_j.

4 Second best

In the previous, section, the objective was to construct a complete ranking of allocations, independently of the information that was needed to rank them. In this section, we introduce the information constraints facing the redistribution designer, that is, we switch to the classical second-best context, whose formalism dates back to Mirrlees (1971). We assume that only earned income $y_i = w_i\ell_i$ is observed, so that redistribution is made via a tax function $\tau(y_i)$.[7] This tax is a subsidy when $\tau(y_i) < 0$. In this context, meeting the incentive compatibility constraints is equivalent to letting individuals choose their labor time in the budget set modified by the tax schedule. Each agent i chooses ℓ_i knowing that it yields a consumption level $w_i\ell_i - \tau(w_i\ell_i)$.

Of course, the optimal tax functions are to be found among those that allow all agents to consume more than the poverty line, that is, those for which $y - \tau(y) \geq p$ for all y, so that, in particular, $-\tau(0) \geq p$. In most economies, though, implementing such tax schemes is budgetary (and, sometimes, politically) too costly. This is why a criterion needs to be developed

[6]Maximin results are frequent in the literature on social preferences based on fairness properties. See Fleurbaey and Maniquet (2011) for a detailed presentation.

[7]See e.g. Stiglitz (1987), p. 1002-1004, or Boadway and Keen (2000), p. 737-738, for simple presentations of this second-best setting.

to evaluate tax functions that do not succeed in bringing everybody above the poverty line. We derive this criterion now.

Given the information available to the tax function designer, the relevant space becomes the earnings-consumption space. Individual preferences in that space are denoted R_i^*, and they are derived from ordinary preferences over labor-consumption bundles by:

$$(y, c)\ R_i^*\ (y', c') \Leftrightarrow \left(\frac{y}{w_i}, c\right)\ R_i\ \left(\frac{y'}{w_i}, c'\right).$$

It is a notably difficult task to characterize the optimal tax function when agents differ in wage rates and in preferences (see Lehmann and Jacquet, 2015, for a recent solution to that problem). We escape the need of such heavy derivations here thanks to the leximin nature of our objective. Indeed, it is sufficient for our purpose to deduce which agent in the population has the lowest W^p well-being index, or, more precisely, it is sufficient to identify the earning level chosen by the agent with the lowest index.[8]

We derive our main result under the following assumption, which we discuss and justify at the end of the section. It requires that, whatever the tax function we are contemplating, there is at least one agent with the minimal wage rate who chooses the pre-tax income level that is just sufficient to reach a consumption level equal to the poverty line, provided such a pre-tax income level exists. If no such pre-tax income level exists, because after-tax incomes are too low, then there is at least one minimal wage agent working full time.

Assumption 1 *For each τ, if there exists $y^p \leq w_m$ such that $y^p - \tau(y^p) = p$, then there exists $i \in \{1, \ldots, n\}$ such that $w_i = w_m$ and $y_i = y^p$. If $y - \tau(y) < p$ for all $y \leq w_m$, then there exists $i \in \{1, \ldots, n\}$ such that $y_i = w_m$.*

The argument leading to the result is illustrated in Fig. 6. Let us assume that τ is such that there exists $y^p \leq w_m$ such that

$$y^p - \tau(y^p) = p. \tag{3}$$

First, let us restrict our attention to minimal-wage agents, that is, agents with $w_i = w_m$. In the figure, if $w_2 = w_m$, then $W^p(z_2, R_2) = -\frac{y^p}{w_m}$. This

[8] A similar simplification due to the leximin nature of the social objective is used in Fleurbaey and Maniquet (2006).

type of agent always exists, by assumption 1. The picture describes the entire indifference curve through bundle (y^p, p), but we need not know the entire shape of the curve. Knowing that this agent has chosen y^p is enough to compute $W^p(z_2, R_2)$. Also, if $w_1 = w_m$, $W^p(z_1, R_1) > -\frac{y^p}{w_m}$. Indeed, choosing an earning level, say y_1, below y^p reveals a preference for $(y_1, y_1 - \tau(y_1))$ over (y^p, p). If $w_3 = w_m$, $W^p(z_3, R_3) > -\frac{y^p}{w_m}$ as well, by the same revealed preference argument. That proves that any minimal-wage agent choosing an earning level different from y^p has a higher $W^p(z_i, R_i)$ than agent 2. This agent has the lowest W^p index among the minimal-wage agent. Second let us consider agents with $w_i > w_m$. If $w_2 > w_m$, then, $W^p(z_2, R_2) = -\frac{y^p}{w_2} > -\frac{y^p}{w_m}$. Again, applying the same revealed preference argument as above, if $w_1 > w_m$, $W^p(z_1, R_1) > -\frac{y^p}{w_1} > -\frac{y^p}{w_m}$, and if $w_3 > w_m$, $W^p(z_3, R_3) > -\frac{y^p}{w_3} > -\frac{y^p}{w_m}$. That proves that the lowest $W^p(z_i, R_i)$ is the one of the minimal-wage agent earning y^p.

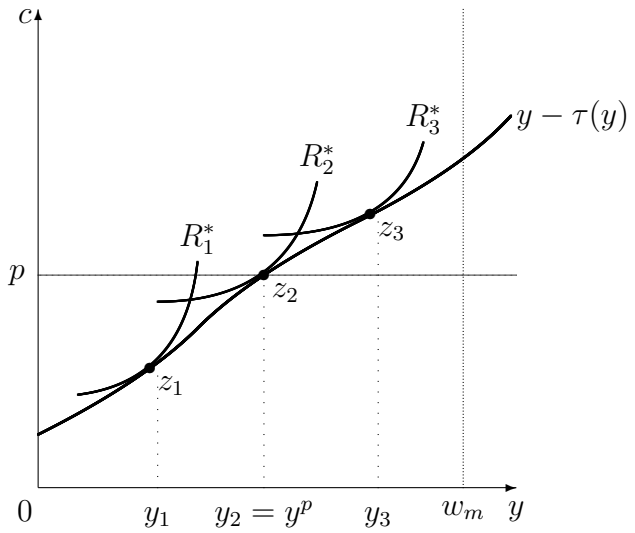


Figure 6: Illustration of Proposition 2

Let us assume, now, that no y^p satisfying Eq. 3 exists. Then, either $-\tau(0) > p$ or $w_m - \tau(w_m) < p$. In the former case, the well-being index of an agent who does not work is $-\tau(0) - p > 0$. Agents having a strictly positive earning reveal strict preference of their bundle over $(0, -\tau(0))$ so that their well-being index satisfies $W^p(z_i, R_i) > -\tau(0) - p$. This means that the lowest well-being index has a strictly positive value so that this allocation is socially

preferred to any allocation in which a y^p satisfying Eq. 3 exists. In the latter case, because assumption 1 guarantees the existence of some agent i such that $w_i = w_m$ and $y_i = w_m$, we have $W^p(z_i, R_i) = w_m - \tau(w_m) - p - 1 < -1$, that is, the lowest well-being index has a strictly lower value than -1, so that this allocation is socially worse than any allocation in which a y^p satisfying Eq. 3 exists.

We can summarize the result in the following proposition.

Proposition 2 *Let τ and τ' be two tax functions. Under assumption 1, the allocation generated by τ is socially better than that generated by τ' if one of the following conditions hold:*

1. *all agents have an after-tax income above the poverty line in the allocation generated by τ but not by τ',*
2. *the earning level that is just necessary to obtain an after-tax income equal to the poverty line is lower than the minimal wage and is lower in the allocation generated by τ than by τ',*
3. *the earning level that is just necessary to obtain an after-tax income equal to the poverty line is lower than the minimal wage in the allocation generated by τ but larger by τ'.*

Proposition 2 is illustrated in Fig. 7. Four tax functions are represented in the earning-after-tax income space. Tax τ^1 yields an allocation that is socially better than all the others, according to condition 1 in the Proposition. Condition 2 amounts to claiming that the allocation generated by τ^2 is socially better than the one generated by τ^3. Condition 3 amounts to claiming that τ^4 is the worse tax function among the four ones represented in the figure.

We have proven that the evaluation criterion of tax functions is the pre-tax income that is just necessary to reach a consumption level equal to the poverty line, because it reveals the labor time it takes to the minimal-wage agents to reach that level. Consequently, a reform of a tax function aiming at increasing social welfare should decrease the tax rate at this precise pre-tax income level.

We conclude the theory part of the paper with two comments, one on our third property, *independence*, and one on Assumption 1. First, the well-being index that proposition 1 teaches us to use gives us a numerical representation of preferences that only depends on the indifference curve through the

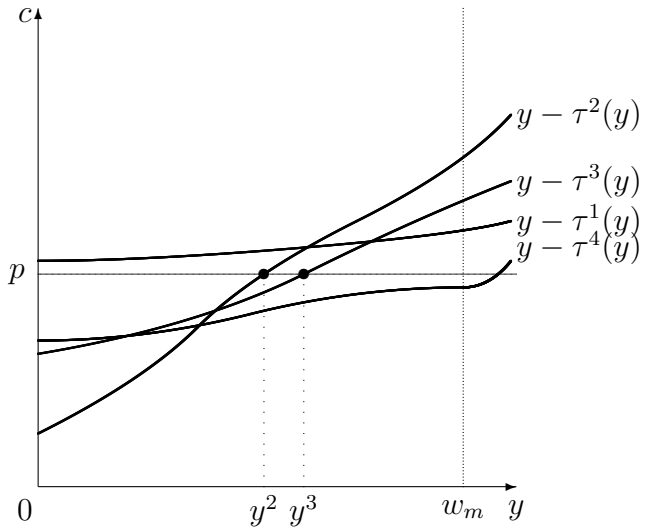


Figure 7: Illustration of Proposition 2

bundle we are contemplating. This immediately follows from *independence*, which prevents social judgements about bundle z_i to depend on preference information outside that difference curve. This is the reason why we succeed in deriving Proposition 2. Indeed, the only fact that one agent, agent 2 in the reasoning above, chooses to earn y^p reveals enough of her well-being index to help us conclude that she is the worst-off. With social preferences violating independence, we might have had first to check for preferences of agent 2 over bundles not contained in her indifference curve through her chosen bundle and such a check would have typically been impossible to do, because we cannot collect more information than agent 2's best bundle in her budget. It would not have been possible to derive a second-best criterion corresponding to social preferences requiring more information than indifference curves through the contemplated bundles. The surprise, actually, comes from the fact that *it is* possible to derive a criterion from R^{lex}. Indeed, agent 2's choice does even not reveal her entire indifference curve. Fortunately, the local information it reveals turns out to be sufficient to make the criterion work.

Second, Proposition 2 is derived under an unusual assumption that bears on both the types of the agents and the shape of the tax function. We discuss and justify it now. Let us assume that, contrary to the assumption,

no minimal-wage agent chooses earning level y^p. There can be two cases. Either no agent chooses y^p, or only higher-wage agent choose it.

In the first case, the part of the tax function around y^p is irrelevant. That means that τ can be adjusted, decreased, until it becomes relevant, that is, until some agents find it interesting to earn pre-tax incomes around y^p. This exercise can actually be done until all points of τ are relevant, that is, until it coincides with the lower envelop of agents' indifference curves through their chosen bundles. After τ is adjusted, either the assumption is satisfied, or we have reached the second case.

That is, let us assume that all agents earning y^p have a higher wage than w_m. In theory, it can be justified to increase the tax at y^p to collect more money and redistribute it towards worse-off agents. In practice, though, this is irrelevant, because it amounts to claim that the tax rate at some $y < w_m$ should be computed by the fiscal authority on the basis that this authority is certain that all agents earning that amount are high-skill agents. It does not sound like a plausible justification, because the typical information a fiscal authority has about the distribution of wages and earnings does not allow it to exclude the possibility that agents with a given wage work a given fraction of their time. We view Assumption 1, actually, as reflecting the imprecision of the available information, with the consequence that the tax designer cannot exclude the existence of an agent having the characteristics mentioned in the formal statement above.

5 Methodology: constructing budget curves with the OECD Tax-Benefit Calculator

In order to apply the criterion developed above, we need to draw the actual budgets that agents face. Given that the tax function typically depends on the composition of the households, we partition the population in household types. Budgets are drawn using the OECD Tax-Benefit Calculator which takes account of all relevant regulatory aspects that transform pre-tax incomes into after-tax incomes in OECD countries.

Our baseline scenario, in Section 6 below, consists in evaluating the ability of tax-benefit policies to alleviate poverty given the way countries themselves define poverty and define the parameters of the policies they use. That is, we will stick to the official poverty lines, the official notion of disposable income

and the legal minimal wage. As a result, we restrict our sample to the EU15 countries that have a legal minimum wage and the US and calculate budgets for tax-benefit rules of 2013, the most recent year available.

Some of the policies we are interested in are conditional. Given that the social preferences we apply are of the leximin type, our strategy is to try to draw the budgets of the poorest people. As a result, when a benefit is means-tested, we assume that the conditions are satisfied. When the conditions do not bear on means, we assume they are not satisfied. Precisely, the tax-benefit policies that are taken into account are income support and social assistance (SA)[9], family and child benefits (FB)[10], housing benefits (HB), in-work benefits (IW), labor income taxes (IT) and social insurance contributions (SC).[11] Unemployment *insurance* benefits are not taken into account as they are typically conditional on past labor force participation and social contributions. As a result, young or long-term unemployed people typically do not benefit from it. Unemployment *assistance* benefits, which are not based on previous contributions, are considered to be part of SA. Finally, because we want to evaluate tax-benefit policies as states define them, we assume full take-up of benefits and no fiscal evasion.

When taxes or benefits vary by region, the OECD Calculator chooses a region that it considers typical. For instance, the whole tax-benefit system of Michigan is used to represent the United States. The state of Michigan is used to represent a typical manufacturing region.

Our assumptions about household types are as follows. We consider six household types: singles and couples without children, with one child (aged 10), with two children (aged 10 and 12). All adults belong to the working age population and we assume away any specific needs for adults or children due to, e.g., disability, sickness or invalidity. The wage earned is the legal

[9]Eligibility to SA might be conditional on behavior, especially whether the individual is actively searching for work. It is assumed that all individuals fulfill all requirements for full social assistance benefits to be received. In some countries, additional benefits can be paid conditional on participation in active labour market programs. Such benefits are not taken into account.

[10]Childcare benefits for parents with children in externally provided childcare and the costs of that care can be modeled with the OECD Tax-Benefit Calculator but are not implemented here.

[11]Only personal income tax and employees' social security contributions payable in respect of earnings and benefits are included. Central, state and local government income taxes are included, but council tax in the United Kingdom is excluded. In general, only standard tax reliefs are included when calculating tax payments.

minimum wage in each country in 2013 (see column 5 in table 1). For single earners, disposable income is derived on the basis of the labor income that is earned when increasing hours worked from zero to full-time. Given that the OECD Tax Benefit Calculator can only assume a fixed number of hours worked for the second earner, we construct the budget curves for couples combining two different situations. First, we assume that the second earner does not work at all and the principal earner varies her hours from zero to full-time, such that the household earns once the minimal wage. Second, at this point the curve is continued under the assumption that the second earner works full-time and the principal earner again varies her hours from zero to full-time, such that the household earns twice the minimal wage.

Further assumptions are made within the OECD Tax-Benefit Calculator to implement the different policies. Due to lack of space, neither all those assumptions nor the detailed policies can be listed here. They are documented on the website of the OECD Tax-Benefit Calculator as well as in the country chapters of the OECD series "Benefits and Wages".[12] We nevertheless report a few main assumptions in the following.

All tax and benefit amounts relate to the year of 2013 and are generally computed using the rules and regulations that were in force on July 1, 2013. All taxes, benefits and net incomes are thus determined for a particular month. Benefits and income taxes, which depend on annual incomes, are determined in relation to the annualized amounts (i.e. multiplied by 12). Any time-lags delaying the assessment of claimants' entitlement or the payment of benefits are disregarded, as are differences in the timing of benefits.

Cash housing benefits are particularly complex and might depend on various characteristics such as the rent, the size of dwelling and the region, amongst several others. Given this complexity, it is not possible to derive typical or average HB for low-income households with the OECD Tax Benefit Calculator. Due to our income definition above, in our baseline scenario we nevertheless include HB and make the following simplified assumptions (amongst others): families live in privately rented accommodation; the level of rent for single (couple) households is 30% (40%) of full-time gross earnings (for both spouses), regardless of income level; where size is relevant and where housing benefits vary by region, we have sticked to the default choice of the OECD Calculator. In an alternative scenario presented in Section 7, we

[12] See http://www.oecd.org/els/benefits-and-wages-models.htm and http://www.oecd.org/social/benefits-and-wages.htm.

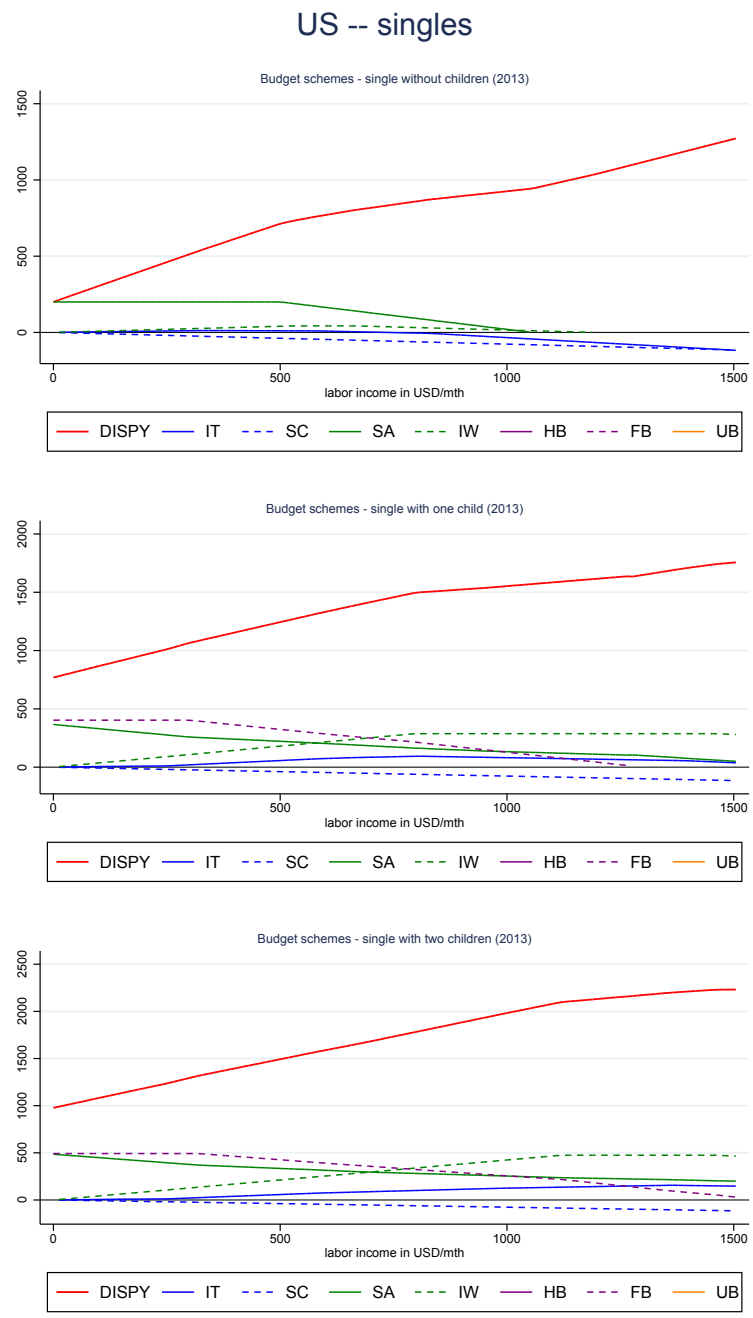
remove HB from the calculation of the schemes.

By construction of the policy, in-work benefits are paid only to those with earnings or those who have worked more than a certain number of hours per week. Thus, our assumptions made above about hours worked and wages earned determine the level of employment-conditional benefits. Delays in payment of benefit (which can be long for IW) are ignored.

Figures 8 and 9 show exemplarily for the United States, how budget curves for the different household types evolve as the sum of the different tax-benefit policies.[13]. SA mainly includes the Supplemental Nutrition Assistance Program (SNAP), formerly known as food stamps, which is designed primarily to increase the food purchasing power of eligible low-income households. As can be seen from the graphs, it is dependent on household size and there is an income threshold for receiving SNAP. IW refers to the Earned Income Tax Credit (EITC) which is a refundable tax credit. Eligible for EITC are working families with children under 24 and childless working persons aged between 25 and 65 that meet certain income thresholds. There is no direct tax-benefit instrument in the US to support rental payments, i.e. no HB. FB refer to the Temporary Assistance for Needy Families (TANF) which are granted to families in need. Each State may establish its own benefit levels and determine its own benefit calculation. Michigan TANF are increasing in the number of family members and are somewhat above the average for all states. TANF is income dependent and Michigan applies certain income disregards in the calculation of the benefit. Concerning IT, there are several tax reliefs applicable, especially related to children. In the graphs, this leads to a partly negative income tax scheme for households with children. In 2013, the rate for employee SC was 7.65 per cent.

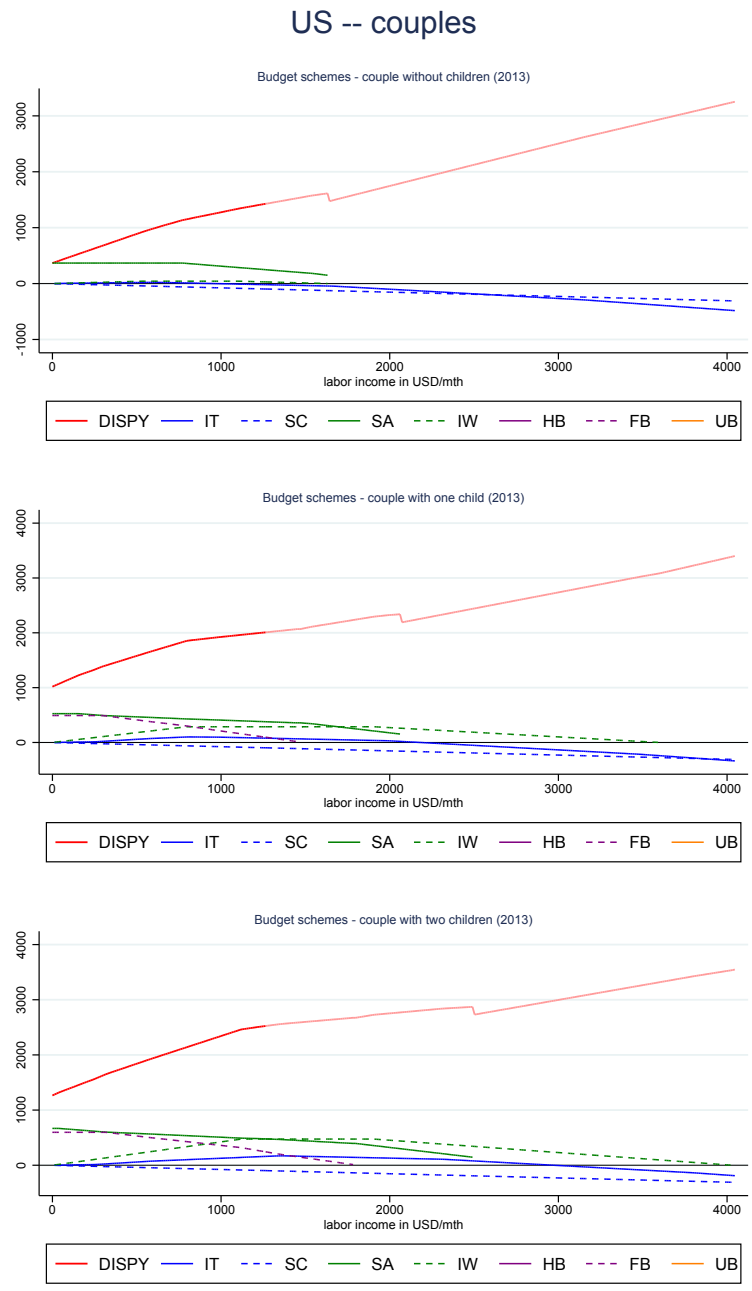
Finally, in order to evaluate budget schemes with respect to poverty alleviation, we have to introduce poverty thresholds. As the needs of a household increase with the number of its household members in a non-proportional way due to economies of scale in consumption, we will also have to adjust poverty thresholds in an appropriate way to households of different size. In our baseline scenario, we stick to official poverty thresholds that are published by national governments. For the European countries in our analysis, this threshold is 60% of median equivalized disposable income, reported by Eurostat and also used as the at-risk-of-poverty indicator within the Europe

[13]Accordant graphs for all other countries under analysis are available from the authors upon request.



Disp. inc. and tax-benefit comp. in USD/mth; all values non-equivalized.

Figure 8: Budget curve decomposition for US single households



Combined schemes of spouse not working (red) + working fulltime (light red)
Disp. inc. and tax-benefit comp. in USD/mth; all values non-equivalized.

Figure 9: Budget curve decomposition for US couple households

2020 strategy.[14] The equivalence scale applied is the modified OECD equivalence scale which assigns a value of 1 to the household head, of 0.5 to each additional adult member and of 0.3 to each child. Thus, the poverty threshold for a single parent household with two children, for instance, will equal the poverty threshold of a pure single household multiplied by a factor of 1.6. For the US, we rely on the so-called Supplemental Poverty Measure and a specific and more complex three-parameter equivalence scale to account for family size.[15] In an alternative application, we will compare our results to the assumption of a poverty line that is 50% of the mean income in each country.

6 Evaluating budget curves

6.1 Measuring and decomposing social welfare

The first column of Table 1 shows us $W^p_{\min}$, the measure of social welfare according to R^{lex}, in percentage of a full time job, for single-parent households with two children. Those households are known to be at high risk of poverty. For instance, the worst-off households of that type in the US are those who work 60.13% of their time (where 100% of one's time means a full-time job). The percentage of labor time needed to reach the poverty line varies from 0% in Ireland and the Netherlands, where even those who do not work have the opportunity to get out of poverty, to countries in which it is impossible for low-skill individuals to get out of poverty even by working full-time, namely Greece, Portugal and Spain. The second column shows the pre-tax income corresponding to this measure, $W^p_{\min} w_m$. For instance, the worst-off in the US, according to R^{lex}, are those who work at the minimal wage and earn 760.60 USD. This is the main result delivered by our approach: if a policy-maker in the US is interested in the normative property of *poverty reduction* defined above, she should modify the tax benefit system in order to increase, in the limit of what is feasible, the disposable income of those earning 760.60 USD.

[14] See for an overview: http://ec.europa.eu/eurostat/statistics-explained/index.php/Europe_2020_headline_indicators.

[15] Detailed information about this measure and its derivation for 2013 can be found in Short (2014). A historical comparison of official poverty measures used in the US up to the Supplemental Poverty Measure can be found in Meyer and Sullivan (2012).

	$-W^p_{\min}$,	$-W^p_{\min} w_m$	b	τ	w_m	p	$\frac{w_m}{p}$
US	60.13	760.60	545.51	7.14	1,264.96	1,739.71	72.71
BE	92.05	1,836.02	1,373.60	84.42	1,994.57	2,284.44	87.31
FR	92.06	1,748.66	982.34	60.39	1,899.48	2,225.79	85.34
GE	6.12	89.38	1,232.02	73.62	1,459.86	2,076.62	70.30
GR	107.09	972.47	135.04	12.58	908.10	890.04	102.03
IR	0.00	0.00	1,355.26	66.20	1,941.48	2,025.62	95.85
LU	14.05	349.66	2,027.55	84.62	2,489.11	3,538.16	70.35
NL	0.00	0.00	1,401.42	66.56	1,951.51	2,213.57	88.16
PT	111.95	841.27	312.55	61.36	751.48	868.79	86.50
SP	146.97	1,469.54	482.27	70.32	999.86	1,436.79	69.59
UK	0.00	0.00	1,245.05	77.35	1,659.93	1,986.20	83.57

Note: All monetary values in monthly USD. Germany introduced a legal minimum wage in 2015. w_m for Germany is this minimum wage deflated to 2013.

Table 1: US and EU15 countries with legal minimal wage: Decomposing social welfare based on the official poverty line for single parent households with two children

The next columns of the table decompose that key statistic into the three policy parameters that determine it. The first one is the basic income, b, that is, the disposable income of those who don't earn anything. It gives us the *level* of the opportunity set available to poor people. The second one is the rate τ at which low incomes are effectively taxed. That is, given any additional dollar earned, how much of it is taken away by the tax-benefit system, on average, below the minimal wage. This tool gives us a summary of the *shape* of the opportunity set of low-skill individuals, that is, of how their labor is rewarded. The third one is the minimal wage itself, w_m, in column 5, but its effect on our key statistic is best seen when it is expressed as a percentage of the poverty line, $\frac{w_m}{p}$ which is done in the last column. It shows that, except in Greece, no household of the type we are looking at could reach the poverty line without benefits.

If the marginal tax rate were constant over low incomes at τ, then our measure of social welfare would satisfy the equation $p = b + W^p_{\min} w_m (1 - \tau)$, which gives us

$$W^p_{\min} = \frac{p - b}{w_m (1 - \tau)},$$

illustrating how the combination of the three policy parameters, b, τ and

w_m determine social welfare and how they can be used to increase it. Social assistance, family benefits and housing benefits typically determine b. How these benefits fade out when gross income increases, in-work transfers, income tax and social security payments together determine τ. Finally, w_m is determined by law.

6.2 Cross-country comparisons

In Fig. 10, we draw the budget frontiers facing the single parent households with two children in all countries we look at. Country specific budget frontiers are made comparable by rescaling the axes. All minimal wages (resp. poverty lines) correspond to coordinate 1 along the horizontal (resp. vertical) axis.

The figure clearly shows the large variety of policies across countries. The budget frontiers in Belgium, Luxembourg, the Netherlands and Spain are characterized by a 100% tax rate on the lowest incomes, sometimes after a small interval of lower tax rates. This 100% tax rate is produced by the one dollar decrease in social assistance following any one dollar increase in gross income over that interval. The budget lines in Germany, France and Portugal are strictly increasing, but at a slow rate, illustrating the way social assistance and, in the case of France and Germany, housing benefits, fade out as gross income increases. The budget lines in Ireland, the UK and the US are steeper, illustrating the in-work benefits existing in these three countries, even if its implementation gives a much smoother budget frontier in the US than in the other two countries. The budget line in Greece is characterized by the disappearance of basically all social assistance programs, except modest family benefits, coupled with very low income tax rates.

The figure shows that there is no clear relationship between the set of policies that are implemented and our measure of social welfare. Ireland, the UK, the Netherlands and Germany are doing the best in terms of $W^p_{\min}$, and they offer budgets of very different shapes. The basic income is much lower in the US than in France or Belgium, but yet, because of its low effective income tax rates, it requires a lower labor time in the US for low-skill workers to get out of poverty than in France and Belgium. In spite of the low effective income tax rate in Greece, working even full time does not allow low-skill people to get out of poverty.

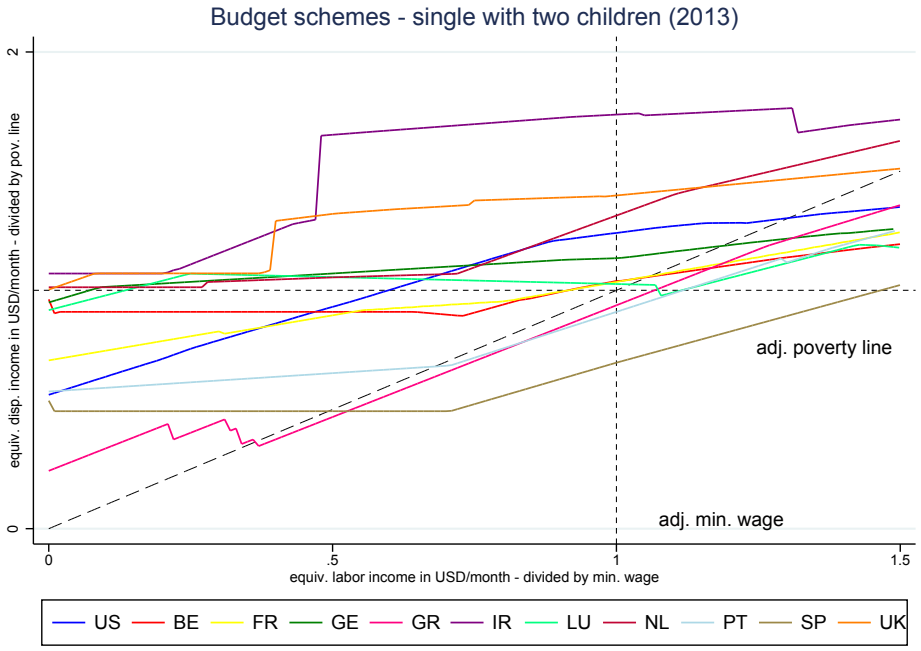


Figure 10: US and EU15 countries with legal minimal wage: Cross-country comparisons of budget sets for single parent households with two children

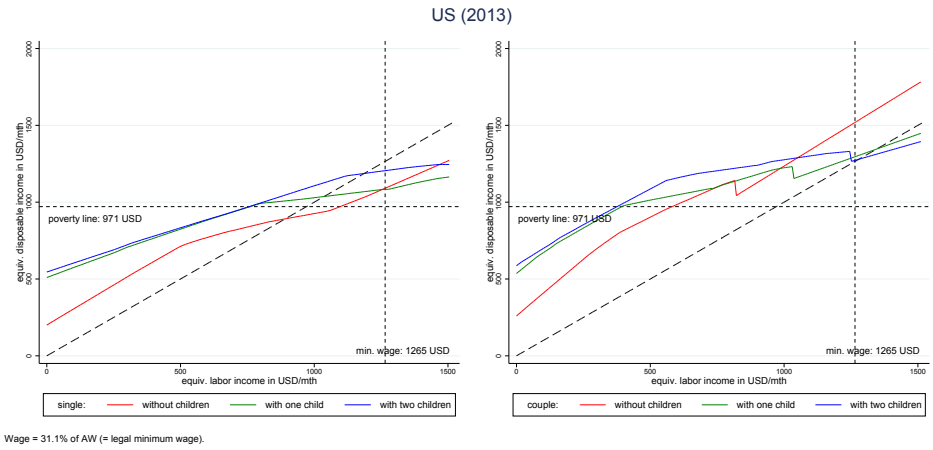


Figure 11: Budget frontiers in the US for six household types: single and couple households with zero, one or two children.

6.3 Does it help to have kids?

Let us compare our well-being measure across household types, to see whether it is easier for households with children to get out of poverty. We do that by rescaling budgets so as to measure equivalized disposable income on the vertical axis. Germany, the UK and the US are the only countries in which social welfare is unambiguously higher among households with children.

Fig. 11 describes the households of singles on the left and couples on the right, with zero, one and two children, for the US. In both cases, the labor time that is required for those households to reach a disposable income equal to the poverty line is lower when there are children in the households. Maybe surprisingly, that does not come from differences in the shape of the budget line, but from differences in the basic income. That is, the in-work benefits (the Earned Income Tax Credits) simply adjust the shape of the budget frontiers to the equivalence scale, so that they are almost parallel to each other. The difference comes from the family benefits on the one hand and the social assistance that is more generous for households with children on the other hand (see decomposition, above, Fig. 8 and 9).

Greece, Portugal and Spain are countries in which social welfare is unambiguously higher among households without children. In all cases, it is due to the simplicity and lack of generosity of the tax and benefit system.

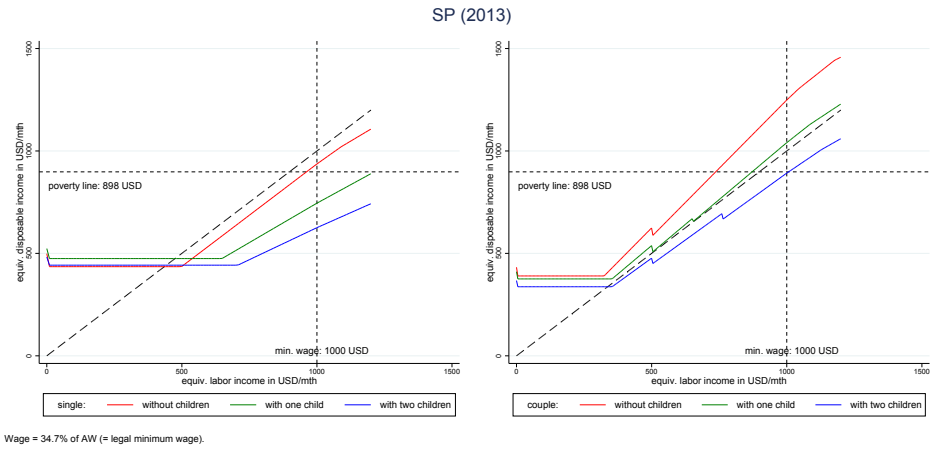


Figure 12: Budget frontiers in Spain for six household types: single-parent households and couples with zero, one or two children.

Fig. 12 describes the budget frontiers of all types of households for Spain. The system is characterized by limited social assistance, which fades out at a tax rate of 100%, above which no benefit exists anymore. The difference in slopes that we can see on the graphs reflect the rescaling of disposable incomes according to the equivalence scale.

The other countries do not offer any clear pattern. In France, for instance, family benefits are more generous for the second child of a single-parent household than for the first child, but social assistance is lower for single parents with two children than with one. All in all, the budget frontier of a single-parent with one child makes it easier for that household to get out of poverty than for singles without or with two children, as can be seen in Fig. 13. For couples, on the other hand, the labor time it takes to get out of poverty increases with the number of children.

7 Extensions

The applications that we provided in the previous sections were all developed under the assumptions that the relevant poverty line was the official one and the assumption that individual labor supplies were not rationed. We remove these two assumptions in this section.

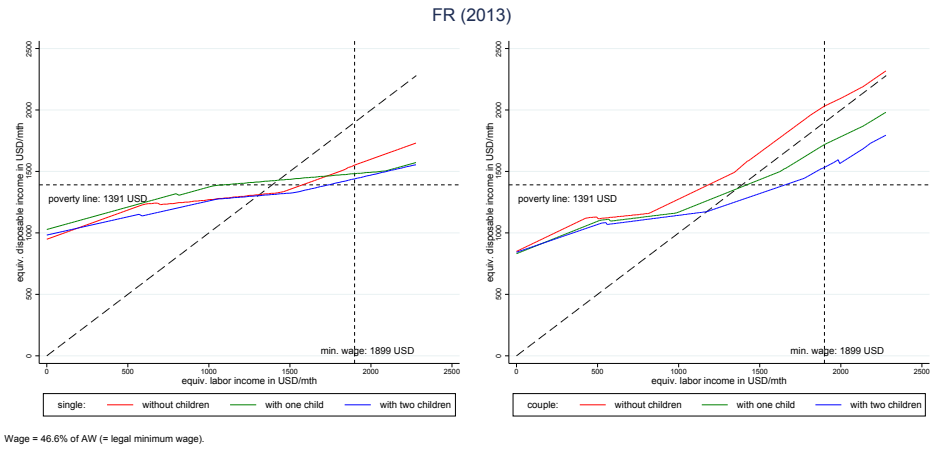


Figure 13: Budget frontiers in France for six household types: single and couple households with zero, one or two children.

7.1 Mean income poverty lines

All official European poverty lines are defined as 60% of the equivalized median income. Median income is determined both by the general living of standards of a country and by the income inequality in that country. When inequality is low, median income is a larger share of mean income and poverty is unduly likely to be considered larger. To take that into account, we change the poverty line from the official ones to 50% of the mean income. The application is now substantially different. In the previous section, we were evaluating how tax and benefit systems are giving opportunities to people to get out of poverty, *according to the way governments themselves define poverty*. Here we compare opportunities given to poor people among countries as a function of those countries' own ability to create income and not their way of defining poverty.

Fig. 14 is the variant of Fig. 10 when the vertical axis is rescaled to take country mean income into account. The consequence is that countries with larger income inequality are doing worse than previously. The striking difference between the two figures is that now the budget set of single parents with two children in the US is the lowest one, dominated even by Greece (except in a small interval of incomes), Portugal and Spain in which welfare systems have suffered a lot from the sovereign debt crisis.

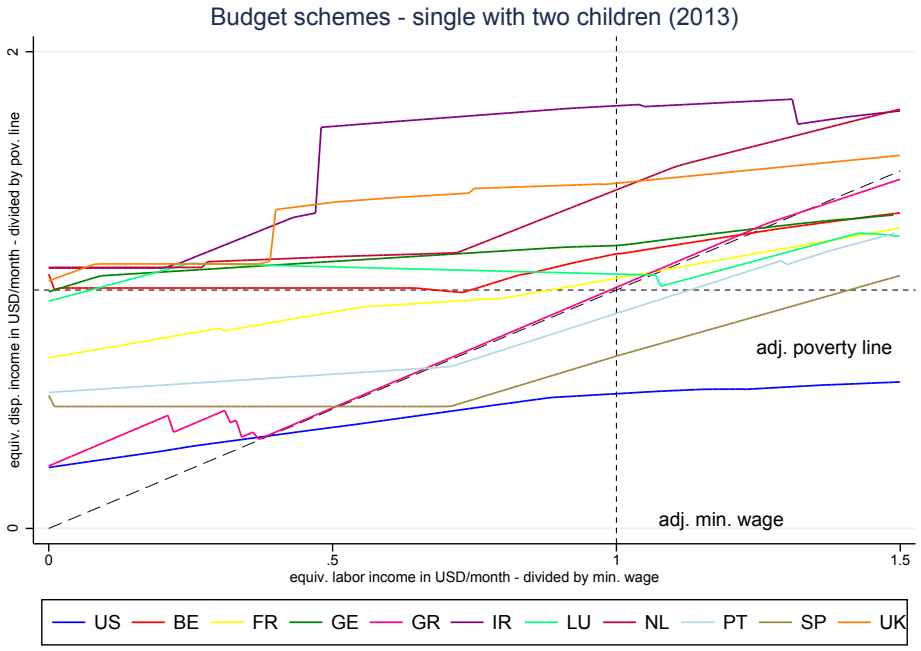


Figure 14: US and EU15 countries with legal minimal wage: Cross-country comparisons of budget sets for single parent households with two children when the poverty line is adjusted to mean income

Budget sets for other types of households display slightly different patterns, with the US always belonging to the set of the two or three worst performing countries.

7.2 Removing housing benefits

	$-W^p_{\min}$,	$-W^p_{\min} w_m$	b	τ	w_m	p	$\frac{w_m}{p}$
FR	116.95	2,221.49	740.99	53.18	1,899.48	2,225.79	85.34
GE	79.08	1,154.48	868.40	55.32	1,459.86	2,076.62	70.30
IR	13.99	271.59	1,106.42	45.89	1,941.48	2,025.62	95.85
LU	20.95	521.53	1,928.99	84.62	2,489.11	3,538.16	70.35
NL	78.02	1,522.61	1,236.31	66.56	1,951.51	2,213.57	88.16
UK	39.94	663.06	933.88	58.52	1,659.93	1,986.20	83.57

Note: All monetary values in monthly USD. Germany introduced a legal minimum wage in 2015. w_m for Germany is this minimum wage deflated to 2013.

Table 2: Decomposing social welfare based on the official poverty line for single parent households with two children when disposable income does not include housing benefits, in countries that do provide housing benefits

Among the tax-benefit policies that we simulate, housing benefits require most assumptions (housing cost, housing size, reference region, fulfillment of eligibility conditions). It is also the policy with the most demanding eligibility conditions. In addition to that, it may induce a bias in our country comparisons, because countries that do not offer housing benefits in cash may have a large and generously subsidized social housing (such as Belgium) directed towards the poorest. This subsidy is not added to the disposable income in those countries.

It may be interesting, therefore, to apply our criterion to a notion of disposable income that does not include housing benefits. Table 2 reproduces table 1 for the six countries providing housing benefits. Removing housing benefits from the computation of disposable incomes has two effects. First, it decreases b, the disposable income of those who do not work. That effect is quite large (between 248,84 USD and 363.62 USD of equivalized income) in the four countries with the most generous policy (France, Germany, Ireland and the UK), and modest in the other two countries (Luxembourg and The Netherlands).

Second, it decreases τ, because housing benefits typically fades out as income increases. This effect is the weakest in The Netherlands, where housing benefits do not depend on income as long as income are below a threshold much larger than the minimal wage. It is the largest in Ireland, where housing benefits decrease with earned income at a one-to-one dollar rate, decreasing the effective tax rates on low incomes by 20.31 percentage points.

As a result, the effect on social welfare (first column) is the largest for The Netherlands. Because this country has a 100% marginal tax rate on low incomes, its performance on our measure of social welfare is extremely sensitive to whether or not the basic income is lower or larger than the poverty line. When we subtract housing benefits from the definition of disposable income, it decreases below the poverty line, making it much harder for low-skill lone parents with two children to earn sufficient income to reach the poverty line.

At the other extreme, because housing benefits fade out even for low incomes in Ireland, the labor time it takes to reach the poverty line does not increase by much (13.99%) when those benefits are not taken into account (or not received by the targeted households). This illustrates that social welfare as we measure it in this paper is much more sensitive to changes in the definition of disposable income when tax-benefits policies are designed in such a way that τ is low. This also means that partial take-up or variations in eligibility conditions is more likely to have limited impact on social welfare when τ is low.

7.3 Taking unemployment rates into account

We have made the assumption throughout this paper that individuals are free to choose their labor time. There are many reasons why this could not be the case, in particular in countries experiencing a high unemployment rate. The figures that we provided up to now should then be read as evaluating the ability of tax and benefit systems to help people get out of poverty *conditional on them having found a job*. This is consistent with the idea that tax and benefit systems should not depend on short-term fluctuations on the labor market. Consequently, as mentioned above, unemployment insurance benefits are not taken into account in the budget curves presented in this paper.

One may argue, though, that the ability to find jobs should be part of the evaluations of the opportunities that are given to individuals who are

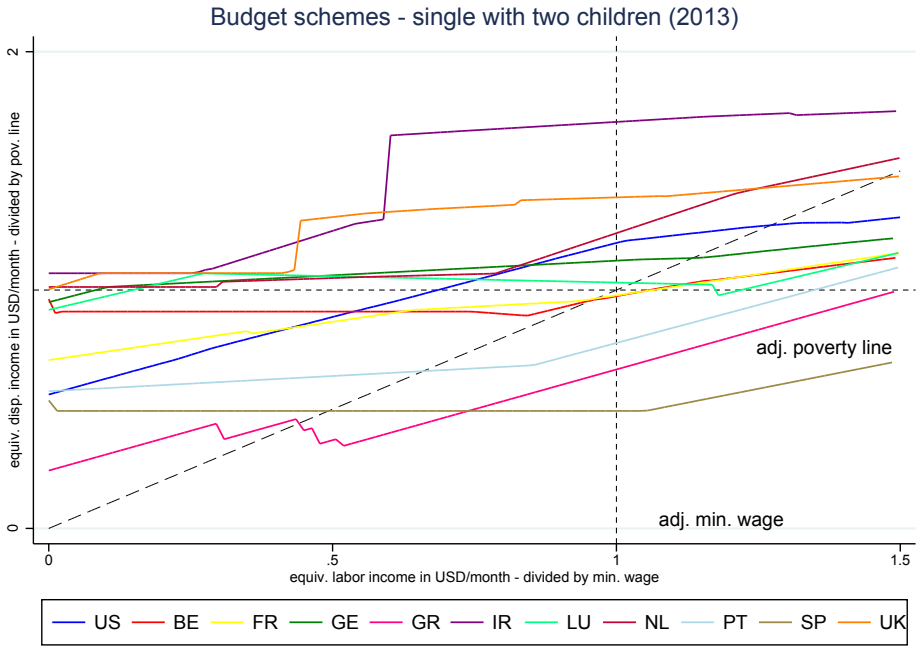


Figure 15: US and EU15 countries with legal minimal wage: Cross-country comparisons of budget sets for single parent households with two children when the minimal wage is adjusted to take unemployment rate into account

ready to work hard to get out of poverty. We apply that idea in this section. Our strategy consists in considering the worst possible scenario, that is, the scenario of an individual whose past labor market participation does not allow her to claim unemployment insurance. The expected gross income of such an individual willing to work, say, full time at the minimal wage is simply the minimal wage weighted by the probability to find a job, approximated in our application by one minus the unemployment rate of low-skill individuals.

Unemployment rates for low-skill individuals are given in table 10 in the Appendix. Low skill means not having completed high school. Fig. 15 is the variant of Fig. 10 when the horizontal axis is rescaled to take country-specific unemployment rates of low-skill individuals into account. The main lesson is that the picture does not change much: the countries with the tax benefit system offering the least opportunities to get out of poverty, Greece, Portugal and Spain, are also the countries experiencing the highest unemployment rates. On the other hand, the countries providing the highest social welfare in terms of our criterion defined above are also those offering high basic incomes, and they are not affected when unemployment rates are taken into account. Countries with an intermediary level of social welfare do not differ much in their unemployment rate of low-skill individuals.

8 Conclusion

In this paper, we have proposed a normative property to make social preferences sensitive to poverty in a way that is compatible with the Pareto criterion. We have used that property, in combination with the Pareto criterion and an independence property to characterize specific social preferences. These preferences work by applying the leximin aggregator to some well-being representation of individual preferences. That representation looks at the labor time that leaves an agent indifferent between her actual bundle and a consumption level equal to the poverty line at that labor time.

We have shown that those social preferences give a very simple criterion to evaluate tax functions. The key statistics consists in the pre-tax income that is just necessary to obtain a disposable income equal to the poverty line.

We have applied that criterion to evaluate tax benefit systems in the US and in the EU, restricting our attention to countries with a legal minimal wage. This application has required to construct budget sets for each country and a selection of typical households, as a function of the following policies:

social assistance, family benefits, housing benefits, in-work benefits, income taxes and social security contributions. In each country and for each type of households (singles or couples with zero, one or two children) applying our criterion has allowed us to identify the level of pre-tax income around which a reform could improve social welfare. Whichever the policy tool that is used, such a reform would consist in increasing the disposable income associated to those levels of pre-tax incomes.

Budget sets of poor people vary considerably across countries, even after rescaling them to take differences in minimal wages and differences in poverty lines into account. We can observe differences in basic income, that is, the transfer to individuals who do not earn any income, in the marginal rates of taxation, and in the size of the minimal wage relative to the poverty line. There is no clear relationship, though, between a country's combination of policy tools and its performance in terms of social welfare.

We also showed that our application was highly dependent on the definition of the poverty line. We sticked to the official lines, but we showed that a poverty line expressed as a fraction of mean income changes the assessment of its tax and benefit system. In particular, the US have a particularly low poverty line in view of its high mean income. When budget sets are rescaled to take it into account, budget sets in the US turn out to be among the lowest in our sample of countries and even the lowest one for some household types.

This paper illustrates that it is possible to do normative policy evaluation by starting at the level of the definition of desirable normative properties and finishing at the level of precise identification of social welfare improving policy reforms. It is important to note that our exercise did not require the detour through the derivation of a formula describing the optimal tax and benefit system, the kind of formula on which the literature in this field has devoted its largest effort so far. The normative property we have studied in this paper is certainly not the only one worth investigating, even if one concentrates on poverty alleviation.

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9 Appendix: proof of Proposition 1

Proof. We let the easy proof that R^{lex} satisfies *poverty reduction*, *Pareto* and *independence* to the reader. We concentrate on the second statement. The proof is divided in three steps. In the first two steps, we prove that social preferences satisfying the three properties we have defined also satisfy other, stronger, properties. We begin by proving that if social preferences R satisfy *poverty reduction*, *Pareto* and *independence*, then they satisfy the following strengthening of *poverty reduction*.

Property 4 *For all economy $(R_1, ..., R_n)$, for all pairs of allocations $z = (z_1, ..., z_n)$ and $z' = (z'_1, ..., z'_n)$, if, for two agents j and k*

$$\begin{aligned} \ell_j = \ell'_j = \ell'_k = \ell_k, \\ c_j < c'_j < p < c'_k < c_k, \end{aligned}$$

whereas $z_i = z'_i$ for all other agents, then $z' \; P \; z$.

We prove this claim in two steps, corresponding to two cases for the value of $\ell_j = \ell'_j = \ell_k = \ell'_k$. Case 1: $\ell_j = \ell'_j = \ell_k = \ell'_k = 1$. Then there exist $z'' = (z''_1, ..., z''_n)$ and $z''' = (z'''_1, ..., z'''_n)$ such that $z''_j \; I_j \; z_j$, $z'''_j \; I_j \; z'_j$, $z''_k \; I_k \; z_k$ and $z'''_k \; I_k \; z'_k$,

$$\begin{aligned} \ell''_j = \ell'''_j = \ell''_k = \ell'''_k < 1, \\ c''_j < c'''_j < p < c'''_k < c''_k, \end{aligned}$$

and $z''_i = z'''_i = z_i = z'_i$ for all other agents. By Pareto, $z'' \; I \; z$ and $z''' \; I \; z'$. By transitivity, we now need to prove that $z''' \; P \; z''$, which corresponds to case 2.

Case 2: $\ell_j = \ell'_j = \ell_k = \ell'_k < 1$. Let us assume, contrary to the claim, that

$$z \, R(R_1, ..., R_n) \, z' \tag{4}$$

where social preferences are written $R(R_1, ..., R_n)$ to denote their dependence on the profile of individual preferences. Let bundles z_j^1, z_j^2 and z_j^3 be defined by

$$
\begin{aligned}
& z'_j \, P_j \, z_j^3 \, I_j \, z_j^2 \, P_j \, z_j^1 \, I_j \, z_j, \\
& \ell_j^1 = l_j^2 = l'_j, \\
& \ell_j^3 = l_j.
\end{aligned}
$$

Now, we construct $z''_k, z_k^1, z_k^2, z_k^3$ and R''_k in such a way that

$$
\begin{aligned}
& l''_k = l_k^3 = l_k, l_k^1 = l_k^2 = l'_k, \\
& c''_k > c_k, c_k^3 - c'_k = c'_j - c_j^3, c_k^2 - c_k^1 = c_j^1 - c_j^2, \\
& z''_k \, I''_k \, z_k^1 \, P''_k \, z_k^2 \, I''_k \, z_k^3 \, P''_k \, z'_k, \text{ and} \\
& I(z'_k, R''_k) = I(z'_k, R_k).
\end{aligned}
$$

The construction of those bundles and preferences are illustrated in Fig. 16.

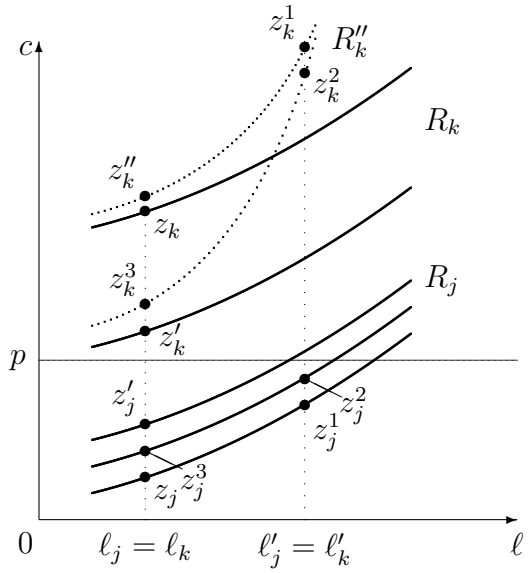

Figure 16: Illustration of the proof of the first claim

Let preferences R'_k be such that

$$\begin{aligned} I(z_k, R'_k) &= I(z_k, R_k) \\ I(z'_k, R'_k) &= I(z'_k, R_k) \\ I(z''_k, R'_k) &= I(z''_k, R''_k). \end{aligned}$$

By *independence*, Eq. 4 implies that

$$z\, R(R_1, \ldots, R'_k, \ldots, R_n)\, z'. \tag{5}$$

Let us define $z'' = (z''_1, ..., z''_n)$ by fixing $z''_i = z_i$ for all $i \neq k$ (z''_k is defined above). By *Pareto*,

$$z''\, P(R_1, \ldots, R'_k, \ldots, R_n)\, z. \tag{6}$$

By transitivity, Eq. 5 and 6 imply

$$z''\, P(R_1, \ldots, R'_k, \ldots, R_n)\, z'. \tag{7}$$

By *independence*,

$$z''\, P(R_1, \ldots, R''_k, \ldots, R_n)\, z'. \tag{8}$$

We define z^1, z^2 and z^3 by: $z^1_i = z^2_i = z^3_i = z_i$ for all $i \neq j, k$ and the corresponding bundles of j and k are defined above. By *Pareto*, we can deduce that

$$z^1\, I(R_1, \ldots, R''_k, \ldots, R_n)\, z''. \tag{9}$$

By *poverty reduction*,

$$z^2\, P(R_1, \ldots, R''_k, \ldots, R_n)\, z^1. \tag{10}$$

By *Pareto*,

$$z^3\, I(R_1, \ldots, R''_k, \ldots, R_n)\, z^2. \tag{11}$$

By *poverty reduction*,

$$z'\, P(R_1, \ldots, R''_k, \ldots, R_n)\, z^3. \tag{12}$$

Gathering Eq. 8, 9, 10, 11 and 13, we obtain

$$z'\, P(R_1, \ldots, R''_k, \ldots, R_n)\, z', \tag{13}$$

the desired contradiction.

We proceed by proving that social preferences satisfying *poverty reduction*, *Pareto* and *independence*, and, therefore, *property 4*, also satisfy the following property.

Property 5 *For all economy* $(R_1, ..., R_n)$*, for all pairs of allocations* $z = (z_1, ..., z_n)$ *and* $z' = (z'_1, ..., z'_n)$*, if, for two agents* j *and* k

$$\ell_j > \ell'_j > \ell'_k > \ell_k,$$
$$c_j = c'_j = c'_k = c_k = p,$$

whereas $z_i = z'_i$ *for all other agents, then* $z' \; P \; z$*.*

Let us assume, contrary to the claim, that

$$z \, R \, z'. \tag{14}$$

The following construction is illustrated in Fig. 17. Let $\delta^1_k > 0$ be such that there exists $c^1_k > p$ such that

$$(\ell_k + \delta^1_k, c^1_k) \, I_k \, z_k.$$

The idea is to choose δ^1_k as close as possible to $\ell'_k - \ell_k$, but the shape of the indifference curve through z_k may be such that δ^1_k is bounded above. Let c^1_j be defined by

$$(\ell_k + \delta^1_k, c^1_j) \, I_j \, z_j.$$

Such a c^1_j always exists, thanks to our assumption that consumption is necessary. This is the only role played by this assumption. By *Pareto*,

$$(z_1, \ldots, (\ell_k + \delta^1_k, c^1_j), \ldots, (\ell_k + \delta^1_k, c^1_k), \ldots, z_n) \, I \, z. \tag{15}$$

Then, we can choose δ^1_j such that

$$\frac{\delta^1_j}{\delta^1_k} < \frac{\ell_j - \ell'_j}{\ell'_k - \ell_k} \tag{16}$$

and define c^{1*}_j by

$$(\ell_k + \delta^1_k, c^{1*}_j) \, I_j \, (\ell_j - \delta^1_j, p).$$

By *property 4*,

$$(z_1, \ldots, (\ell_k + \delta^1_k, c^{1*}_j), \ldots, (\ell_k + \delta^1_k, p), \ldots, z_n)$$

$$P \, (z_1, \ldots, (\ell_k + \delta^1_k, c^1_j), \ldots, (\ell_k + \delta^1_k, c^1_k), \ldots, z_n). \tag{17}$$

By *Pareto*,

$$(z_1, \ldots, (\ell_j - \delta^1_j, p), \ldots, (\ell_k + \delta^1_k, p), \ldots, z_n)$$

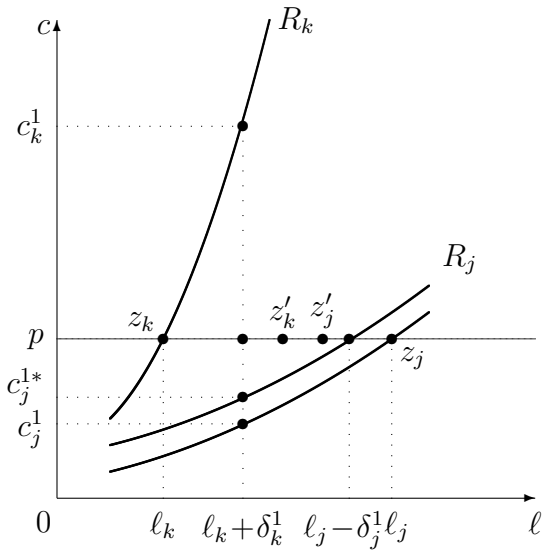

Figure 17: Illustration of the proof of the second claim

$$P\,(z_1,\ldots,(\ell_k+\delta_k^1,c_j^{1*}),\ldots,(\ell_k+\delta_k^1,p),\ldots,z_n). \tag{18}$$

By transitivity, Eq. 15 17 and 18 imply

$$(z_1,\ldots,(\ell_j-\delta_j^1,p),\ldots,(\ell_k+\delta_k^1,p),\ldots,z_n)\,P\,z'. \tag{19}$$

Iterating this chain of social indifference and strict preference a finite number of times and constructing $\delta_k^2,\delta_j^2,\ldots,\delta_k^M,\delta_j^M$ that satisfy Eq. 16 at each iteration, we arrive at labor time

$$\begin{aligned}\ell_j'' &= \ell_j-\sum_{m=1}^{M}\delta_j^m>\ell_j',\\ \ell_k'' &= \ell_k+\sum_{m=1}^{M}\delta_k^m>\ell_k',\end{aligned}$$

and allocation

$$z''=(z_1,\ldots,(\ell_j'',p),\ldots,(\ell_k'',p),\ldots,z_n)$$

such that, by iteration of the sequence leading to Eq. 19,

$$z''\,P\,z',$$

violating *Pareto*, because $\ell''_i \geq \ell'_i$ for all i, $\ell''_j > \ell'_j$ and $\ell''_k > \ell'_k$.

We now complete the proof of the Proposition. In this third step, all allocations will be composed of bundles containing a consumption level equal to p. Only labor times will vary, so that the objects we define are essentially unidimensional. The resulting proof boils down to the adaptation to this setting of Hammond (1982)'s characterization of the leximin in utility. Let economy $(R_1, ..., R_n)$ and allocations $z = (z_1, ..., z_n)$ and $z' = (z'_1, ..., z'_n)$ be such that

$$\min_i W^p(z_i) > \min_i W^p z'_i.$$

We need to prove that $z\, P\, z'$. For all $i \in \{1, \ldots, n\}$, let us define $z''_i = (W^p(z_i, R - i), p)$ and $z'''_i = (W^p(z'_i, R - i), p)$. By *Pareto*, $z''\, I\, z$ and $z'''\, I\, z'$, so that, by transitivity, $z'''\, R\, z''$. Without loss of generality, let us fix

$$\begin{aligned} z_1 &= \min_i W^p(z_i, R_i) \\ z_2 &= \min_i W^p(z'_i, R_i). \end{aligned}$$

Let us choose ℓ^*_i for each $i \in \{1, \ldots, n\}$ such that $\ell^*_i \geq \ell^*_1$ and $\ell_1 < \ell^*_1 \leq \ell_i < \ell'_2$. Let $z^* = ((\ell^*_i), \ldots, (\ell^*_n))$. Using *property 5* iteratively for each $i \in \{2, \ldots, n\}$, we get the conclusion that $z^*\, P\, z''$. By transitivity, $z^*\, P\, z''$, violating *Pareto*. ■

10 Appendix: unemployment rates for low-skilled

	Unemp. rate
US	0.13
BE	0.14
FR	0.14
GE	0.12
GR	0.29
IR	0.20
LU	0.09
NL	0.09
PT	0.17
SP	0.33
UK	0.10

Table 3: Unemployment rates for low-skilled (below upper secondary education) aged 25-64 (*Source:* OECD)