Long-term care social insurance. How to avoid big losses?*

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Abstract

Long-term care (LTC) needs are expected to rapidly increase in the next decades and at the same time the main provider of LTC, namely the family is stalling. This calls for more involvement of the state that today covers less than 20% of these needs and most often in an inconsistent way.

Besides the need to help the poor dependent, there is a mounting concern in the middle class that a number of dependent people are incurring costs that could force them to sell all their assets. In this paper we study the design of a social insurance that meets this concern. Following Arrow (1963), we suggest a policy that is characterized by complete insurance above a deductible amount.

Keywords: capped spending, Arrow's theorem, long-term care insurance.

1 Introduction

Long-term care (LTC) is becoming a major concern for policy makers. Following the rapid aging of our societies, the needs for LTC are expected to grow and yet there is a lot of uncertainty as how to finance those needs; see Norton (2000) and Cremer, Pestieau and Ponthière (2012) for an overview. Family solidarity, which has been the main provider of LTC, is reaching a ceiling, and the market remains rather thin. Not surprisingly, one would expect that the state takes the relay.

The state plays already some role in most countries but this role is still modest and inconsistent. In a recent report for the UK, Andrew Dilnot (2011) sketches the features of what can be considered as an ideal social program for LTC. This would be a two-tier program. The first tier would concern those who cannot afford paying for their LTC. It would be a means-test program. The second tier would address the fears of most dependents in the middle class that they might incur costs that would force them to sell all their assets and prevent them from bequeathing any of them. This concern is not met by current LTC practices.

In this paper we want to study the design of a social insurance that would cover those with a modest level of assets (for example 300,000 euros) who can face losing up to their entirety to pay for

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care costs. To do that we explore Dilnot's suggestion that individuals' contribution to their long-term care costs should be capped at a certain amount, after which they will be eligible for full state support. We are thus in the spirit of Arrow's (1963) theorem on insurance deductible. To recall, this theorem states that "if an insurance company is willing to offer any insurance policy against loss desired by the buyer at a premium which depends only on the policy's actuarial value, then the policy chosen by a risk-averting buyer will take the form of 100% coverage above a deductible minimum" (Arrow, 1963). Our paper explores whether and how this idea can be applied to LTC social insurance.

We focus on the concerns of the middle class assets and thus abstract from the protection of the low income dependent. We look at a welfare maximizing government which faces a society consisting of people who differ in their earning and face the risk of dependence. Following Arrow, we assume that insurance is not costless; we thus introduce a loading factor that is at the heart of his theorem. We assume that this is true for both private and social insurance but consider the possibility that the government might face lower costs than private insurers. We study the design of a non-linear optimal social LTC insurance and show that this insurance features a deductible as long as there is a loading cost. We then ask ourselves whether we can obtain maximum social welfare by restricting public policy to income taxation and not interfering in the choice of insurance by individuals, which would be in line with Atkinson and Stiglitz (1976). As it will appear, this result of non interference with the insurance choice of individuals will hold only if individuals have the same probability of losses and the same level of losses. As soon as we depart from this assumption, Atkinson-Stiglitz proposition does not apply and we can tax or subsidize private insurance purchases to improve social welfare. In this paper, we consider two types of individuals: skilled and unskilled. They face a probability of becoming dependent and would like to buy some insurance. When the losses incurred by the skilled are higher than that of the unskilled, there is a case for taxing the premium paid by the unskilled. This tax allows for relaxing the self-selection constraint that the skilled are not tempted to mimic the non skilled. We also use the idea that the higher needs of the skilled are somehow whimsical and thus are taken seriously by the social planner in his design of optimal policy.

It will be seen in the analysis that the interference or not with individual insurance choices will have an important impact on the way optimal deductibles for skilled and non skilled individuals are designed, but an important role will also be played by absolute risk aversion exhibited by individual preferences.

An insurance policy with deductible is not the only possible type of contract. One of the most common practices today is to provide flat payments. Concretely, the insured individuals are entitled to a (periodic) lump-sum payment conditional on their (observable) degree of dependency. This practice has been justified by Kessler (2008) on the basis of alleged huge ex-post moral hazard and by Lozachmeur et al. (2015) on the basis of family solidarity that acts as a last resort payer. Finally, note that in this paper, we adopt a very simple specification of dependency. We do not explicitly account for the time dimension, namely for the fact that the loss incurred by a dependent depends on the yearly cost of dependency times the number of years of dependency. This number is the difference between the age of death and the age at which an irreversible dependency occurs. For an extension of Arrow's theorem to such a temporal framework, see Drèze et al. (2015).

2 The model

We consider a society consisting of two types of individuals: skilled (i.e. those with a high productivity/wage denoted by w_h) and unskilled (i.e. those with a low productivity/wage $w_l < w_h$). Before their retirement, individuals provide labour supply, respectively l_h and l_l, on the labour market and thus earn respectively $y_h = w_h l_h$ and $y_l = w_l l_l$. By working, the individuals experience a disutility of labour $v(l_i)$ $(i = h, l)$, with $v'(l_i) > 0$ and $v''(l_i) > 0$.

When they reach their old age and retire, the individuals face the risk of becoming dependent. With probability π_1, they experience a low severity level of dependence in which case they have LTC needs (expressed in terms of costs incurred) L_{1i} $(i = h, l)$, with probability π_2, they face a heavy dependence with LTC needs $L_{2i} > L_{1i}$ $(i = h, l)$, and with probability $1 - \pi_1 - \pi_2$, they remain healthy. At each severity level, the two types of individuals can have different LTC needs (i.e. $L_{1h} \neq L_{1l}$ and $L_{2h} \neq L_{2l}$) or these needs can be the same (i.e. $L_{1h} = L_{1l}$ and $L_{2h} = L_{2l}$); we will discuss these cases separately.

The individuals can purchase private LTC insurance which charges a premium $\widehat{P}_i$ and reimburses a fraction $\widehat{\alpha}_{1i}$ of the needs in state 1 and $\widehat{\alpha}_{2i}$ in state 2 $(0 \leq \widehat{\alpha}_{1i} \leq 1$ and $0 \leq \widehat{\alpha}_{2i} \leq 1; i = h, l)$.[1]

For simplicity, we do not model explicitly the individuals' consumption and saving choices made before the retirement; we rather assume that the individuals save a constant share β of their income left after paying the insurance premium and consume the rest. To simplify even more, we focus on the post-retirement stage and abstract from the individuals' utility of consumption before the retirement. We thus normalize β to 1 and consider that the individuals arrive to the post-retirement stage with a wealth equal to $y_i - \widehat{P}_i$.

The expected utility of an individual i $(i = h, l)$ can thus be written as follows:

$$EU_i = \pi_1 u\left(c_i^{D_1}\right) + \pi_2 u\left(c_i^{D_2}\right) + (1 - \pi_1 - \pi_2)\, u\left(c_i^I\right) - v\left(\frac{y_i}{w_i}\right) \tag{1}$$

where
$c_i^{D_1} = y_i - \widehat{P}_i - (1 - \widehat{\alpha}_{1i})L_{1i}$,
$c_i^{D_2} = y_i - \widehat{P}_i - (1 - \widehat{\alpha}_{2i})L_{2i}$
and $c_i^I = y_i - \widehat{P}_i$ are individual wealth levels in the three states of nature[2]
and $\widehat{P}_i = \pi_1(1 + \widehat{\lambda})\widehat{\alpha}_{1i}L_{1i} + \pi_2(1 + \widehat{\lambda})\widehat{\alpha}_{2i}L_{2i}$, with $\widehat{\lambda} > 0$ being the loading cost of private insurance.

3 The *laissez-faire*

In the *laissez-faire*, the problem of an individual i $(i = h, l)$ is to determine his pre-retirement labour supply l_i (or, equivalently, his earnings y_i) and to choose an insurance policy characterized

[1] Following Drèze and Schokkaert (2013), we will show that the equilibrium insurance policy is in line with Arrow's theorem of the deductible.

[2] Individuals can obviously decide how to allocate their wealth between, e.g., their old age consumption and bequests left to their children. We do not model these choices explicitly but rather focus on individuals' total wealth. As long as bequests are considered as normal goods, wealthier individuals will leave higher bequests.

by a premium $\widehat{P}_i$ and insurance rates $\widehat{\alpha}_{1i}$ and $\widehat{\alpha}_{2i}$ ($0 \leq \widehat{\alpha}_{1i} \leq 1$ and $0 \leq \widehat{\alpha}_{2i} \leq 1$). The Lagrangean of this problem can be written as follows:

$$\begin{aligned}
\mathcal{L} &= \pi_1 u\left(c_i^{D_1}\right) + \pi_2 u\left(c_i^{D_2}\right) + (1-\pi_1-\pi_2)\, u\left(c_i^I\right) - v\left(\frac{y_i}{w_i}\right) - \\
&\quad -\mu_i\left[\widehat{P}_i - \pi_1(1+\widehat{\lambda})\widehat{\alpha}_{1i}L_{1i} - \pi_2(1+\widehat{\lambda})\widehat{\alpha}_{2i}L_{2i}\right]
\end{aligned}$$

where, as defined before,
$c_i^{D_1} = y_i - \widehat{P}_i - (1-\widehat{\alpha}_{1i})L_{1i}$,
$c_i^{D_2} = y_i - \widehat{P}_i - (1-\widehat{\alpha}_{2i})L_{2i}$,
$c_i^I = y_i - \widehat{P}_i$
and μ_i is the Lagrange multiplier associated with the constraint defining the insurance premium.

The FOCs with respect to the choice variables are the following:

$$\frac{\partial \mathcal{L}}{\partial y_i} = \pi_1 u'(c_i^{D_1}) + \pi_2 u'(c_i^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_i^I) - \frac{v'\left(\frac{y_i}{w_i}\right)}{w_i} = 0 \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial \widehat{P}_i} = -\pi_1 u'(c_i^{D_1}) - \pi_2 u'(c_i^{D_2}) - (1-\pi_1-\pi_2)\, u'(c_i^I) - \mu_i = 0 \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial \widehat{\alpha}_{1i}} = u'(c_i^{D_1}) + \mu_i(1+\widehat{\lambda}) \leq 0, \quad \widehat{\alpha}_{1i}\frac{\partial \mathcal{L}}{\partial \widehat{\alpha}_{1i}} = 0 \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial \widehat{\alpha}_{2i}} = u'(c_i^{D_2}) + \mu_i(1+\widehat{\lambda}) \leq 0, \quad \widehat{\alpha}_{2i}\frac{\partial \mathcal{L}}{\partial \widehat{\alpha}_{2i}} = 0 \tag{5}$$

Following Drèze and Schokkaert (2013), we will now show that the equilibrium insurance policy is in line with Arrow's theorem of the deductible. To see this, first note that from (4), we have that either $\widehat{\alpha}_{1i} = 0$ or $u'(c_i^{D_1}) = -\mu_i(1+\widehat{\lambda})$. It can be easily verified that the second equality is equivalent to

$$(1-\widehat{\alpha}_{1i})L_{1i} = y_i - \widehat{P}_i - u'^{-1}(-\mu_i(1+\widehat{\lambda})).$$

Similarly, from (5), we have that either $\widehat{\alpha}_{2i} = 0$ or $u'(c_i^{D_2}) = -\mu_i(1+\widehat{\lambda})$, and from the second equality we can get

$$(1-\widehat{\alpha}_{2i})L_{2i} = y_i - \widehat{P}_i - u'^{-1}(-\mu_i(1+\widehat{\lambda})).$$

Denoting $y_i - \widehat{P}_i - u'^{-1}(-\mu_i(1+\widehat{\lambda})) \equiv \widehat{D}_i$, we can write

$$\widehat{\alpha}_{1i} = \max\left[0; \frac{L_{1i} - \widehat{D}_i}{L_{1i}}\right]$$

and

$$\widehat{\alpha}_{2i} = \max\left[0; \frac{L_{2i} - \widehat{D}_i}{L_{2i}}\right]$$

Thus, if the needs are lower than $\widehat{D}_i$, it is optimal for the individual to have zero insurance coverage and to bear all the costs himself, whereas if the needs are higher than $\widehat{D}_i$, the optimal insurance is such that the individual actually pays the amount $\widehat{D}_i$ and the rest is covered by the insurer. This is thus exactly what is stated by Arrow's theorem of the deductible.

We therefore have that if the needs are higher than the deductible at both severity levels of dependence (i.e. if all the solutions are interior), the marginal utilities in the two dependence states of nature will be equalized. To compare these marginal utilities with the marginal utility in the state of independence, combining (3) with (4) and (5), we get

$$\frac{u'(c_i^I)}{u'(c_i^{D_1})} = \frac{u'(c_i^I)}{u'(c_i^{D_2})} = \frac{1 - \pi_1(1+\widehat{\lambda}) - \pi_2(1+\widehat{\lambda})}{(1 - \pi_1 - \pi_2)(1+\widehat{\lambda})} < 1 \tag{6}$$

We can see that as long as $\widehat{\lambda} > 0$, insurance is not full and thus the deductible is always strictly positive.

Focusing further on interior solutions, in Appendix A we derive the comparative statics of equilibrium earnings y_i and deductible $\widehat{D}_i$ with respect to changes in the individual's wage/productivity w_i, LTC needs L_{1i} and L_{2i} and insurance loading cost $\widehat{\lambda}$. We show that while y_i always increases with the level of w_i, this is not necessarily true for $\widehat{D}_i$. In particular, the reaction of $\widehat{D}_i$ to a change in w_i depends on the absolute risk aversion (ARA) exhibited by the utility function. More specifically, $\widehat{D}_i$ is increasing in w_i under decreasing absolute risk aversion (DARA), decreasing in w_i under increasing absolute risk aversion (IARA) and constant in w_i under constant absolute risk aversion (CARA) preferences.[3] This is in line with the deductible insurance theory showing that under DARA (resp. IARA and CARA) the deductible increases (resp. decreases and remains constant) when the initial wealth goes up.[4] Indeed, in our setting, an increase in w_i implies an increase in y_i, which can also be seen as an increase in the initial wealth.

As far as changes in LTC needs are concerned, an increase in L_{1i} or L_{2i} fosters labour supply and increases earnings y_i.The effect on the deductible $\widehat{D}_i$ again depends on risk aversion but is opposite to the effect of w_i: an increase in L_{1i} or L_{2i} decreases (resp. increases and does not affect) $\widehat{D}_i$ under DARA (resp. IARA and CARA) preferences. The reason for this is that, as shown in Appendix A, the increase of y_i due to higher needs is not sufficient to offset the increase of the insurance premium resulting from these needs. This means that a rise in LTC needs causes an overall decrease in wealth, which explains the implications for $\widehat{D}_i$ under the different types of ARA.

Finally, the effect of a change in the loading cost $\widehat{\lambda}$ is also dependent on the type of ARA exhibited by the preferences, and in this case it is true not only for $\widehat{D}_i$ but also for y_i. In particular, y_i is increasing in $\widehat{\lambda}$ under DARA and CARA, whereas the effect is undetermined under IARA. $\widehat{D}_i$, on the contrary, is increasing in $\widehat{\lambda}$ under IARA and CARA, while there is ambiguity in the case of DARA.

[3] DARA (resp. IARA and CARA) means that absolute risk aversion decreases (resp. increases and remains constant) when wealth increases. For more details, see Appendix A.

[4] See, for instance, Seog (2010). For the intuition of this result, note that a higher deductible means less insurance; thus, since under DARA (resp. IARA) wealthier people are less (resp. more) risk averse, they require less (resp. more) insurance.

To understand the intuition of this result (which is also consistent with the deductible insurance theory[5]), we should first note that an increase in $\widehat{\lambda}$ is an increase in the price of insurance which can be decomposed into the substitution and wealth effects. When $\widehat{\lambda}$ goes up, the substitution effect pushes for buying less insurance (i.e. for a higher deductible), but the wealth effect has different consequences depending on ARA. Since wealth decreases when $\widehat{\lambda}$ goes up, under IARA we have a decrease in risk aversion as well, which also pushes for less insurance and so a higher deductible. The two effects thus drive the deductible to the same direction under IARA. On the other hand, under DARA, a decrease in wealth increases risk aversion and so pushes for a lower deductible. The wealth and substitution effects thus go to opposite directions, which creates the ambiguity under DARA preferences. Finally, the wealth effect plays no role under CARA, in which case $\widehat{D}_i$ increases simply due to the substitution effect.

Concluding the discussion of the *laissez-faire*, it should be noted that obviously the *laissez-faire* choices are made separately by each type of individuals and there is thus no redistribution between the two types. One can however expect this situation to be suboptimal from the social point of view. Moreover, one can also expect the government to be able to provide insurance at a lower cost than private insurers, as it is the case with health insurance and pension schemes.[6] For these reasons, we now investigate what would be an optimal scheme of social LTC insurance. As it will be seen from the analysis, the conclusions drawn are quite different depending on whether the two types of individuals have the same LTC needs or not. We therefore study these two cases separately. We begin with the case of identical needs.

4 Social insurance: identical needs

In this section, we assume that, at each severity level of dependence, the two types of individuals have the same LTC needs, i.e. $L_{1h} = L_{1l} = L_1$ and $L_{2h} = L_{2l} = L_2$, with $L_1 < L_2$. We consider a social LTC insurance characterized by premiums P_i $(i = h, l)$ paid by each type of individuals and fractions α_{1i} $(i = h, l)$ and α_{2i} $(i = h, l)$ of LTC needs covered by the government in state 1 and state 2, with $0 \leq \alpha_{1i} \leq 1$ and $0 \leq \alpha_{2i} \leq 1$. Moreover, we assume that providing insurance is not costless for the government, i.e. the government faces loading costs $\lambda > 0$ which reflect, for instance, the associated administrative expenses. However, we also allow for the fact that insurance provision might be less costly for the government than for private insurers, i.e. we consider $\lambda \leqslant \widehat{\lambda}$.

We first assume that the government has full information and derive the first-best optimal allocation as well as discuss its possible decentralization. Afterwards, we turn to the second-best scenario where the government cannot observe individual types.

[5] See also Seog (2010).

[6] Regarding the relative costs of private and public health insurance and pension schemes see Diamond (1992) and Mitchell (1998). Both argue that public costs tend to be lower than private ones. For the high loading costs in the private LTC insurance market, see Brown and Finkelstein (2007).

4.1 First-best

To derive the first-best optimal allocation, we assume that the government has full information about the economy and in particular, it can observe individual types. The government maximizes the (utilitarian)[7] social welfare function subject to the resource constraint. The Lagrangean of the government's problem can be written in the following way:

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{i=h,l} n_i \left[\pi_1 u\left(c_i^{D_1}\right) + \pi_2 u\left(c_i^{D_2}\right) + (1-\pi_1-\pi_2)\, u\left(c_i^{I}\right) - v\left(\frac{y_i}{w_i}\right)\right] - \\
& -\mu \sum_{i=h,l} n_i \left[P_i - \pi_1(1+\lambda)\alpha_{1i}L_1 - \pi_2(1+\lambda)\alpha_{2i}L_2\right]
\end{aligned}
$$

where
$c_i^{D_1} = y_i - P_i - (1-\alpha_{1i})L_1$,
$c_i^{D_2} = y_i - P_i - (1-\alpha_{2i})L_2$,
$c_i^{I} = y_i - P_i$
and n_i is the share of type i ($i = h, l$) individuals in the society ($n_h + n_l = 1$), whereas μ is the Lagrange multiplier associated with the resource constraint.

The FOCs for P_i, α_{1i}, α_{2i} and y_i write as follows:

$$\frac{\partial \mathcal{L}}{\partial P_i} = -\pi_1 u'(c_i^{D_1}) - \pi_2 u'(c_i^{D_2}) - (1-\pi_1-\pi_2)\, u'(c_i^{I}) - \mu = 0 \tag{7}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{1i}} = u'(c_i^{D_1}) + \mu(1+\lambda) \leq 0, \qquad \alpha_{1i}\frac{\partial \mathcal{L}}{\partial \alpha_{1i}} = 0 \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{2i}} = u'(c_i^{D_2}) + \mu(1+\lambda) \leq 0, \qquad \alpha_{2i}\frac{\partial \mathcal{L}}{\partial \alpha_{2i}} = 0 \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial y_i} = \pi_1 u'(c_i^{D_1}) + \pi_2 u'(c_i^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_i^{I}) - \frac{v'\left(\frac{y_i}{w_i}\right)}{w_i} = 0 \tag{10}$$

Using equations (8) and (9) and proceeding in a similar way as in the *laissez-faire*, we can now define $D_i \equiv y_i - P_i - u'^{-1}(-\mu(1+\lambda))$ and we have that

$$\alpha_{1i} = \max\left[0; \frac{L_1 - D_i}{L_1}\right]$$

and

$$\alpha_{2i} = \max\left[0; \frac{L_2 - D_i}{L_2}\right]$$

[7] Note however that the results would also hold with a more general social welfare function using Pareto efficient weights.

We thus can see that optimal social insurance also features a deductible for each individual type i. In other words, it is socially optimal to equalize for each type i his marginal utilities in the states of nature where LTC needs are higher than D_i. Thus, if the needs are higher than D_i at both severity levels of dependence, $u'(c_i^{D_1}) = u'(c_i^{D_2})$ will hold, while we will have

$$\frac{u'(c_i^I)}{u'(c_i^{D_1})} = \frac{u'(c_i^I)}{u'(c_i^{D_2})} = \frac{1 - \pi_1(1+\lambda) - \pi_2(1+\lambda)}{(1 - \pi_1 - \pi_2)(1+\lambda)} < 1$$

as long as $\lambda > 0$. It can immediately be seen that if $\lambda = \widehat{\lambda}$, these tradeoffs are the same as in the private market.

To investigate more the allocations of each type, we can first note that combining equations (7) and (10) gives $\frac{v'\left(\frac{y_h}{w_h}\right)}{w_h} = \frac{v'\left(\frac{y_l}{w_l}\right)}{w_l}$, which implies $\frac{y_h}{w_h} > \frac{y_l}{w_l}$ and also $y_h > y_l$. However, it can be noted that equation (10) corresponds exactly to the FOC for y_i in the *laissez-faire*, which means that individual labour supply tradeoffs are not distorted. Further, using equations (7), (8) and (9), it can be verified that at the optimum we must have $y_h - P_h = y_l - P_l$. From the definition of D_i it then follows that we also have $D_h = D_l$. This therefore implies that wealth levels in each state of nature are equalized between the two types, even though the more productive type works (and earns) more than the less productive one. It can be verified that such an outcome is not achieved in the *laissez-faire* where type h always has a higher wealth.[8]

This brings us to the question of how the first-best allocation can be decentralized in our economy. If $\lambda < \widehat{\lambda}$, i.e. if providing insurance is less costly for the government than for private insurers, it is clearly more efficient to provide social insurance than to rely on the private market. In that case, social insurance with the above defined premiums P_h and P_l and deductibles D_h and D_l should be introduced. On the other hand, if $\lambda = \widehat{\lambda}$, it was just seen above that the socially optimal tradeoffs between the marginal utilities in different states of nature are the same as the ones arising in the private market. In other words, individual choices of insurance are efficient, and the only suboptimality in the *laissez-faire* economy comes from the absence of redistribution between the two types. In that case, introduction of social LTC insurance is not necessary: the task of insurance can be left to the private market while the government only needs to redistribute wealth from type h to type l using lump-sum transfers.

The conclusions of this subsection can be summarized in the following proposition:

Proposition 1 *Assume that high and low productivity individuals have the same LTC needs. As long as providing insurance is costly for the government (i.e.* $\lambda > 0$*), the first-best optimal social LTC insurance features a deductible which is the same for both types of individuals. The first-best optimality also requires to equalize wealth between the two types in each of the three states of nature. Social LTC insurance should be introduced if the government faces a lower loading cost than private insurers (i.e.* $\lambda < \widehat{\lambda}$*). If* $\lambda = \widehat{\lambda}$*, insurance can be left to the private market provided that lump-sum transfers from high to low productivity individuals are used by the government.*

[8] Using the comparative statics derived in Appendix A, it can be shown that individual wealth c_i in each state of nature is increasing in w_i.

4.2 Second-best

After having discussed the first-best optimality, we now drop the assumption of full information and study the case where the government cannot observe individual types. To be more precise, we assume that the government cannot observe individual productivity/wage w_i and individual labour supply l_i while it can observe the gross income y_i. In that case, the government has to make sure that the two types of individuals self-select and thus it has to respect the incentive compatibility constraints. It can be easily seen that the first-best optimal allocation clearly violates the incentive compatibility constraint of type h. This constraint will therefore be binding in the second-best.

The Lagrangean of the government can be written as follows:

$$\begin{aligned}\mathcal{L} \quad = \quad & \sum_{i=h,l} n_i \left[\pi_1 u\left(c_i^{D_1}\right) + \pi_2 u\left(c_i^{D_2}\right) + (1-\pi_1-\pi_2)\, u\left(c_i^I\right) - v\left(\frac{y_i}{w_i}\right)\right] - \\ & -\mu \sum_{i=h,l} n_i \left[P_i - \pi_1(1+\lambda)\alpha_{1i}L_1 - \pi_2(1+\lambda)\alpha_{2i}L_2\right] - \\ & -\gamma[\pi_1 u\left(c_h^{D_1}\right) + \pi_2 u\left(c_h^{D_2}\right) + (1-\pi_1-\pi_2)\, u\left(c_h^I\right) - v\left(\frac{y_h}{w_h}\right) - \\ & -\pi_1 u\left(c_l^{D_1}\right) - \pi_2 u\left(c_l^{D_2}\right) - (1-\pi_1-\pi_2)\, u\left(c_l^I\right) + v\left(\frac{y_l}{w_h}\right)]\end{aligned}$$

where
$c_i^{D_1} = y_i - P_i - (1-\alpha_{1i})L_1$,
$c_i^{D_2} = y_i - P_i - (1-\alpha_{2i})L_2$,
$c_i^I = y_i - P_i$
and γ is the Lagrange multiplier associated with type $h's$ incentive compatibility constraint.

The FOCs for P_i, α_{1i}, α_{2i} and y_i are given in Appendix B. We will now discuss the results obtained from these FOCs.

We can first note that the FOC for y_h (equation (62)) can be rearranged to get exactly the first-best (and, in turn, also the *laissez-faire*) FOC for y_h which implies

$$\frac{\frac{v'\left(\frac{y_h}{w_h}\right)}{w_h}}{\left[\pi_1 u'(c_h^{D_1}) + \pi_2 u'(c_h^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_h^I)\right]} = 1.$$

The labour supply choice of type h is thus not distorted. On the other hand, the FOC for y_l (equation (63)) can be rearranged to get

$$\frac{\frac{v'\left(\frac{y_l}{w_l}\right)}{w_l}}{\left[\pi_1 u'(c_l^{D_1}) + \pi_2 u'(c_l^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_l^I)\right]} = \frac{n_l + \gamma}{n_l + \gamma \frac{v'\left(\frac{y_l}{w_h}\right)/w_h}{v'\left(\frac{y_l}{w_l}\right)/w_l}} < 1.$$ [9]

[9] Note that $\gamma < 0$ and $\frac{v'\left(\frac{y_l}{w_h}\right)/w_h}{v'\left(\frac{y_l}{w_l}\right)/w_l} < 1$.

We thus see that labour supply of type l is distorted downwards, which helps to relax the incentive constraint of type h.

Turning to insurance and again proceeding in a similar way as before, from equations (58) and (59) we can now define $D_h \equiv y_h - P_h - u'^{-1}\left(\frac{-\mu(1+\lambda)n_h}{(n_h-\gamma)}\right)$ and from equations (60) and (61) we can now define $D_l \equiv y_l - P_l - u'^{-1}\left(\frac{-\mu(1+\lambda)n_l}{(n_l+\gamma)}\right)$, such that

$$\alpha_{1i} = \max\left[0; \frac{L_1 - D_i}{L_1}\right]$$

and

$$\alpha_{2i} = \max\left[0; \frac{L_2 - D_i}{L_2}\right]$$

Optimal social insurance thus features a deductible in the second-best as well and marginal utilities are again equalized in the states of nature where LTC needs exceed D_i. These marginal utilities are however no longer equalized between individuals. In particular, we now have $u'(c_h^{D_1}) = u'(c_h^{D_2}) = \frac{-\mu(1+\lambda)n_h}{(n_h-\gamma)} < u'(c_l^{D_1}) = u'(c_l^{D_2}) = \frac{-\mu(1+\lambda)n_l}{(n_l+\gamma)}$.[10] Moreover, using these expressions in (56) and (57), we get

$$u'(c_h^I) = \frac{-\mu n_h\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]}{(1-\pi_1-\pi_2)(n_h-\gamma)} < u'(c_l^I) = \frac{-\mu n_l\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]}{(1-\pi_1-\pi_2)(n_l+\gamma)}$$

which implies that $y_h - P_h > y_l - P_l$. Thus, unlike in the first-best, the redistribution of wealth between the two types is not complete. In other words, the second-best requires to leave some informational rent to type h.

On the other hand, it can be easily seen that for both types we have

$$\frac{u'(c_i^I)}{u'(c_i^{D_1})} = \frac{u'(c_i^I)}{u'(c_i^{D_2})} = \frac{1-\pi_1(1+\lambda)-\pi_2(1+\lambda)}{(1-\pi_1-\pi_2)(1+\lambda)} < 1$$

which are the same tradeoffs as in the first-best and the *laissez-faire* when $\lambda = \widehat{\lambda}$. We thus see that even in the second-best, redistribution does not require to distort insurance tradeoffs.

One might also ask what implications the second-best has for the socially optimal deductibles of the two types. While it is not possible to compare D_h and D_l in the general case, it can be seen using specific utility functions that the first-best result $D_h = D_l$ does not necessarily hold in the second-best. For instance, we show in Appendix C that we can have $D_h > D_l$ if the utility function is logarithmic. However, it is important to note that this result is only indirectly related to self-selection and redistribution. In fact, a logarithmic function is a function exhibiting DARA and since, as we have seen above, self-selection requires to leave some rent to type h, it is not surprising that, being wealthier, this type has a higher deductible. Indeed, we also show in Appendix C that if instead we assume an exponential utility function, which is a function exhibiting CARA, it becomes optimal to have $D_h = D_l$ as in the first-best. This implies that differences in the deductibles for the two types are due to risk aversion and not to distortions required by the second-best, which is also confirmed by the tradeoffs derived above.

[10] Note that $\mu < 0$ and $\gamma < 0$.

We can now discuss the implementation of the second-best optimal allocation. First, if $\lambda < \widehat{\lambda}$, similarly to the case of the first-best, the implementation should rely on social rather than on private LTC insurance. In that case, social insurance premiums and deductibles should be based on individual income y_i and the income of type l should be taxed at the margin. On the other hand, if $\lambda = \widehat{\lambda}$, the implementation can involve private insurance and it has to be stressed that no interference with individual insurance choices is needed. The only public intervention required is the introduction of a non-linear income tax, with a marginal tax for type l. This result is in fact in line with the classical result of Atkinson and Stiglitz (1976).

The above discussed findings can be summarized in the following proposition:

Proposition 2 *Assume that high and low productivity individuals have the same LTC needs. The second-best optimal allocation features a downward distortion of low productivity individuals' labour supply and an informational rent left to high productivity individuals, whereas insurance tradeoffs are not distorted. As long as providing insurance is costly for the government (i.e. $\lambda > 0$), the second-best optimal social insurance features a deductible which may be different for high and for low productivity individuals due to possibly different absolute risk aversion caused by incomplete redistribution between the two types. If the government faces a lower loading cost than private insurers (i.e. $\lambda < \widehat{\lambda}$), the implementation of the second-best optimum should rely on income-based social insurance with a marginal tax on low productivity individuals' income. If $\lambda = \widehat{\lambda}$, the second-best optimum can be implemented by introducing a non-linear income tax with a marginal tax on low productivity individuals' income and leaving insurance to the private market without any interference with individual choices.*

5 Social insurance: different needs

Having discussed the case where both types of individuals have the same LTC needs, we now consider the possibility that these needs (and thus the costs incurred) differ between the two types.[11] More precisely, we adopt a quite intuitive idea that more productive individuals might be somewhat more "spoiled" by their life, used to higher quality and more comfort or even feel obliged to comply with "standards" related to their social status, which might translate into their LTC needs being higher than those of the less productive type.[12] We thus assume in this section that, at each severity level of dependence, individuals of type h have higher LTC needs than individuals of type l (i.e. $L_{1h} > L_{1l}$ and $L_{2h} > L_{2l}$).[13] Moreover, we consider two possible positions that the government may have facing these differences in needs. In the first part of this section, we study the case where the government recognizes all needs as legitimate and thus accepts the fact that type h individuals need more. This is what we call a non-paternalistic case. On the other hand, the government might act in a paternalistic way in the sense of considering the higher needs of type h as being whimsical

[11] The dependence probabilities are assumed to remain the same for both types.

[12] For instance, these individuals might require more comfort or even "luxury" in a nursing home or want to go to a more "prestigious" nursing home.

[13] Apart from assuming that h has higher needs than l in both dependence states of nature, we do not impose any structure on their need differences in the two states: we allow for $L_{1h} - L_{1l} \lesseqgtr L_{2h} - L_{2l}$ and discuss the implications of these different cases.

and thus recognizing only a certain level of "legitimate" needs. We analyze this case in the second part of the section.

It should be also noted at this point that in the setting of type h having higher LTC costs than type l, it might be possible to have a *laissez-faire* outcome with type h being worse-off than type l, which, assuming that the government accepts all the needs, would require to redistribute resources from type l to type h. However, we focus on the (realistic) case where the costs of type h individuals are not too high and, due to their higher productivity, they still remain better-off in the *laissez-faire.*

5.1 No paternalism

In this subsection, we assume that the government fully recognizes the higher needs of type h and considers them as legitimate. We first study the first-best optimal solution under full information and then turn to the second-best setting with unobservable types.

5.1.1 First-best

In the first-best, the problem of the government in fact writes in the same way as in the first-best case of identical needs, except that the levels of LTC needs are now indexed by i (i.e. we now have L_{1i} and L_{2i} instead of L_1 and L_2). Solving the problem, we obtain again that, as long as $\lambda > 0$, it is optimal to have a deductible for each individual type i and that all tradeoffs are the same as in the private market if $\lambda = \widehat{\lambda}$. When all solutions are interior, we can easily verify that, just like with identical needs, at the optimum we must have $y_h - P_h = y_l - P_l$, which then implies $D_h = D_l$. We thus have that wealth levels in each state of nature are equalized between the two types, which means that resources are redistributed from type h to type l (given the above mentioned assumption that type h is better-off in the *laissez-faire* despite his higher costs). However, it should be noted that the government now pays more for LTC of type h than for that of type l (the amount above the deductible to be covered is larger for type h). Moreover, in this case it might be also possible to have a solution with $y_h - P_h > y_l - P_l$, $D_h > D_l$ and the needs in the low severity state of dependence lower than the deductible for type l or for both types (corner solutions). Indeed, from equation (7) we can see that the first-best requires to have

$$\pi_1 u'(c_h^{D_1}) + \pi_2 u'(c_h^{D_2}) + (1 - \pi_1 - \pi_2)\, u'(c_h^I) =$$

$$= \pi_1 u'(c_l^{D_1}) + \pi_2 u'(c_l^{D_2}) + (1 - \pi_1 - \pi_2)\, u'(c_l^I) \tag{11}$$

When solutions are interior, it is optimal to equalize between the two types the wealth levels in each state of nature because individuals pay the amount of the deductible at both severity levels and thus it is possible to make both types pay the same amount by giving them equal deductibles. However, when we have corner solution(s) in the low severity state, individuals have to pay the real costs in that state and thus the amounts they pay can no longer be equalized. Since type h has higher costs, to achieve the balance required by (11) it becomes optimal to redistribute less from h to l, but then type h has to pay a higher deductible so that he covers more of his LTC costs himself. We, however, focus more attention on interior solutions which, especially in the analysis of the second-best that will follow, allow to have more tractability. Nevertheless, summarizing the first-best case, it can be noted that, whether the solutions are purely interior or not, the common conclusion is that the first-best requires redistribution from type h to type l, but type h is also given some "compensation" (either because the government pays more for his LTC or because the redistribution is smaller) due to the fact that he has higher LTC costs.

As far as decentralization is concerned, similarly to the case of identical needs, social insurance should be provided if $\lambda < \widehat{\lambda}$. If $\lambda = \widehat{\lambda}$, insurance can be left to the private market and the government only needs to introduce appropriate lump-sum transfers from h to l, but these transfers are now lower than in the case of identical needs.

The above results are summarized in Proposition 3:

Proposition 3 *Assume that high productivity individuals have higher LTC needs than low productivity ones but these needs still allow them to remain better-off in the laissez-faire. Assume also that the government recognizes all needs as legitimate. As long as providing insurance is costly for the government (i.e. $\lambda > 0$), the first-best optimal social LTC insurance features a deductible. The first-best optimality requires to redistribute resources from high to low productivity individuals, but high productivity individuals are also given certain compensation due to the fact that they have higher needs. If the government faces a lower loading cost than private insurers (i.e. $\lambda < \widehat{\lambda}$), social insurance should be introduced, whereas if $\lambda = \widehat{\lambda}$, insurance can be left to the private market after appropriate lump-sum transfers are made by the government. These transfers are lower than in the case of identical needs.*

5.1.2 Second-best

We now turn to the setting where the government cannot observe individual types. In the case of different needs, this means that not only individual productivity and labour supply but also true LTC needs are not observable to the government. More precisely, we assume that the government can observe the severity level of dependence (which can generally be objectively assessed according to specially designed scales such as, for instance, the Katz scale) but cannot observe the true needs that a certain individual has at this severity level. The setting of different needs also implies that an individual of type h who wants to mimic an individual of type l not only has to earn income y_l but also has to accept the fact that his insurance will be based on the needs of type l. This obviously affects type $h's$ incentive compatibility constraint. In particular, this constraint now writes as follows:

$$\begin{aligned}
&\pi_1 u\left(y_h - P_h - (1-\alpha_{1h})L_{1h}\right) + \pi_2 u\left(y_h - P_h - (1-\alpha_{2h})L_{2h}\right) + \\
&+ (1-\pi_1-\pi_2)\, u\left(y_h - P_h\right) - v\left(\frac{y_h}{w_h}\right) \geqslant \pi_1 u\left(y_l - P_l - (1-\alpha_{1l})L_{1l} - \widehat{L}_{1h}\right) + \\
&+\pi_2 u\left(y_l - P_l - (1-\alpha_{2l})L_{2l} - \widehat{L}_{2h}\right) + (1-\pi_1-\pi_2)\, u\left(y_l - P_l\right) - v\left(\frac{y_l}{w_h}\right) \qquad (12)
\end{aligned}$$

where $\widehat{L}_{1h} = L_{1h} - L_{1l} > 0$ and $\widehat{L}_{2h} = L_{2h} - L_{2l} > 0$ are the differences between the needs of type h and type l. It should be noted that in this case of no paternalism, it is possible that type $h's$ incentive compatibility constraint will not be violated at the first-best optimal allocation because, as we saw above, even though the first-best requires redistribution from h to l, some compensation is given to h because of his higher needs which are fully accepted by the government. It might thus be possible that this compensation outweighs the fact that type h is required to work more and give some of his resources to type l. This suggests that in the case of no paternalism it might be possible to achieve the first-best even under asymmetric information about individual types. Nevertheless, this is not certain and we therefore focus on the case where the first-best allocation does not satisfy type $h's$ incentive compatibility and thus type $h's$ incentive constraint is binding in the second-best. We will see in the next subsection that this will always be the case with paternalism. It is therefore important to derive the second-best policy in this "benchmark" case of no paternalism to be able to compare the two.

The Lagrangean of the government's problem can thus be written as follows:

$$\begin{aligned}\mathcal{L} \;=\; & \sum_{i=h,l} n_i \left[\pi_1 u\left(c_i^{D_1}\right) + \pi_2 u\left(c_i^{D_2}\right) + (1-\pi_1-\pi_2)\, u\left(c_i^I\right) - v\left(\frac{y_i}{w_i}\right)\right] - \\ & -\mu \sum_{i=h,l} n_i \left[P_i - \pi_1(1+\lambda)\alpha_{1i}L_{1i} - \pi_2(1+\lambda)\alpha_{2i}L_{2i}\right] - \\ & -\gamma[\pi_1 u\left(c_h^{D_1}\right) + \pi_2 u\left(c_h^{D_2}\right) + (1-\pi_1-\pi_2)\, u\left(c_h^I\right) - v\left(\frac{y_h}{w_h}\right) - \\ & -\pi_1 u\left(\widetilde{c}_l^{D_1}\right) - \pi_2 u\left(\widetilde{c}_l^{D_2}\right) - (1-\pi_1-\pi_2)\, u\left(c_l^I\right) + v\left(\frac{y_l}{w_h}\right)]\end{aligned}$$

where
$c_i^I = y_i - P_i$,
$c_i^{D_1} = y_i - P_i - (1-\alpha_{1i})L_{1i}$,
$c_i^{D_2} = y_i - P_i - (1-\alpha_{2i})L_{2i}$,
$\widetilde{c}_l^{D_1} = y_l - P_l - (1-\alpha_{1l})L_{1l} - \widehat{L}_{1h}$,
$\widetilde{c}_l^{D_2} = y_l - P_l - (1-\alpha_{2l})L_{2l} - \widehat{L}_{2h}$, with $\widetilde{c}_l^{D_1}$ and $\widetilde{c}_l^{D_2}$ denoting the wealth levels of type h mimicking type l.

While the FOCs of this problem are given in Appendix D, we will now discuss their implications.

Let us first consider the FOCs for labour supply. As in the case of identical needs, the FOC for y_h (equation (70)) can be rearranged to get the first-best (and, in turn, also the *laissez-faire*) FOC for y_h. We thus again have no distortion of labour supply of type h. In contrast, the FOC for y_l (equation (71)) can be rearranged to get

$$\begin{aligned}&\frac{\frac{v'\left(\frac{y_l}{w_l}\right)}{w_l}}{\left[\pi_1 u'(c_l^{D_1}) + \pi_2 u'(c_l^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_l^I)\right]} = \frac{n_l+\gamma}{n_l + \gamma\frac{v'\left(\frac{y_l}{w_h}\right)/w_h}{v'\left(\frac{y_l}{w_l}\right)/w_l}} + \\ &+ \frac{\gamma\pi_1\left[u'(\widetilde{c}_l^{D_1}) - u'(c_l^{D_1})\right] + \gamma\pi_2\left[u'(\widetilde{c}_l^{D_2}) - u'(c_l^{D_2})\right]}{\left[n_l + \gamma\frac{v'\left(\frac{y_l}{w_h}\right)/w_h}{v'\left(\frac{y_l}{w_l}\right)/w_l}\right]\left[\pi_1 u'(c_l^{D_1}) + \pi_2 u'(c_l^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_l^I)\right]} < 1 \qquad (13)\end{aligned}$$

There is thus a downward distortion of labour supply of type l.

To discuss insurance, we can first note that using equations (66) and (67), we can define $D_h \equiv y_h - P_h - u'^{-1}\left(\frac{-\mu(1+\lambda)n_h}{(n_h-\gamma)}\right)$ such that

$$\alpha_{1h} = \max\left[0; \frac{L_{1h} - D_h}{L_{1h}}\right]$$

and

$$\alpha_{2h} = \max\left[0; \frac{L_{2h} - D_h}{L_{2h}}\right]$$

Thus, for type h, optimal social insurance as before features a deductible and type $h's$ marginal utilities are again equalized in the states of nature where LTC needs exceed D_h. If type $h's$ needs are higher than D_h at both severity levels of dependence, we will therefore have $u'(c_h^{D_1}) = u'(c_h^{D_2})$, while, moreover, we can check that

$$\frac{u'(c_h^I)}{u'(c_h^{D_1})} = \frac{u'(c_h^I)}{u'(c_h^{D_2})} = \frac{1 - \pi_1(1+\lambda) - \pi_2(1+\lambda)}{(1 - \pi_1 - \pi_2)(1+\lambda)} < 1$$

will hold as long as $\lambda > 0$. As before, if $\lambda = \widehat{\lambda}$, these tradeoffs are the same as in the private market. Type h thus faces no distortion of insurance tradeoffs.

Let us now turn to type l. As we are now going to show, optimal insurance for this type is in this case rather different from the cases studied before. To see this, let us first note that from (68) we have that either $\alpha_{1l} = 0$ or

$$n_l u'(c_l^{D_1}) + \mu n_l(1+\lambda) + \gamma u'(\widetilde{c}_l^{D_1}) = 0$$

$$\Longleftrightarrow$$

$$n_l u'(y_l - P_l - (1-\alpha_{1l})L_{1l}) + \mu n_l(1+\lambda) + \gamma u'(y_l - P_l - (1-\alpha_{1l})L_{1l} - \widehat{L}_{1h}) = 0 \qquad (14)$$

Moreover, from (69) we have that either $\alpha_{2l} = 0$ or

$$n_l u'(c_l^{D_2}) + \mu n_l(1+\lambda) + \gamma u'(\widetilde{c}_l^{D_2}) = 0$$

$$\Longleftrightarrow$$

$$n_l u'(y_l - P_l - (1-\alpha_{2l})L_{2l}) + \mu n_l(1+\lambda) + \gamma u'(y_l - P_l - (1-\alpha_{2l})L_{2l} - \widehat{L}_{2h}) = 0 \qquad (15)$$

We can verify from (14) and (15) that now we no longer have a state-independent deductible as we had in the previous cases. In fact, we now have that the deductible paid by type l will generally have to be different at each severity level of dependence and this difference will depend on the comparison of $\widehat{L}_{1h}$ and $\widehat{L}_{2h}$, i.e. the differences between the needs of type h and type l at each severity level.

To see this, let us first assume that $\widehat{L}_{2h} > \widehat{L}_{1h}$, i.e. that the difference between the needs of type h and type l is larger when the severity level of dependence is high (state 2) than when it is low (state 1). Let us also assume that $\alpha_{1l}^0 > 0$ is a solution to equation (14) and denote $(1-\alpha_{1l}^0)L_{1l} \equiv D_{1l}$. We can also define α'_{2l} such that $(1-\alpha'_{2l})L_{2l} = D_{1l}$. It can then be checked that the left-hand side of (15) evaluated at α'_{2l} is negative, which means that the optimal value of α_{2l} is lower than α'_{2l}. Denoting this value by α_{2l}^0 and defining $(1-\alpha_{2l}^0)L_{2l} \equiv D_{2l}$, we have that $D_{2l} > D_{1l}$.

Similarly, if $\widehat{L}_{2h} < \widehat{L}_{1h}$, we get that $D_{2l} < D_{1l}$. Only if $\widehat{L}_{2h} = \widehat{L}_{1h}$, we will have $D_{2l} = D_{1l} = D_l$.

These results are in fact quite intuitive. As mentioned above, if an individual of type h wants to mimic an individual of type l, he has to accept the fact that his insurance will be based on the needs of type l and thus to fully cover himself his additional needs (i.e. $\widehat{L}_{1h}$ or $\widehat{L}_{2h}$). Nevertheless, insurance based on type $l's$ needs still helps to balance to some extent type $h's$ wealth in the two dependence states of nature. To achieve a better balance, type h would prefer this insurance to be more generous in the state where his additional needs are larger. Therefore, to make type $l's$ allocation less attractive to type h, type $l's$ allocation is designed exactly in the opposite way: in the state of nature where the additional needs of type h are larger, type l gets a higher deductible (and thus less insurance) than in the state of nature where the additional needs of type h are smaller.

In other words, this means that it is generally no longer optimal to equalize type $l's$ marginal utilities in the two dependence states of nature even when all solutions are interior. Indeed, from the discussion above it follows that $\frac{u'(c_l^{D_1})}{u'(c_l^{D_2})} < 1$ if $\widehat{L}_{2h} > \widehat{L}_{1h}$, $\frac{u'(c_l^{D_1})}{u'(c_l^{D_2})} > 1$ if $\widehat{L}_{2h} < \widehat{L}_{1h}$, and only we have $\frac{u'(c_l^{D_1})}{u'(c_l^{D_2})} = 1$ if $\widehat{L}_{2h} = \widehat{L}_{1h}$. Moreover, using (14), (15) and (65) and performing some manipulations, we can get

$$\frac{u'(c_l^I)}{u'(c_l^{D_1})} = \frac{-\mu n_l^2(1+\lambda)}{\left[-\mu n_l^2(1+\lambda) - \gamma n_l\left(u'(\widetilde{c}_l^{D_1}) - u'(c_l^{D_1})\right)\right]} \frac{[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)]}{(1-\pi_1-\pi_2)(1+\lambda)} < \\ < \frac{1-\pi_1(1+\lambda)-\pi_2(1+\lambda)}{(1-\pi_1-\pi_2)(1+\lambda)} \tag{16}$$

and

$$\frac{u'(c_l^I)}{u'(c_l^{D_2})} = \frac{-\mu n_l^2(1+\lambda)}{\left[-\mu n_l^2(1+\lambda) - \gamma n_l\left(u'(\widetilde{c}_l^{D_2}) - u'(c_l^{D_2})\right)\right]} \frac{[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)]}{(1-\pi_1-\pi_2)(1+\lambda)} < \\ < \frac{1-\pi_1(1+\lambda)-\pi_2(1+\lambda)}{(1-\pi_1-\pi_2)(1+\lambda)} \tag{17}$$

We can thus see that the ratios $\frac{u'(c_l^I)}{u'(c_l^{D_1})}$ and $\frac{u'(c_l^I)}{u'(c_l^{D_2})}$ are also distorted and in particular, they are lower than the first-best ones. This means that, besides the distortion of the tradeoff between insurance in the two dependence states arising when $\widehat{L}_{2h} \neq \widehat{L}_{1h}$, there is also a downward distortion of type $l's$ insurance in general, and this distortion is present even if $\widehat{L}_{2h} = \widehat{L}_{1h}$. Indeed, if $\widehat{L}_{2h} = \widehat{L}_{1h}$, the tradeoff between the two dependence states will not be distorted, but the tradeoffs between each dependence state and the healthy state will still be subject to a downward distortion. This result is again quite intuitive: since type h has higher needs, he values insurance more than type l; thus, to make type $l's$ allocation less attractive it is optimal to distort his insurance downwards. It is also interesting to note that the ratios $\frac{u'(c_l^I)}{u'(c_l^{D_1})}$ and $\frac{u'(c_l^I)}{u'(c_l^{D_2})}$ would be smaller than 1 even with $\lambda = 0$, which means that type l would face a deductible even if the government had no loading costs.

In addition to the discussed distortions, it can be also verified that, as in the second-best with identical needs, type h is again given some informational rent: using equations (64)-(69) it can be checked that in the case of interior solutions type h now has lower marginal utilities than type l in all states of nature, in contrast to the equality of marginal utilities of the two types in the first-best.

As with identical needs, we can also ask ourselves about the implications of the second-best optimality to the comparison of optimal deductibles between the two types. These implications again depend on the specification of individual utility functions. The most informative case is that of an exponential utility function which, as mentioned above, exhibits CARA. It can be shown that with this utility function the state-independent deductible given to type h is lower than each of the state-dependent deductibles given to type l, i.e. $D_h < D_{1l}$ and $D_h < D_{2l}$. In this "pure" case in which absolute risk aversion does not depend on wealth, the comparison of optimal deductibles exactly reflects the downward distortion of type $l's$ insurance. On the other hand, with different utility functions the influence of this distortion is less clearly seen because a role is also played by differences in absolute risk aversion caused by the differences in wealth present in the second-best. For instance, with DARA preferences the comparison of optimal deductibles between the two types is not clear since the lower wealth of type l pushes for a lower deductible for this type while the insurance distortion requires a higher one.

Finally, we can discuss how the above defined second-best optimum could be implemented. Again, if $\lambda < \widehat{\lambda}$, the implementation should rely on social rather than on private LTC insurance. Social insurance should be based on individual income y_i and designed in the way described above. Individuals of type l should face a marginal tax on their income. On the other hand, if $\lambda = \widehat{\lambda}$, private insurance can be involved in the implementation; however, interference with individual choices of type l is now needed. First of all, insurance of type l has to be taxed at the margin. Nevertheless, a marginal tax on type $l's$ insurance premium will generally not be sufficient since, when $\widehat{L}_{2h} \neq \widehat{L}_{1h}$, an additional instrument is needed to distort the tradeoff between the two dependence states of nature. This means that a marginal tax or subsidy has to be introduced on type $l's$ private insurance deductible (or on the benefit received from the insurer) in at least one dependence state of nature. In addition to this, a non-linear income tax is needed and type $l's$ income has to be taxed at the margin.

We now summarize the above derived results in the following proposition:

Proposition 4 *Assume that high productivity individuals have higher LTC needs than low productivity ones but these needs still allow them to remain better-off in the laissez-faire. Assume also that the government recognizes all needs as legitimate. The second-best optimal allocation features an informational rent left to high productivity individuals and a downward distortion of low productivity individuals' labour supply as well as of their insurance coverage. Moreover, if the difference between the needs of high and low productivity individuals is not the same at both severity levels of dependence (i.e.* $\widehat{L}_{2h} \neq \widehat{L}_{1h}$*), low productivity individuals also face a distortion of insurance tradeoff between the two severity levels. Optimal social LTC insurance features a deductible for high productivity individuals as long as providing insurance is costly for the government (i.e.* $\lambda > 0$*), whereas* *low productivity individuals face a deductible even when $\lambda = 0$. High productivity individuals face a state-independent deductible, while the deductible for low productivity individuals is state-dependent as long as $\widehat{L}_{2h} \neq \widehat{L}_{1h}$. If the government faces a lower loading cost than private insurers (i.e. $\lambda < \widehat{\lambda}$), the implementation of the second-best optimum should rely on income-based social insurance with a marginal tax on low productivity individuals' income. If $\lambda = \widehat{\lambda}$, private insurance can be involved, but this requires a marginal tax on low productivity individuals' insurance premiums and, when $\widehat{L}_{2h} \neq \widehat{L}_{1h}$, a marginal tax or subsidy on their deductibles in at least one dependence state of nature. A marginal tax on low productivity individuals' income is also required.*

5.2 Paternalism

We now turn to the idea that fully recognizing the higher needs of the somewhat "spoiled" type h might be an inappropriate approach for the government. Therefore, in this subsection we assume that the government recognizes as legitimate only a certain level of needs: $\overline{L}_1$ when the severity level of dependence is low and $\overline{L}_2 > \overline{L}_1$ when the severity level is high. For simplicity, we assume that the legitimate levels of needs coincide with the needs of type l, i.e. $\overline{L}_1 = L_{1l} < L_{1h}$ and $\overline{L}_2 = L_{2l} < L_{2h}$.

The fact that the government accepts only legitimate needs translates into social insurance being based on these legitimate needs for all individuals (and not on their higher needs for type h). For individuals of type h this therefore means that, in addition to the part of legitimate needs not covered by the government (the deductible), they will also have to fully cover their additional needs, whereas individuals of type l will only have to pay the part of legitimate needs not covered by the government. In other words, the expected utility of type l in the presence of the government's policy can be written as

$$\pi_1 u\left(y_l - P_l - (1-\alpha_{1l})\overline{L}_1\right) + \pi_2 u\left(y_l - P_l - (1-\alpha_{2l})\overline{L}_2\right) + \\ + (1-\pi_1-\pi_2)\, u\left(y_l - P_l\right) - v\left(\frac{y_l}{w_l}\right) \tag{18}$$

whereas the expected utility of type h writes as

$$\pi_1 u\left(y_h - P_h - (1-\alpha_{1h})\overline{L}_1 - \widehat{L}_{1h}\right) + \pi_2 u\left(y_h - P_h - (1-\alpha_{2h})\overline{L}_2 - \widehat{L}_{2h}\right) + \\ + (1-\pi_1-\pi_2)\, u\left(y_h - P_h\right) - v\left(\frac{y_h}{w_h}\right) \tag{19}$$

where $\widehat{L}_{1h}$ and $\widehat{L}_{2h}$ are defined as before as the differences between the needs of type h and type l which are now also equivalent to the differences between the needs of type h and the legitimate needs.

Moreover, since the government considers the needs $\overline{L}_1$ and $\overline{L}_2$ as sufficient, only these needs are taken into account in its objective function. In other words, the objective function of the government does not consider the additional costs $\widehat{L}_{1h}$ and $\widehat{L}_{2h}$ borne by type h to satisfy his higher needs.

Given this setting, we will now discuss the first-best optimal allocation achieved under full information and then we will look at the second-best with unobservable types.

5.2.1 First-best

In the first-best, the Lagrangean of the government can be written as follows:

$$\begin{aligned}\mathcal{L} &= \sum_{i=h,l} n_i \left[\pi_1 u\left(\overline{c}_i^{D_1}\right) + \pi_2 u\left(\overline{c}_i^{D_2}\right) + (1-\pi_1-\pi_2)\, u\left(c_i^I\right) - v\left(\frac{y_i}{w_i}\right)\right] - \\ & -\mu \sum_{i=h,l} n_i \left[P_i - \pi_1(1+\lambda)\alpha_{1i}\overline{L}_1 - \pi_2(1+\lambda)\alpha_{2i}\overline{L}_2\right]\end{aligned}$$

where
$c_i^I = y_i - P_i$,
$\overline{c}_i^{D_1} = y_i - P_i - (1-\alpha_{1i})\overline{L}_1$,
$\overline{c}_i^{D_2} = y_i - P_i - (1-\alpha_{2i})\overline{L}_2$.

The bar above the wealth levels denotes the fact that the government considers only $\overline{L}_1$ and $\overline{L}_2$. Note that for type l, $\overline{c}_l^{D_1} = c_l^{D_1}$ and $\overline{c}_l^{D_2} = c_l^{D_2}$, but for type h, $\overline{c}_h^{D_1} > c_h^{D_1} = y_h - P_h - (1-\alpha_{1h})\overline{L}_1 - \widehat{L}_{1h}$ and $\overline{c}_h^{D_2} > c_h^{D_2} = y_h - P_h - (1-\alpha_{2h})\overline{L}_2 - \widehat{L}_{2h}$.

The FOCs of the government's problem write as follows:

$$\frac{\partial \mathcal{L}}{\partial P_i} = -\pi_1 u'(\overline{c}_i^{D_1}) - \pi_2 u'(\overline{c}_i^{D_2}) - (1-\pi_1-\pi_2)\, u'(c_i^I) - \mu = 0 \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{1i}} = u'(\overline{c}_i^{D_1}) + \mu(1+\lambda) \leq 0, \qquad \alpha_{1i}\frac{\partial \mathcal{L}}{\partial \alpha_{1i}} = 0 \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{2i}} = u'(\overline{c}_i^{D_2}) + \mu(1+\lambda) \leq 0, \qquad \alpha_{2i}\frac{\partial \mathcal{L}}{\partial \alpha_{2i}} = 0 \tag{22}$$

$$\frac{\partial \mathcal{L}}{\partial y_i} = \pi_1 u'(\overline{c}_i^{D_1}) + \pi_2 u'(\overline{c}_i^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_i^I) - \frac{v'\left(\frac{y_i}{w_i}\right)}{w_i} = 0 \tag{23}$$

The solution of the government's problem is in fact analogous to the first-best solution in the case of identical needs except that here the needs considered by the government are the "legitimate" needs (which are lower than the true needs of type h). In particular, we can define $D_i \equiv y_i - P_i - u'^{-1}(-\mu(1+\lambda))$ and we have that

$$\alpha_{1i} = \max\left[0; \frac{\overline{L}_1 - D_i}{\overline{L}_1}\right]$$

and

$$\alpha_{2i} = \max\left[0; \frac{\overline{L}_2 - D_i}{\overline{L}_2}\right]$$

Optimal social insurance again features a deductible, but the deductible now has to be compared with the "legitimate" needs and, if these needs exceed the deductible at both severity levels, we now have the equalities $u'(\bar{c}_i^{D_1}) = u'(\bar{c}_i^{D_2})$ and

$$\frac{u'(c_i^I)}{u'(\bar{c}_i^{D_1})} = \frac{u'(c_i^I)}{u'(\bar{c}_i^{D_2})} = \frac{1 - \pi_1(1+\lambda) - \pi_2(1+\lambda)}{(1 - \pi_1 - \pi_2)(1+\lambda)} < 1$$

which do not necessarily coincide with the tradeoffs in terms of the true marginal utilities of individuals. In fact, they obviously do coincide for type l but do not coincide for type h. Indeed, when $u'(\bar{c}_h^{D_1}) = u'(\bar{c}_h^{D_2})$ holds, for type h we have

$$\frac{u'(c_h^{D_1})}{u'(c_h^{D_2})} = \frac{u'(y_h - P_h - D_h - \widehat{L}_{1h})}{u'(y_h - P_h - D_h - \widehat{L}_{2h})} \gtrless 1 \text{ if } \widehat{L}_{1h} \gtrless \widehat{L}_{2h} \tag{24}$$

as well as

$$\frac{u'(c_h^I)}{u'(c_h^{D_1})} < \frac{1 - \pi_1(1+\lambda) - \pi_2(1+\lambda)}{(1 - \pi_1 - \pi_2)(1+\lambda)} \tag{25}$$

and

$$\frac{u'(c_h^I)}{u'(c_h^{D_2})} < \frac{1 - \pi_1(1+\lambda) - \pi_2(1+\lambda)}{(1 - \pi_1 - \pi_2)(1+\lambda)}. \tag{26}$$

Therefore, for type l, the first-best optimal allocation implies that the tradeoffs between his true marginal utilities are the same as in the first-best with identical needs or in the first-best with different needs and no paternalism. Moreover, if $\lambda = \widehat{\lambda}$, these tradeoffs are the same as the ones achieved in the private market.

For type h, however, the tradeoffs between his true marginal utilities are different from the first-best ones obtained with identical needs or with different needs and no paternalism. In particular, we can see from (25) and (26) that now type h is not insured against his LTC needs as well as before. Indeed, since his needs are now higher than accepted by the government, a part of his needs is not taken into account in the determination of socially optimal insurance, which results in him being insured against his true needs more "poorly" than before. Moreover, since the government does not take into account a part of his needs, the socially optimal insurance does not properly balance his wealth in the two dependence states of nature if the parts of the needs which are not accounted for are different in these two states, as it can be seen in (24). In fact, if $\widehat{L}_{1h} \neq \widehat{L}_{2h}$, the first-best allocation implies that type h implicitly faces state-dependent deductibles: in addition to the state-independent social insurance deductible D_h, he has to pay $\widehat{L}_{1h}$ in state 1 and $\widehat{L}_{2h}$ in state 2, which means that the total amount paid in the two states is different. It can be also noted that when $\lambda = \widehat{\lambda}$, differently from the case of type l, type $h's$ tradeoffs implied by the first-best allocation are different from the private market ones. The optimal allocation thus implies a "correction" of type $h's$ insurance choices.

However, not only insurance choices of type h need to be corrected. From equation (23), we have that the optimal tradeoff for type $h's$ labour supply is

$$\frac{\frac{v'\left(\frac{y_h}{w_h}\right)}{w_h}}{\left[\pi_1 u'(\overline{c}_h^{D_1}) + \pi_2 u'(\overline{c}_h^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_h^I)\right]} = 1.$$

This implies that in terms of type $h's$ true marginal utilities we have

$$\frac{\frac{v'\left(\frac{y_h}{w_h}\right)}{w_h}}{\left[\pi_1 u'(c_h^{D_1}) + \pi_2 u'(c_h^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_h^I)\right]} =$$

$$= 1 - \frac{\pi_1 \left[u'(c_h^{D_1}) - u'(\overline{c}_h^{D_1})\right] + \pi_2 \left[u'(c_h^{D_2}) - u'(\overline{c}_h^{D_2})\right]}{\left[\pi_1 u'(c_h^{D_1}) + \pi_2 u'(c_h^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_h^I)\right]} < 1 \qquad (27)$$

This means that, compared to the *laissez-faire*, type $h's$ labour supply is "corrected" downwards. Indeed, individuals of type h consider higher expected LTC needs than the government and thus they find it necessary to earn more to be ready to face those needs. From the point of view of the government, however, lower needs are sufficient and thus there is no necessity to exert too much work effort to cover "unnecessary" additional needs. Nevertheless, using equations (23) and (20), it can be checked that even though the first-best level of type $h's$ labour supply is lower than the *laissez-faire* one, it is still higher than the first-best labour supply of type l. The more productive type still has to work more than the less productive one. Moreover, it can be easily seen that the first-best does not require any corrections of labour supply choices of type l.

We can now look at redistributional issues. Using equations (20), (21) and (22), it can be verified that at the optimum we always have $y_h - P_h = y_l - P_l$ and $D_h = D_l$. There is thus redistribution of wealth from type h to type l. In addition to this, it is important to note that, in contrast to the case of no paternalism, the government now covers the same amount of LTC costs for both types (i.e. the difference between the legitimate needs and the deductible which is the same for both types). Consequently, in the two dependence states of nature, the (true) wealth level of type h is lower than that of type l because type h incurs additional costs which are not considered by the government. Type h is thus no longer given any compensation for the fact that he has higher needs.

Turning to the decentralization of the first-best optimum, again, if $\lambda < \widehat{\lambda}$, social insurance characterized above should be introduced instead of relying on the private market. In addition to this, a marginal tax on type $h's$ income is needed since from the government's point of view, type h works too much. On the other hand, if $\lambda = \widehat{\lambda}$, private insurance can be involved, but this requires certain additional instruments to correct for the choice of type h. In particular, since the government does not recognize the full needs of type h, from its point of view, type h buys too much insurance, which implies that type $h's$ insurance purchases have to be taxed. This means that a marginal tax on type $h's$ insurance premium is needed. Nevertheless, this tax alone is generally not enough since, as we saw above, the first-best allocation implies that type $h's$ marginal utilities in the two dependence states of nature are not equalized as long as $\widehat{L}_{1h} \neq \widehat{L}_{2h}$, which requires an additional tax or subsidy applied to the private insurance deductible in one of the dependence states of nature. Indeed, the policy has to correct for the fact that type h takes into account "unnecessary" needs which exceed the sufficient (legitimate) needs and so a different extent of correction is needed in the states of nature where the legitimate needs are exceeded by different amounts. This actually means that in the private market type h is forced to buy insurance with state-dependent deductibles. Finally, lump-sum transfers have to be used to redistribute resources from h to l and a marginal tax on type $h's$ income is needed to discourage type h from working "too much".

The above discussion is summarized in the following proposition:

Proposition 5 *Assume that high productivity individuals have higher LTC needs than low productivity ones but these needs still allow them to remain better-off in the laissez-faire. Assume also that the government does not accept these higher needs as legitimate. As long as providing insurance is costly for the government (i.e. $\lambda > 0$), the first-best optimal social LTC insurance features a deductible which is the same for both types of individuals. The first-best allocation equalizes the wealth levels of the two individual types in the healthy state of nature but implies high productivity individuals having lower wealth in both dependence states of nature since their higher needs are not taken into account. If the government faces a lower loading cost than private insurers (i.e. $\lambda < \widehat{\lambda}$), the decentralization of the first-best optimum should rely on social LTC insurance. If $\lambda = \widehat{\lambda}$, private insurance can be involved, but this requires a marginal "corrective" tax on high productivity individuals' insurance premiums and, when $\widehat{L}_{2h} \neq \widehat{L}_{1h}$, a marginal tax or subsidy on their deductibles in at least one dependence state of nature. Lump-sum transfers from high to low productivity individuals are also needed. Moreover, in both cases of loading costs, a marginal tax on high productivity individuals' income is required since they work too much from the paternalistic point of view.*

5.2.2 Second-best

Let us now study the paternalistic case when individual types are not observable to the government. In this case, we have the following incentive compatibility constraint for type h:

$$\begin{aligned}
&\pi_1 u\left(y_h - P_h - (1-\alpha_{1h})\overline{L}_1 - \widehat{L}_{1h}\right) + \pi_2 u\left(y_h - P_h - (1-\alpha_{2h})\overline{L}_2 - \widehat{L}_{2h}\right) + \\
&+ (1-\pi_1-\pi_2)\, u\left(y_h - P_h\right) - v\left(\frac{y_h}{w_h}\right) \geqslant \pi_1 u\left(y_l - P_l - (1-\alpha_{1l})\overline{L}_1 - \widehat{L}_{1h}\right) + \\
&+ \pi_2 u\left(y_l - P_l - (1-\alpha_{2l})\overline{L}_2 - \widehat{L}_{2h}\right) + (1-\pi_1-\pi_2)\, u\left(y_l - P_l\right) - v\left(\frac{y_l}{w_h}\right) \qquad (28)
\end{aligned}$$

It can be easily seen that this constraint is always violated by the first-best optimal allocation, which implies that it will be binding in the second-best. We can therefore write the following Lagrangean of the government's problem:

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{i=h,l} n_i \left[\pi_1 u\left(\overline{c}_i^{D_1}\right) + \pi_2 u\left(\overline{c}_i^{D_2}\right) + (1-\pi_1-\pi_2)\, u\left(c_i^I\right) - v\left(\frac{y_i}{w_i}\right)\right] - \\
& -\mu \sum_{i=h,l} n_i \left[P_i - \pi_1(1+\lambda)\alpha_{1i}\overline{L}_1 - \pi_2(1+\lambda)\alpha_{2i}\overline{L}_2\right] - \\
& -\gamma[\pi_1 u\left(c_h^{D_1}\right) + \pi_2 u\left(c_h^{D_2}\right) + (1-\pi_1-\pi_2)\, u\left(c_h^I\right) - v\left(\frac{y_h}{w_h}\right) - \\
& -\pi_1 u\left(\widetilde{c}_l^{D_1}\right) - \pi_2 u\left(\widetilde{c}_l^{D_2}\right) - (1-\pi_1-\pi_2)\, u\left(c_l^I\right) + v\left(\frac{y_l}{w_h}\right)]
\end{aligned}
$$

where
$c_i^I = y_i - P_i$,
$\overline{c}_i^{D_1} = y_i - P_i - (1-\alpha_{1i})\overline{L}_1$,
$\overline{c}_i^{D_2} = y_i - P_i - (1-\alpha_{2i})\overline{L}_2$,
$c_h^{D_1} = y_h - P_h - (1-\alpha_{1h})\overline{L}_1 - \widehat{L}_{1h}$,
$c_h^{D_2} = y_h - P_h - (1-\alpha_{2h})\overline{L}_2 - \widehat{L}_{2h}$,
$\widetilde{c}_l^{D_1} = y_l - P_l - (1-\alpha_{1l})\overline{L}_1 - \widehat{L}_{1h}$,
$\widetilde{c}_l^{D_2} = y_l - P_l - (1-\alpha_{2l})\overline{L}_2 - \widehat{L}_{2h}$.

The FOCs are provided in Appendix E and we now discuss their implications.
Let us first consider the FOCs for y_h and y_l (equations (78) and (79)). From (78), we have that

$$
\begin{aligned}
& \frac{\frac{v'\left(\frac{y_h}{w_h}\right)}{w_h}}{\left[\pi_1 u'(c_h^{D_1}) + \pi_2 u'(c_h^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_h^I)\right]} = \\
& = 1 - \frac{n_h}{(n_h-\gamma)}\left[\frac{\pi_1\left[u'(c_h^{D_1}) - u'(\overline{c}_h^{D_1})\right] + \pi_2\left[u'(c_h^{D_2}) - u'(\overline{c}_h^{D_2})\right]}{\left[\pi_1 u'(c_h^{D_1}) + \pi_2 u'(c_h^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_h^I)\right]}\right] < 1 \qquad (29)
\end{aligned}
$$

Thus, as in the first-best, labour supply of type h still requires a correction with respect to the *laissez-faire* choice. However, comparing the tradeoff in equation (29) to the tradeoff obtained in the first-best (equation (27)), we can see that, all other things being equal, the second-best tradeoff is closer to 1 and thus implies a smaller distortion of type $h's$ labour supply. This shows that even though the government still corrects type $h's$ choice, it has to go into a certain compromise in order to make type $h's$ allocation more desirable to himself and thus prevent him from mimicking type l. Moreover, using (79), it can be checked that, differently from the first-best, labour supply of type l is now also distorted downwards.[14] This also helps to prevent mimicking by making type $l's$ allocation less attractive to type h.

[14] Equation (79) in fact implies the same tradeoff as the one obtained in the case with no paternalism (see equation (13)).

Turning to insurance, it can first be easily verified that all the results concerning type l are in this case the same as in the case with no paternalism. As in that case, type $l's$ insurance is distorted to make type $l's$ allocation less desirable to type h.

Let us now look at type h. From (74), we have that either $\alpha_{1h} = 0$ or

$$n_h u'(\bar{c}_h^{D_1}) + \mu n_h (1+\lambda) - \gamma u'(c_h^{D_1}) = 0$$

$$\Longleftrightarrow$$

$$n_h u'(y_h - P_h - (1-\alpha_{1h})\overline{L}_1) + \mu n_h (1+\lambda) - \gamma u'(y_h - P_h - (1-\alpha_{1h})\overline{L}_1 - \widehat{L}_{1h}) = 0 \qquad (30)$$

Moreover, from (75), we have that either $\alpha_{2h} = 0$ or

$$n_h u'(\bar{c}_h^{D_2}) + \mu n_h (1+\lambda) - \gamma u'(c_h^{D_2}) = 0$$

$$\Longleftrightarrow$$

$$n_h u'(y_h - P_h - (1-\alpha_{2h})\overline{L}_2) + \mu n_h (1+\lambda) - \gamma u'(y_h - P_h - (1-\alpha_{2h})\overline{L}_2 - \widehat{L}_{2h}) = 0 \qquad (31)$$

Equations (30) and (31) imply that, differently from the cases analyzed before, optimal social insurance now generally features a state-dependent deductible for type h. To see this, assume first that $\widehat{L}_{2h} > \widehat{L}_{1h}$. Also assume that α_{1h}^0 is a solution to equation (30) and denote $(1-\alpha_{1h}^0)\overline{L}_1 \equiv D_{1h}$. We can also define α'_{2h} such that $(1-\alpha'_{2h})\overline{L}_2 = D_{1h}$. It can then be verified that the left-hand side of (31) evaluated at α'_{2h} is positive, which means that the optimal value of α_{2h} is higher than α'_{2h}. Denoting this value by α_{2h}^0 and defining $(1-\alpha_{2h}^0)\overline{L}_2 \equiv D_{2h}$, we have that $D_{2h} < D_{1h}$.

Similarly, if $\widehat{L}_{2h} < \widehat{L}_{1h}$, we get that $D_{2h} > D_{1h}$. Only if $\widehat{L}_{2h} = \widehat{L}_{1h}$, we will have $D_{2h} = D_{1h} = D_h$.

Let us recall that type l also faces a state-dependent deductible; however, it is interesting to note that for type h, the comparison of the deductibles in the two dependence states of nature is opposite to their comparison for type l. Indeed, in contrast to type l, type h gets a lower deductible (and thus more insurance) in the state of nature where the difference between his true needs and the legitimate needs is higher than in the state where this difference is lower. The intuition for this result is quite simple. Since the paternalistic government provides insurance only against legitimate needs, type h has to fully cover his additional costs himself. In the first-best, these additional costs are not at all taken into account by the government and social insurance thus features a unique deductible which equalizes wealth in the two dependence states of nature given that there are no additional needs. However, if $\widehat{L}_{2h} \neq \widehat{L}_{1h}$, for type h this means that his true wealth is not equalized. On the other hand, in the second-best, the government has to ensure incentive compatibility and so, similarly to the case of labour supply, it has to make a certain concession. Thus, even though it still bases its insurance on the legitimate needs, the insurance is designed to allow a better (although still not perfect) balance of the true wealth levels of type h by providing a better protection against the legitimate needs in the state of nature where the uncovered needs are higher.

We can also look at the tradeoffs between the dependence states of nature and the healthy state. Using equations (72), (30) and (31), we can get the following tradeoffs in terms of the marginal utilities as considered by the government:

$$\frac{u'(c_h^I)}{u'(\overline{c}_h^{D_1})} = \frac{-\mu n_h^2(1+\lambda)}{\left[-\mu n_h^2(1+\lambda) - \gamma n_h \left(u'(\overline{c}_h^{D_1}) - u'(c_h^{D_1})\right)\right]} \frac{[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)]}{(1-\pi_1-\pi_2)(1+\lambda)} >$$

$$> \frac{1-\pi_1(1+\lambda)-\pi_2(1+\lambda)}{(1-\pi_1-\pi_2)(1+\lambda)}$$

and

$$\frac{u'(c_h^I)}{u'(\overline{c}_h^{D_2})} = \frac{-\mu n_h^2(1+\lambda)}{\left[-\mu n_h^2(1+\lambda) - \gamma n_h \left(u'(\overline{c}_h^{D_2}) - u'(c_h^{D_2})\right)\right]} \frac{[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)]}{(1-\pi_1-\pi_2)(1+\lambda)} >$$

$$> \frac{1-\pi_1(1+\lambda)-\pi_2(1+\lambda)}{(1-\pi_1-\pi_2)(1+\lambda)}.$$

We can thus see that there is an upward distortion of insurance against the legitimate needs compared to the first-best allocation. This comes again from the need to ensure incentive compatibility: even though social insurance is based only on the legitimate needs, to prevent mimicking the government makes a concession by providing a more generous coverage against these needs than in the first-best. Thus, while type h still has additional needs which are not covered at all, he is at least better covered against the legitimate needs. Note also that for type l, insurance against the legitimate needs (which coincide with his true needs) is distorted downwards (see equations (16) and (17)).

If, on the other hand, we look at the true marginal utilities faced by type h, we can see that the better coverage provided against the legitimate needs is still not sufficient to restore the tradeoffs obtained for type h without paternalism. In particular, we have that

$$\frac{u'(c_h^I)}{u'(c_h^{D_1})} = \frac{\mu n_h \gamma(1+\lambda)}{\left[\mu n_h \gamma(1+\lambda) + \gamma n_h \left(u'(\overline{c}_h^{D_1}) - u'(c_h^{D_1})\right)\right]} \frac{[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)]}{(1-\pi_1-\pi_2)(1+\lambda)} <$$

$$< \frac{1-\pi_1(1+\lambda)-\pi_2(1+\lambda)}{(1-\pi_1-\pi_2)(1+\lambda)}$$

and

$$\frac{u'(c_h^I)}{u'(c_h^{D_2})} = \frac{\mu n_h \gamma(1+\lambda)}{\left[\mu n_h \gamma(1+\lambda) + \gamma n_h \left(u'(\overline{c}_h^{D_2}) - u'(c_h^{D_2})\right)\right]} \frac{[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)]}{(1-\pi_1-\pi_2)(1+\lambda)} <$$

$$< \frac{1-\pi_1(1+\lambda)-\pi_2(1+\lambda)}{(1-\pi_1-\pi_2)(1+\lambda)}.$$

Therefore, in terms of type $h's$ true marginal utilities, there is still a downward distortion of his insurance coverage due to the presence of paternalism. Nevertheless, as noted above, type $l's$ insurance is also distorted downwards.

It can also be verified that the second-best setting again implies some informational rent given to type h. It can be shown from equations (72)-(77) that $u'(c_h^I) < u'(c_l^I)$, $u'(\bar{c}_h^{D_1}) < u'(\bar{c}_l^{D_1})$ and $u'(\bar{c}_h^{D_2}) < u'(\bar{c}_l^{D_2})$. It should be noted that if we consider the true marginal utilities of type h in the dependence states of nature, the comparison between the two types becomes less clear and it is not ruled out that type h can still have a lower wealth than type l because of his additional needs; however, type h is now given some advantage compared to the first-best allocation where we had $u'(c_h^I) = u'(c_l^I)$, $u'(\bar{c}_h^{D_1}) = u'(\bar{c}_l^{D_1})$ and $u'(\bar{c}_h^{D_2}) = u'(\bar{c}_l^{D_2})$.

As in the previous cases, we can also discuss the comparison of optimal social insurance deductibles between the two types. This comparison again requires to use specific utility functions and is again the most informative in the case of exponential utility exhibiting CARA. Assuming interior solutions, it can be shown that in that case we have $D_{1h} < D_{1l}$ and $D_{2h} < D_{2l}$. This reflects the above derived result that the second-best requires to provide a better insurance against the legitimate needs to type h than to type l. As discussed before, CARA utility allows to isolate this consideration since it is not influenced by differences in wealth. On the other hand, similarly to the case of no paternalism, with DARA preferences the comparison of the deductibles becomes less clear.

Let us now look at how the second-best allocation could be implemented. If $\lambda < \widehat{\lambda}$, the above characterized social insurance should be introduced. It should be based on individual income and now both types of individuals should face marginal taxes. If $\lambda = \widehat{\lambda}$, private insurance can be involved, but interference with the choices of both individual types is now needed. First, the insurance premiums of both types should be taxed at the margin. Second, if $\widehat{L}_{1h} \neq \widehat{L}_{2h}$, both types should also face an additional tax or subsidy applied to the private insurance deductible in at least one of the dependence states of nature. Moreover, a non-linear income tax is also needed with marginal taxes on both types' income.

The above findings can be summarized as follows:

Proposition 6 *Assume that high productivity individuals have higher LTC needs than low productivity ones but these needs still allow them to remain better-off in the laissez-faire. Assume also that the government does not accept these higher needs as legitimate. The second-best optimal allocation has the following features:*

a) Low productivity individuals face a downward distortion of their labour supply and insurance coverage. Moreover, if the difference between the needs of high and low productivity individuals is not the same at both severity levels of dependence (i.e. $\widehat{L}_{2h} \neq \widehat{L}_{1h}$), low productivity individuals also face a distortion of insurance tradeoff between the two severity levels.

b) As in the first-best, high productivity individuals face paternalistic corrections, but the paternalism is now "softer": there is a smaller correction of their labour supply, a better balance of their true wealth levels in the two states of dependence and a better coverage against the legitimate needs. Moreover, high productivity individuals get informational rent.

If the government faces a lower loading cost than private insurers (i.e. $\lambda < \widehat{\lambda}$), the implementation of the second-best optimum should rely on income-based social LTC insurance with marginal income taxes for both types of individuals. As long as $\widehat{L}_{2h} \neq \widehat{L}_{1h}$, optimal social insurance features state-dependent deductibles for both individual types. If $\lambda = \widehat{\lambda}$, private insurance can be involved, but this requires marginal taxes on the insurance premiums of both types and, when $\widehat{L}_{2h} \neq \widehat{L}_{1h}$, also *marginal taxes or subsidies on their private insurance deductibles in at least one dependence state of nature. Marginal income taxes for both individual types are also required.*

6 Conclusion

In this paper, we have studied the design of an optimal social LTC insurance which would address the growing concerns of many (especially middle class) people that LTC costs might force them to spend down all their wealth. Recent suggestions made by Dilnot's Commission (2011) in the UK raise the idea of capping individual LTC spending. While this idea is very much in the spirit of Arrow's (1963) theorem of the deductible, we were interested in exploring more formally whether this well-known result of (private) insurance theory can be applied to social LTC insurance and how such a social policy should be designed. To do this, we considered a model in which two types of individuals, skilled and unskilled, face the risk of becoming dependent, and their dependence can have a low or a high degree of severity. We first looked at the individual choices in the *laissez-faire* and then investigated optimal social insurance under different scenarios. In particular, we studied separately the case where, at each severity level of dependence, both types of individuals have the same LTC needs and the case where these needs are higher for high productivity (skilled) individuals. In the latter case, we considered two different positions that could be taken by the government: a non-paternalistic scenario where the government recognizes all needs as legitimate and a paternalistic case where the government does not accept the "whims" of high productivity individuals. In all the cases, we first looked at the first-best setting with full information and then considered the second-best situation when the government cannot observe individual types.

Our results show that, as long as providing insurance is not costless for the government, optimal social LTC insurance indeed features a deductible. In the first-best setting when the government has full information about individual types, it is optimal to give the same deductible to both types of individuals because wealth is perfectly equalized between the two types. In the second-best, the situation is somewhat different due to the presence of self-selection constraints. Moreover, the influence of self-selection constraints is also rather different depending on whether the two types of individuals have the same or different LTC needs. With identical needs, the second-best optimality does not require any distortions of insurance tradeoffs. In fact, if in that case loading costs of private and social insurance are the same and if optimal non-linear income taxation is introduced, the government can leave the task of insurance to the private market without any need to interfere with individual choices, which is in line with the classical result of Atkinson and Stiglitz (1976). The absence of insurance distortions, however, does not necessarily mean that optimal deductibles will be the same for both individual types: due to asymmetric information, the redistribution of resources is incomplete and thus wealth differences remain, which implies different absolute risk aversion for the two types of individuals under DARA or IARA preferences. This in turn results in different deductibles being optimal for the two types. Nevertheless, equal deductibles remain optimal under CARA.

Insurance distortions, however, come into play when skilled individuals have higher LTC needs than the unskilled. In that case, self-selection requires to distort downwards the insurance coverage of unskilled individuals, which among other things means that they will face a positive deductible even if insurance is costless for the government. Moreover, if the difference between the needs of skilled and unskilled individuals is not the same at both severity levels of dependence, unskilled individuals also face a distortion of insurance tradeoff between the two severity levels, which again helps to make their allocation less attractive to the skilled. In other words, this means that generally it becomes optimal to give the unskilled state-dependent deductibles rather than a unique one as before. This constitutes a departure from a straightforward application of Arrow's theorem, even though it still remains optimal to have a deductible at each severity level.

These distortions for the unskilled apply in both the paternalistic and the non-paternalistic case. On the other hand, skilled individuals face no distortions in the non-paternalistic case but this is no longer true in the paternalistic one. In the paternalistic case, there is a mismatch between socially optimal and the skilled type's individual tradeoffs already in the first-best because the government considers different needs than skilled individuals do. In that case, one has to make a distinction between social insurance explicitly provided by the government (and based on the legitimate needs) and the "true" level of insurance that is implied for skilled individuals who have additional needs which they must fully cover themselves. Indeed, even though in the first-best social insurance features the same deductible for both types of individuals, skilled individuals effectively pay higher amounts which are equal to the social insurance deductible plus their additional costs. Moreover, if the additional costs are not the same at both severity levels, skilled individuals effectively face state-dependent deductibles even though the explicit social insurance deductible is state-independent. Consequently, if the first-best outcome is to be decentralized using private insurance, "corrections" of skilled individuals' choices are needed because in the private market they want to buy too much insurance from the social point of view.

The need for paternalistic corrections remains in the second-best as well; however, the presence of the self-selection constraint forces the government to "soften" its paternalism. Social insurance becomes more generous in the sense that it provides a better coverage against the legitimate needs than in the first-best (and than the coverage provided to unskilled individuals). Moreover, if the difference between the needs of skilled individuals and the legitimate needs is not the same at both severity levels of dependence, it becomes optimal to have state-dependent social insurance deductibles for skilled individuals too. The idea is to allow skilled individuals to achieve a better balance between their wealth levels in the two dependence states as these levels are not equalized because of differences in uncovered additional costs.

While there is a number of differences between the paternalistic and the non-paternalistic case, the comparison of second-best social insurance deductibles between the two individual types has a similar pattern in both cases. The downward distortion of unskilled individuals' insurance coverage present in both cases and complemented in the paternalistic case by the upward distortion of skilled individuals' coverage against the legitimate needs implies that at each severity level, the skilled face lower deductibles than the unskilled under CARA preferences. The equality obtained with identical needs is thus no longer valid. On the other hand, under different types of preferences, the influence of insurance distortions becomes less obvious since differences in absolute risk aversion then come into play as well. The comparison of optimal deductibles then becomes less clear.

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Appendix A: comparative statics in the *laissez-faire*

Focusing on interior solutions, the problem of an individual i $(i = h, l)$ can be rewritten in the following way:

$$\max_{y_i, \widehat{D}_i} \left\{ \pi_1 u(c_i^{D_1}) + \pi_2 u(c_i^{D_2}) + (1 - \pi_1 - \pi_2)\, u(c_i^I) - v\left(\frac{y_i}{w_i}\right) \right\} \tag{32}$$

where
$c_i^{D_1} = c_i^{D_2} = y_i - \widehat{P}_i - \widehat{D}_i$;

$c_i^I = y_i - \widehat{P}_i$
and $\widehat{P}_i = \pi_1(1+\widehat{\lambda})(L_{1i} - \widehat{D}_i) + \pi_2(1+\widehat{\lambda})(L_{2i} - \widehat{D}_i)$.

The FOC for $\widehat{D}_i$ can be written as

$$(1-\pi_1-\pi_2)\,u'(c_i^I)\left[\pi_1(1+\widehat{\lambda}) + \pi_2(1+\widehat{\lambda})\right] - \\ - \left[1-\pi_1(1+\widehat{\lambda}) - \pi_2(1+\widehat{\lambda})\right]\left[\pi_1 u'(c_i^{D_1}) + \pi_2 u'(c_i^{D_2})\right] = 0 \tag{33}$$

and the FOC for y_i writes as

$$\pi_1 u'(c_i^{D_1}) + \pi_2 u'(c_i^{D_2}) + (1-\pi_1-\pi_2)\,u'(c_i^I) - \frac{v'\left(\frac{y_i}{w_i}\right)}{w_i} = 0 \tag{34}$$

Fully differentiating (33) and (34) with respect to w_i, we get respectively

$$\frac{\partial y_i}{\partial w_i}\left[(1-\pi_1-\pi_2)\,u''(c_i^I)(1+\widehat{\lambda})(\pi_1+\pi_2) - \left[1-(1+\widehat{\lambda})(\pi_1+\pi_2)\right]\left[\pi_1 u''(c_i^{D_1}) + \pi_2 u''(c_i^{D_2})\right]\right] + \\ + \frac{\partial \widehat{D}_i}{\partial w_i}\left[(1-\pi_1-\pi_2)\,u''(c_i^I)(1+\widehat{\lambda})^2(\pi_1+\pi_2)^2 + \left[1-(1+\widehat{\lambda})(\pi_1+\pi_2)\right]^2\left[\pi_1 u''(c_i^{D_1}) + \pi_2 u''(c_i^{D_2})\right]\right] = 0 \tag{35}$$

and

$$\frac{\partial y_i}{\partial w_i}\left[\pi_1 u''(c_i^{D_1}) + \pi_2 u''(c_i^{D_2}) + (1-\pi_1-\pi_2)\,u''(c_i^I) - \frac{v''\left(\frac{y_i}{w_i}\right)}{w_i^2}\right] + \\ + \frac{\partial \widehat{D}_i}{\partial w_i}\left[(1-\pi_1-\pi_2)\,u''(c_i^I)(1+\widehat{\lambda})(\pi_1+\pi_2) - \left[1-(1+\widehat{\lambda})(\pi_1+\pi_2)\right]\left[\pi_1 u''(c_i^{D_1}) + \pi_2 u''(c_i^{D_2})\right]\right] + \\ + \frac{v''\left(\frac{y_i}{w_i}\right) y_i}{w_i^3} + \frac{v'\left(\frac{y_i}{w_i}\right)}{w_i^2} = 0 \tag{36}$$

For ease of exposition, let us define the following:

$$[1] \equiv \left[(1-\pi_1-\pi_2)\,u''(c_i^I)(1+\widehat{\lambda})(\pi_1+\pi_2) - \left[1-(1+\widehat{\lambda})(\pi_1+\pi_2)\right]\left[\pi_1 u''(c_i^{D_1}) + \pi_2 u''(c_i^{D_2})\right]\right] \tag{37}$$

$$[2] \equiv \left[(1-\pi_1-\pi_2)\, u''(c_i^I)(1+\widehat{\lambda})^2(\pi_1+\pi_2)^2 + \left[1-(1+\widehat{\lambda})(\pi_1+\pi_2)\right]^2 \left[\pi_1 u''(c_i^{D_1}) + \pi_2 u''(c_i^{D_2})\right] \right] < 0 \tag{38}$$

$$[3] \equiv \left[\pi_1 u''(c_i^{D_1}) + \pi_2 u''(c_i^{D_2}) + (1-\pi_1-\pi_2)\, u''(c_i^I) - \frac{v''\left(\frac{y_i}{w_i}\right)}{w_i^2} \right] < 0 \tag{39}$$

$$[4] \equiv \frac{v''\left(\frac{y_i}{w_i}\right) y_i}{w_i^3} + \frac{v'\left(\frac{y_i}{w_i}\right)}{w_i^2} > 0 \tag{40}$$

Solving the system of equations (35) and (36) for $\frac{\partial y_i}{\partial w_i}$ and $\frac{\partial \widehat{D}_i}{\partial w_i}$, we obtain

$$\frac{\partial y_i}{\partial w_i} = \frac{[4] \cdot [2]}{-[2] \cdot [3] + [1]^2} > 0^{15} \tag{41}$$

and

$$\frac{\partial \widehat{D}_i}{\partial w_i} = \frac{\frac{\partial y_i}{\partial w_i} \cdot [1]}{-[2]} \tag{42}$$

Since $\frac{\partial y_i}{\partial w_i} > 0$ and $-[2] > 0$, the sign of $\frac{\partial \widehat{D}_i}{\partial w_i}$ depends on the sign of $[1]$. The sign of $[1]$ is however ambiguous in the general case and differs depending on the absolute risk aversion (ARA) exhibited by the utility function. In particular, we are now going to show that $[1] > 0$ under decreasing absolute risk aversion (DARA), $[1] < 0$ under increasing absolute risk aversion (IARA) and $[1] = 0$ under constant absolute risk aversion (CARA).

To see this, let us first note that DARA (resp. IARA and CARA) means that

$$ARA(c) = \frac{-u''(c)}{u'(c)} < (\text{resp.} > \text{ and } =)\ ARA(d) = \frac{-u''(d)}{u'(d)} \quad \text{for } c > d,$$

where $\frac{-u''(x)}{u'(x)}$ is the Arrow-Pratt measure of absolute risk aversion at wealth x.

Thus, noting that with $\widehat{D}_i > 0$, we have $c_i^I > c_i^{D_1}$, under DARA (resp. IARA and CARA) preferences we can write

$$\frac{-u''(c_i^I)}{u'(c_i^I)} < (\text{resp.} > \text{ and } =)\ \frac{-u''(c_i^{D_1})}{u'(c_i^{D_1})}$$

$$\Longleftrightarrow$$

$$u''(c_i^I) > (\text{resp.} < \text{ and } =)\ \frac{u''(c_i^{D_1})}{u'(c_i^{D_1})}\, u'(c_i^I)$$

[15] The sign follows from the fact that the numerator of the expression is negative and the denominator can be verified to be negative as well.

We can then multiply both sides by $(1-\pi_1-\pi_2)\,(1+\widehat{\lambda})(\pi_1+\pi_2)$ and subtract from both sides $\left[1-(1+\widehat{\lambda})(\pi_1+\pi_2)\right]\left[\pi_1 u''(c_i^{D_1})+\pi_2 u''(c_i^{D_2})\right]$, which gives

$$(1-\pi_1-\pi_2)\,(1+\widehat{\lambda})(\pi_1+\pi_2)u''(c_i^I)-\left[1-(1+\widehat{\lambda})(\pi_1+\pi_2)\right]\left[\pi_1 u''(c_i^{D_1})+\pi_2 u''(c_i^{D_2})\right]$$

$$> (\text{resp.} < \text{ and } =)\ \frac{u''(c_i^{D_1})}{u'(c_i^{D_1})}\left[\begin{array}{c} u'(c_i^I)\,(1-\pi_1-\pi_2)\,(1+\widehat{\lambda})(\pi_1+\pi_2)- \\ -\left[1-(1+\widehat{\lambda})(\pi_1+\pi_2)\right]\left[\pi_1 u'(c_i^{D_1})+\pi_2 u'(c_i^{D_2})\right] \end{array}\right]=0 \tag{43}$$

where we have used the fact that $c_i^{D_1}=c_i^{D_2}$ and that the expression in the last big bracket is the FOC for $\widehat{D}_i$.

The left-hand side of inequality (43) is exactly the definition of [1] ; we therefore indeed have that under DARA (resp. IARA and CARA), [1] > (resp. < and =) 0. Coming back to $\frac{\partial \widehat{D}_i}{\partial w_i}$ (equation (42)), we can thus conclude that $\frac{\partial \widehat{D}_i}{\partial w_i}$ > (resp. < and =) 0 with DARA (resp. IARA and CARA) preferences.

Fully differentiating (33) and (34) with respect to L_{1i}, we get respectively

$$\frac{\partial y_i}{\partial L_{1i}}\cdot[1]+\frac{\partial \widehat{D}_i}{\partial L_{1i}}\cdot[2]-(1+\widehat{\lambda})\pi_1\cdot[1]=0 \tag{44}$$

and

$$\frac{\partial y_i}{\partial L_{1i}}\cdot[3]+\frac{\partial \widehat{D}_i}{\partial L_{1i}}\cdot[1]-$$
$$-(1+\widehat{\lambda})\pi_1\left[\pi_1 u''(c_i^{D_1})+\pi_2 u''(c_i^{D_2})+(1-\pi_1-\pi_2)\,u''(c_i^I)\right]=0 \tag{45}$$

Defining

$$[5]\equiv\left[\pi_1 u''(c_i^{D_1})+\pi_2 u''(c_i^{D_2})+(1-\pi_1-\pi_2)\,u''(c_i^I)\right]<0 \tag{46}$$

and solving the system of equations (44) and (45) for $\frac{\partial y_i}{\partial L_{1i}}$ and $\frac{\partial \widehat{D}_i}{\partial L_{1i}}$, we obtain

$$\frac{\partial y_i}{\partial L_{1i}}=(1+\widehat{\lambda})\pi_1\frac{[1]^2-[2]\cdot[5]}{[1]^2-[2]\cdot[3]}>0 \tag{47}$$[16]

and

$$\frac{\partial \widehat{D}_i}{\partial L_{1i}}=\frac{\left[\frac{\partial y_i}{\partial L_{1i}}-(1+\widehat{\lambda})\pi_1\right]\cdot[1]}{-[2]} \tag{48}$$

[16] It can be checked that both the numerator and the denominator of the expression are negative, which implies the sign of the expression.

It can be seen that, as in the previous case, the sign of $\frac{\partial \widehat{D}_i}{\partial L_{1i}}$ depends on the sign of [1] which, as we have seen above, changes depending on the type of ARA exhibited by the utility function. In this case, however, the sign of $\frac{\partial \widehat{D}_i}{\partial L_{1i}}$ is opposite to the sign of [1] since, while the denominator is always positive, the first bracket in the numerator is negative as it can be checked that $\frac{[1]^2-[2]\cdot[5]}{[1]^2-[2]\cdot[3]} < 1$, which means that $\frac{\partial y_i}{\partial L_{1i}} < (1+\widehat{\lambda})\pi_1$. Therefore, we have that $\frac{\partial \widehat{D}_i}{\partial L_{1i}} <$ (resp. $>$ and $=$) 0 with DARA (resp. IARA and CARA) preferences.

In the same way, it can be shown that for L_{2i}, we have

$$\frac{\partial y_i}{\partial L_{2i}} = (1+\widehat{\lambda})\pi_2 \frac{[1]^2 - [2]\cdot[5]}{[1]^2 - [2]\cdot[3]} > 0 \tag{49}$$

and

$$\frac{\partial \widehat{D}_i}{\partial L_{2i}} = \frac{\left[\frac{\partial y_i}{\partial L_{2i}} - (1+\widehat{\lambda})\pi_2\right]\cdot[1]}{-[2]} \tag{50}$$

and $\frac{\partial \widehat{D}_i}{\partial L_{2i}} <$ (resp. $>$ and $=$) 0 with DARA (resp. IARA and CARA) preferences.

Finally, fully differentiating (33) and (34) with respect to $\widehat{\lambda}$, we get respectively

$$\begin{aligned} &\frac{\partial y_i}{\partial \widehat{\lambda}}\cdot[1] + \frac{\partial \widehat{D}_i}{\partial \widehat{\lambda}}\cdot[2] + \\ &+(\pi_1+\pi_2)\left[\pi_1 u'(c_i^{D_1}) + \pi_2 u'(c_i^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_i^I)\right] - \\ &-\left[\pi_1(L_{1i}-\widehat{D}_i) + \pi_2(L_{2i}-\widehat{D}_i)\right]\cdot[1] = 0 \end{aligned} \tag{51}$$

and

$$\begin{aligned} &\frac{\partial y_i}{\partial \widehat{\lambda}}\cdot[3] + \frac{\partial \widehat{D}_i}{\partial \widehat{\lambda}}\cdot[1] - \\ &-\left[\pi_1(L_{1i}-\widehat{D}_i) + \pi_2(L_{2i}-\widehat{D}_i)\right]\cdot[5] = 0 \end{aligned} \tag{52}$$

Defining

$$[6] \equiv \left[\pi_1 u'(c_i^{D_1}) + \pi_2 u'(c_i^{D_2}) + (1-\pi_1-\pi_2)\, u'(c_i^I)\right] > 0 \tag{53}$$

and solving the system of equations (51) and (52) for $\frac{\partial y_i}{\partial \widehat{\lambda}}$ and $\frac{\partial \widehat{D}_i}{\partial \widehat{\lambda}}$, we can obtain

$$\frac{\partial y_i}{\partial \widehat{\lambda}} = \left[\pi_1(L_{1i}-\widehat{D}_i) + \pi_2(L_{2i}-\widehat{D}_i)\right]\frac{[1]^2-[2]\cdot[5]}{[1]^2-[2]\cdot[3]} + \frac{(\pi_1+\pi_2)\cdot[6]\cdot[1]}{[2]\cdot[3]-[1]^2} \tag{54}$$

and

$$\frac{\partial \widehat{D}_i}{\partial \widehat{\lambda}} = \frac{(\pi_1+\pi_2)\cdot[6]\cdot[3]}{[1]^2-[2]\cdot[3]} + \frac{[1]\cdot\left[\pi_1(L_{1i}-\widehat{D}_i)+\pi_2(L_{2i}-\widehat{D}_i)\right]}{[2]}\left[1-\frac{[1]^2-[2]\cdot[5]}{[1]^2-[2]\cdot[3]}\right] \tag{55}$$

Let us first discuss $\frac{\partial y_i}{\partial \widehat{\lambda}}$. It can be checked that its first term is always positive, while its second term is positive (resp. negative and zero) with DARA (resp. IARA and CARA) preferences. We thus have $\frac{\partial y_i}{\partial \widehat{\lambda}} > 0$ under DARA and CARA, whereas under IARA, the sign of $\frac{\partial y_i}{\partial \widehat{\lambda}}$ is undetermined.

Turning to $\frac{\partial \widehat{D}_i}{\partial \widehat{\lambda}}$, it can be verified that its first term is always positive as well. Its second term, however, is negative (resp. positive and zero) with DARA (resp. IARA and CARA) preferences. Therefore, we have $\frac{\partial \widehat{D}_i}{\partial \widehat{\lambda}} > 0$ under IARA and CARA, whereas under DARA, the sign of $\frac{\partial \widehat{D}_i}{\partial \widehat{\lambda}}$ is undetermined.

Appendix B: second-best FOCs with identical needs

In the second-best with identical needs, the FOCs of the government's problem write as follows:

$$\frac{\partial \mathcal{L}}{\partial P_h} = -n_h\pi_1 u'(c_h^{D_1}) - n_h\pi_2 u'(c_h^{D_2}) - n_h\left(1-\pi_1-\pi_2\right)u'(c_h^I) - \mu n_h +$$

$$+\gamma\pi_1 u'(c_h^{D_1}) + \gamma\pi_2 u'(c_h^{D_2}) + \gamma\left(1-\pi_1-\pi_2\right)u'(c_h^I) = 0 \tag{56}$$

$$\frac{\partial \mathcal{L}}{\partial P_l} = -n_l\pi_1 u'(c_l^{D_1}) - n_l\pi_2 u'(c_l^{D_2}) - n_l\left(1-\pi_1-\pi_2\right)u'(c_l^I) - \mu n_l -$$

$$-\gamma\pi_1 u'(c_l^{D_1}) - \gamma\pi_2 u'(c_l^{D_2}) - \gamma\left(1-\pi_1-\pi_2\right)u'(c_l^I) = 0 \tag{57}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{1h}} = n_h u'(c_h^{D_1}) + \mu n_h(1+\lambda) - \gamma u'(c_h^{D_1}) \le 0, \qquad \alpha_{1h}\frac{\partial \mathcal{L}}{\partial \alpha_{1h}} = 0 \tag{58}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{2h}} = n_h u'(c_h^{D_2}) + \mu n_h(1+\lambda) - \gamma u'(c_h^{D_2}) \le 0, \qquad \alpha_{2h}\frac{\partial \mathcal{L}}{\partial \alpha_{2h}} = 0 \tag{59}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{1l}} = n_l u'(c_l^{D_1}) + \mu n_l(1+\lambda) + \gamma u'(c_l^{D_1}) \le 0, \qquad \alpha_{1l}\frac{\partial \mathcal{L}}{\partial \alpha_{1l}} = 0 \tag{60}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{2l}} = n_l u'(c_l^{D_2}) + \mu n_l(1+\lambda) + \gamma u'(c_l^{D_2}) \le 0, \qquad \alpha_{2l}\frac{\partial \mathcal{L}}{\partial \alpha_{2l}} = 0 \tag{61}$$

$$\frac{\partial \mathcal{L}}{\partial y_h} = n_h\pi_1 u'(c_h^{D_1}) + n_h\pi_2 u'(c_h^{D_2}) + n_h\left(1-\pi_1-\pi_2\right)u'(c_h^I) - n_h\frac{v'\left(\frac{y_h}{w_h}\right)}{w_h} -$$

$$-\gamma\pi_1 u'(c_h^{D_1}) - \gamma\pi_2 u'(c_h^{D_2}) - \gamma\left(1-\pi_1-\pi_2\right)u'(c_h^I) + \gamma\frac{v'\left(\frac{y_h}{w_h}\right)}{w_h} = 0 \tag{62}$$

$$\frac{\partial\mathcal{L}}{\partial y_l} = n_l\pi_1 u'(c_l^{D_1}) + n_l\pi_2 u'(c_l^{D_2}) + n_l\left(1-\pi_1-\pi_2\right)u'(c_l^I) - n_l\frac{v'\left(\frac{y_l}{w_l}\right)}{w_l} +$$

$$+\gamma\pi_1 u'(c_l^{D_1}) + \gamma\pi_2 u'(c_l^{D_2}) + \gamma\left(1-\pi_1-\pi_2\right)u'(c_l^I) - \gamma\frac{v'\left(\frac{y_l}{w_h}\right)}{w_h} = 0 \tag{63}$$

Appendix C: specific examples with identical needs

Focusing on interior solutions, we have

$$u'(c_h^I) = \frac{-\mu n_h\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]}{\left(1-\pi_1-\pi_2\right)\left(n_h-\gamma\right)}$$

$$\Longleftrightarrow$$

$$c_h^I = u'^{-1}\left(\frac{-\mu n_h\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]}{\left(1-\pi_1-\pi_2\right)\left(n_h-\gamma\right)}\right)$$

$$\Longleftrightarrow$$

$$y_h - P_h = u'^{-1}\left(\frac{-\mu n_h\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]}{\left(1-\pi_1-\pi_2\right)\left(n_h-\gamma\right)}\right)$$

and

$$u'(c_l^I) = \frac{-\mu n_l\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]}{\left(1-\pi_1-\pi_2\right)\left(n_l+\gamma\right)}$$

$$\Longleftrightarrow$$

$$y_l - P_l = u'^{-1}\left(\frac{-\mu n_l\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]}{\left(1-\pi_1-\pi_2\right)\left(n_l+\gamma\right)}\right)$$

Then, using the definitions of D_h and D_l from the text, we have

$$D_h = u'^{-1}\left(\frac{-\mu n_h\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]}{\left(1-\pi_1-\pi_2\right)\left(n_h-\gamma\right)}\right) - u'^{-1}\left(\frac{-\mu(1+\lambda)n_h}{\left(n_h-\gamma\right)}\right)$$

and

$$D_l = u'^{-1}\left(\frac{-\mu n_l\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]}{(1-\pi_1-\pi_2)(n_l+\gamma)}\right) - u'^{-1}\left(\frac{-\mu(1+\lambda)n_l}{(n_l+\gamma)}\right)$$

With $u(x) = \ln x$, we get

$$\begin{aligned} D_h &= \frac{-(1-\pi_1-\pi_2)(n_h-\gamma)}{\mu n_h\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]} + \frac{(n_h-\gamma)}{\mu(1+\lambda)n_h} = \\ &= \frac{(n_h-\gamma)\left[\frac{-\lambda}{(1+\lambda)}\right]}{n_h\mu(1+\lambda)\left[\frac{1}{(1+\lambda)}-\pi_1-\pi_2\right]} \end{aligned}$$

and

$$\begin{aligned} D_l &= \frac{-(1-\pi_1-\pi_2)(n_l+\gamma)}{\mu n_l\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]} + \frac{(n_l+\gamma)}{\mu(1+\lambda)n_l} = \\ &= \frac{(n_l+\gamma)\left[\frac{-\lambda}{(1+\lambda)}\right]}{n_l\mu(1+\lambda)\left[\frac{1}{(1+\lambda)}-\pi_1-\pi_2\right]} < D_h \end{aligned}$$

With $u(x) = -e^{-x}$, we get

$$\begin{aligned} D_h &= -\ln\left[\frac{-\mu n_h\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]}{(1-\pi_1-\pi_2)(n_h-\gamma)}\right] + \ln\left[\frac{-\mu(1+\lambda)n_h}{(n_h-\gamma)}\right] = \\ &= \ln\left[\frac{(1+\lambda)(1-\pi_1-\pi_2)}{\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]}\right] \end{aligned}$$

and

$$\begin{aligned} D_l &= -\ln\left[\frac{-\mu n_l\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]}{(1-\pi_1-\pi_2)(n_l+\gamma)}\right] + \ln\left[\frac{-\mu(1+\lambda)n_l}{(n_l+\gamma)}\right] = \\ &= \ln\left[\frac{(1+\lambda)(1-\pi_1-\pi_2)}{\left[1-\pi_1(1+\lambda)-\pi_2(1+\lambda)\right]}\right] = D_h \end{aligned}$$

Moreover, with $u(x) = -e^{-x}$ it can be shown that $D_h = D_l$ also holds if $\alpha_{1h} = 0$, $\alpha_{1l} = 0$, $\alpha_{2h} > 0$, $\alpha_{2l} > 0$. Furthermore, it can be also shown that with $u(x) = -e^{-x}$ it is not possible to have solutions where in the same state of nature α would be zero for one type but non-zero for the other (for instance, $\alpha_{1h} = 0$, $\alpha_{1l} > 0$, $\alpha_{2h} > 0$, $\alpha_{2l} > 0$ or $\alpha_{1h} = 0$, $\alpha_{1l} > 0$, $\alpha_{2h} = 0$, $\alpha_{2l} > 0$ or $\alpha_{1h} = 0$, $\alpha_{1l} = 0$, $\alpha_{2h} = 0$, $\alpha_{2l} > 0$ are not possible). Thus, with $u(x) = -e^{-x}$ we always have $D_h = D_l$.

Appendix D: second-best FOCs with no paternalism

In the second-best with different needs and no paternalism, the FOCs of the government's problem write as follows:

$$\frac{\partial \mathcal{L}}{\partial P_h} = -n_h \pi_1 u'(c_h^{D_1}) - n_h \pi_2 u'(c_h^{D_2}) - n_h (1-\pi_1-\pi_2) u'(c_h^I) - \mu n_h +$$

$$+\gamma \pi_1 u'(c_h^{D_1}) + \gamma \pi_2 u'(c_h^{D_2}) + \gamma (1-\pi_1-\pi_2) u'(c_h^I) = 0 \tag{64}$$

$$\frac{\partial \mathcal{L}}{\partial P_l} = -n_l \pi_1 u'(c_l^{D_1}) - n_l \pi_2 u'(c_l^{D_2}) - n_l (1-\pi_1-\pi_2) u'(c_l^I) - \mu n_l -$$

$$-\gamma \pi_1 u'(\widetilde{c}_l^{D_1}) - \gamma \pi_2 u'(\widetilde{c}_l^{D_2}) - \gamma (1-\pi_1-\pi_2) u'(c_l^I) = 0 \tag{65}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{1h}} = n_h u'(c_h^{D_1}) + \mu n_h (1+\lambda) - \gamma u'(c_h^{D_1}) \leq 0, \qquad \alpha_{1h} \frac{\partial \mathcal{L}}{\partial \alpha_{1h}} = 0 \tag{66}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{2h}} = n_h u'(c_h^{D_2}) + \mu n_h (1+\lambda) - \gamma u'(c_h^{D_2}) \leq 0, \qquad \alpha_{2h} \frac{\partial \mathcal{L}}{\partial \alpha_{2h}} = 0 \tag{67}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{1l}} = n_l u'(c_l^{D_1}) + \mu n_l (1+\lambda) + \gamma u'(\widetilde{c}_l^{D_1}) \leq 0, \qquad \alpha_{1l} \frac{\partial \mathcal{L}}{\partial \alpha_{1l}} = 0 \tag{68}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{2l}} = n_l u'(c_l^{D_2}) + \mu n_l (1+\lambda) + \gamma u'(\widetilde{c}_l^{D_2}) \leq 0, \qquad \alpha_{2l} \frac{\partial \mathcal{L}}{\partial \alpha_{2l}} = 0 \tag{69}$$

$$\frac{\partial \mathcal{L}}{\partial y_h} = n_h \pi_1 u'(c_h^{D_1}) + n_h \pi_2 u'(c_h^{D_2}) + n_h (1-\pi_1-\pi_2) u'(c_h^I) - n_h \frac{v'\left(\frac{y_h}{w_h}\right)}{w_h} -$$

$$-\gamma \pi_1 u'(c_h^{D_1}) - \gamma \pi_2 u'(c_h^{D_2}) - \gamma (1-\pi_1-\pi_2) u'(c_h^I) + \gamma \frac{v'\left(\frac{y_h}{w_h}\right)}{w_h} = 0 \tag{70}$$

$$\frac{\partial \mathcal{L}}{\partial y_l} = n_l \pi_1 u'(c_l^{D_1}) + n_l \pi_2 u'(c_l^{D_2}) + n_l (1-\pi_1-\pi_2) u'(c_l^I) - n_l \frac{v'\left(\frac{y_l}{w_l}\right)}{w_l} +$$

$$+\gamma \pi_1 u'(\widetilde{c}_l^{D_1}) + \gamma \pi_2 u'(\widetilde{c}_l^{D_2}) + \gamma (1-\pi_1-\pi_2) u'(c_l^I) - \gamma \frac{v'\left(\frac{y_l}{w_h}\right)}{w_h} = 0 \tag{71}$$

Appendix E: second-best FOCs with paternalism

In the second-best with different needs and paternalism, we have the following FOCs:

$$\frac{\partial \mathcal{L}}{\partial P_h} = -n_h \pi_1 u'(\bar{c}_h^{D_1}) - n_h \pi_2 u'(\bar{c}_h^{D_2}) - n_h (1 - \pi_1 - \pi_2) u'(c_h^I) - \mu n_h +$$

$$+\gamma \pi_1 u'(c_h^{D_1}) + \gamma \pi_2 u'(c_h^{D_2}) + \gamma (1 - \pi_1 - \pi_2) u'(c_h^I) = 0 \quad (72)$$

$$\frac{\partial \mathcal{L}}{\partial P_l} = -n_l \pi_1 u'(\bar{c}_l^{D_1}) - n_l \pi_2 u'(\bar{c}_l^{D_2}) - n_l (1 - \pi_1 - \pi_2) u'(c_l^I) - \mu n_l -$$

$$-\gamma \pi_1 u'(\tilde{c}_l^{D_1}) - \gamma \pi_2 u'(\tilde{c}_l^{D_2}) - \gamma (1 - \pi_1 - \pi_2) u'(c_l^I) = 0 \quad (73)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{1h}} = n_h u'(\bar{c}_h^{D_1}) + \mu n_h (1 + \lambda) - \gamma u'(c_h^{D_1}) \leq 0, \qquad \alpha_{1h} \frac{\partial \mathcal{L}}{\partial \alpha_{1h}} = 0 \quad (74)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{2h}} = n_h u'(\bar{c}_h^{D_2}) + \mu n_h (1 + \lambda) - \gamma u'(c_h^{D_2}) \leq 0, \qquad \alpha_{2h} \frac{\partial \mathcal{L}}{\partial \alpha_{2h}} = 0 \quad (75)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{1l}} = n_l u'(\bar{c}_l^{D_1}) + \mu n_l (1 + \lambda) + \gamma u'(\tilde{c}_l^{D_1}) \leq 0, \qquad \alpha_{1l} \frac{\partial \mathcal{L}}{\partial \alpha_{1l}} = 0 \quad (76)$$

$$\frac{\partial \mathcal{L}}{\partial \alpha_{2l}} = n_l u'(\bar{c}_l^{D_2}) + \mu n_l (1 + \lambda) + \gamma u'(\tilde{c}_l^{D_2}) \leq 0, \qquad \alpha_{2l} \frac{\partial \mathcal{L}}{\partial \alpha_{2l}} = 0 \quad (77)$$

$$\frac{\partial \mathcal{L}}{\partial y_h} = n_h \pi_1 u'(\bar{c}_h^{D_1}) + n_h \pi_2 u'(\bar{c}_h^{D_2}) + n_h (1 - \pi_1 - \pi_2) u'(c_h^I) - n_h \frac{v'\left(\frac{y_h}{w_h}\right)}{w_h} -$$

$$-\gamma \pi_1 u'(c_h^{D_1}) - \gamma \pi_2 u'(c_h^{D_2}) - \gamma (1 - \pi_1 - \pi_2) u'(c_h^I) + \gamma \frac{v'\left(\frac{y_h}{w_h}\right)}{w_h} = 0 \quad (78)$$

$$\frac{\partial \mathcal{L}}{\partial y_l} = n_l \pi_1 u'(\bar{c}_l^{D_1}) + n_l \pi_2 u'(\bar{c}_l^{D_2}) + n_l (1 - \pi_1 - \pi_2) u'(c_l^I) - n_l \frac{v'\left(\frac{y_l}{w_l}\right)}{w_l} +$$

$$+\gamma \pi_1 u'(\tilde{c}_l^{D_1}) + \gamma \pi_2 u'(\tilde{c}_l^{D_2}) + \gamma (1 - \pi_1 - \pi_2) u'(c_l^I) - \gamma \frac{v'\left(\frac{y_l}{w_h}\right)}{w_h} = 0 \quad (79)$$