Financing Unemployment Insurance over the Business Cycle*

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Abstract

Should unemployment insurance (UI) benefits be countercyclical or procyclical? The answer relies on the ability of the government to finance these benefits. UI benefits are introduced in an infinite-horizon matching model featuring aggregate shocks to labor productivity, employed and unemployed risk-averse workers as well as an intensive margin. They are financed by taxes on labor and possibly by public debt. Under the Ramsey primal approach, optimal allocation is decentralized by an appropriate fiscal policy.

In a recession, optimal UI benefits should be countercyclical because risk-averse workers require insurance, while the incentive issue might be settled by suitable taxes. However, if the government's budget is severely constrained, UI benefits might be sacrificed and appear as procyclical. On the contrary, in the case of perfect access to financial markets, the insurance and incentive motives are independent from each other.

Key Words : Unemployment insurance benefits, Ramsey primal approach to optimal taxation, financing constraint, contracyclical policy. **JEL :** E24, E32, H21, H22, J64, J65.

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1 Introduction

Should unemployment insurance benefits be countercyclical or procyclical? Two principles guide unemployment benefits determination: insuring workers against the unemployment risk weighs in favor of more generous benefits, while encouraging unemployed workers to search for a new job would require to reduce their outside option and to raise the value from work. However, during a recession, the government has generally to tightly balance its budget. It might therefore be difficult for him to finance policies implementing these objectives. A few articles have dealt with optimal unemployment policies over the business cycle, but none have really focused on how the financing structure of unemployment insurance benefits might impact their cyclicality.

In this article, I develop an infinite-horizon matching model featuring discrete time, rational expectations, aggregate shocks to labor productivity, employed and unemployed workers as well as an intensive margin. Workers are risk-averse, cannot save or borrow on financial markets and are therefore ready to pay for insurance whether they are employed or not. Unemployed workers search for a job but the government is unable to monitor this activity, inducing a moral hazard issue. He has to set unemployment benefits carefully in order to provides incentives to search. Contrary to most articles in the literature taking unemployment benefits as the only available fiscal instrument, the government has also access here to a large variety of taxes to achieve his goals. Firms and workers interact in the context of a competitive search equilibrium, a framework developed by Moen [1997] insuring an efficient surplus-splitting between these two parts. Competitive search equilibrium is chosen over Nash bargaining because the latter would introduce an inefficiency that could disturb the cyclicality of UI benefits. Following the Ramsey primal approach, I prove that an appropriate set of taxes enables the planner to decentralize the competitive equilibrium and I characterize the optimal allocation.

In a baseline scenario where the government faces a strict budget constraint, I find that optimal UI benefits should be countercyclical. The main reason is that risk-averse workers need more insurance when a negative shock occurs. Moreover, in line with Landais et al. [2010], I show that the incentive motive is less important during recessions because the probability to find a new job depends more on the state of the labor market than on the individual search effort. Thanks to labor income taxes, the government is able to disentangle the insurance and the incentive objectives, using UI benefits to achieve the one and taxes to implement the other. However, tax revenue and UI benefits are closely linked in his budget constraint. Thus he has to implement the insurance and the incentive objective successively. The separation of objectives will be strict only when the government can freely borrow on perfect financial markets. In this case, evolutions of taxes and benefits can be decorrelated and each one of these instruments will implement its objective independently from the other. To the contrary, under extreme conditions, he might even have to sacrifice insurance in order to stimulate the labor market, which confers UI benefits a procyclical profile, as in Andersen and Svarer [2011]. In their empirical work, Dolls et al. [2012] show that UI benefits and labor taxes play an important role of stabilization in European Countries because they absorb contractionary shocks, in particular when taxation is progressive, and mainly because they allow liquidity constrained households to smooth their consumption. Consistent with this finding, I show that these instruments help reducing the volatility of aggregates.

Most articles dealing with the cyclicality of unemployment benefits, such as Landais et al. [2010] or Mitman and Rabinovich [2011], specially focus on this instrument and consider a simplistic fiscal policy. As a consequence, UI benefits might be really constrained and their cyclicality might result from some objectives that could be better achieved through a more specific instrument. According to Mitman and Rabinovich [2011], after a negative productivity shock, benefits first immediately rise and then have to decrease. Besides the moral hazard issue, too high UI benefits increase the reservation wage and thus the average wage, which would harm labor demand and dampen the recovery. But this result might stem from the fact they only consider a constant lump-sum production tax. When taxes can vary along the business cycle, I show that they can help providing incentives, leaving more room for UI benefits to specifically insure workers. Jung and Kuester [2011] take into account a more general fiscal policy. Their government is able to finance unemployment benefits and a vacancy subsidy through a production and a layoff tax. They conclude that UI benefits should be countercyclical, but that the rise of UI benefits after a negative shock is small and short-lived. However, their conclusion might stem from their assumption of tight government budget constraint. I show here that, whether the government has access to public debt or not, he will be more or less able to temporally disentangle the insurance and incentive objectives. If he is really constrained, he will have to implement these objectives successively, which might limit the initial rise in UI benefits.

In order to avoid strong assumptions, I consider a general form of labor taxation, consisting of lump-sum and linear taxes on labor income, allowing the social planner to reproduce many kind of taxes and subsidies. As wages are purely flexible, those taxes will affect both employees and firms. I also include an intensive margin to give more importance to marginal tax rates and increase the scope for government action. Subsection 6.3 shows that the assumption of intensive margin matters for the cyclicality of UI benefits.

In many OECD countries, the budget of the Social Security system is tight. In an extreme scenario, if the government has to balance its budget each period, higher unemployment benefits during recessions would require higher tax revenue (Burda and Weder [2010]). This situation widens the gap between the net and the gross wage, increases the cost of a vacancy and has a negative impact on labor demand and search effort. I show that, in this case, an optimal policy consists in firstly increasing UI benefits to provide insurance for workers, then reducing these benefits while financing a tax credit in order to increase the value from work and stimulate search effort. However, if the government is able to freely borrow on perfect financial markets, the financing constraint will be relaxed and public debt might be used as liquidity, helping households smoothing their consumption (Woodford [1990]). In this case, the planner will be able to implement simultanously both objectives.

The present work is related to a vast literature about the objectives of UI benefits. Chetty [2008] shows that the need for liquidity during an unemployment spell is the main theoretical justification for higher unemployment insurance benefits in a recession, as the risk of unemployment grows. However, if the government cannot monitor search activities, a moral hazard issue arise. Too high a level of unemployment insurance benefits might deter search effort and be detrimental for the labor market as well as for public finances. Kroft and Notowidigdo [2011] and Landais et al. [2010] prove that the cost of moral hazard is procyclical, because the probability to find a new job is less dependent on individual search activities and more on the state of the labor market. This advocates for countercyclical unemployment benefits. Andersen and Svarer [2009] add that such type of UI benefits might help reducing distortions compared to stateindependent benefits. However, all of these articles do not consider the possibility of other fiscal instruments.

A second major reference is an important microeconomic literature about the inci-

dence of taxes on the labor market. A rich tax system provides the government with a better tool in order to control the tax wedge, which is a crucial determinant of the unemployment rate (Nickell et al. [2005]). In a static framework, higher marginal tax rates increase employment (Sorensen [1999]), reduce wages and hours worked, because of the "wage moderating" effect of progressive taxation (Lehmann et al. [2013], Hansen [1998]). According to this effect, an rise in the marginal tax rate makes increases in the net wage more costly for firms while increases in the gross wage are less beneficial for employees. So the rise in the marginal tax rate strengthens the effective bargaining power of firms, which are negotiating the wage more aggressively. As a consequence, the wage falls and firms might have more flexibility to raise their labor demand.

The paper is organized as follows. Section 2 presents the model. Section 3 characterizes the optimal policy. Section 4 presents the functional form of the model, sets the steady state and defines calibration of the main parameters. Section 5 analyzes dynamic results. Section 6 develops alternative scenarios. Section 7 concludes.

2 Model

The financing of unemployment insurance benefits is considered an infinite horizon matching model with discrete time, featuring two kinds of agents, employed and unemployed workers, as well as an intensive margin. Productivity is exogenous and is subject to shocks. In order to shed light on the insurance motive of the fiscal policy, I assume no savings. The government set taxes as a wedge between the gross and the net labor income, in order to finance unemployment insurance benefits. As the wage is flexible, taxes weigh on both employed workers and firms.

2.1 Timing

Timing of the model is an important focus in order to catch the main monitoring issue. In particular, it really matters that the matching process happen during the period in order to shed light on how the government chooses the optimal allocation. During one period, the successive steps are:

- 1. n_{t-1} workers enter the period employed, $1 n_{t-1}$ enter the period unemployed.
- 2. A fraction s of jobs are destroyed.

- 3. Unmatched firms pay a vacancy cost and find a new worker with a probability corresponding to the vacancy filling rate.
- 4. Unemployed workers search for a job and a fraction of them, corresponding to the job-finding rate, find a new job. The fraction s of employed workers who just lost their job cannot search before next period.
- 5. n_t workers are now employed: previous employed workers who did not lose their job and previous unemployed workers who find a new job. They bargain their wage and hours worked with the firm that hire them.
- 6. Employed workers consume their net labor income and unemployed workers consume unemployment benefits.

2.2 Matching function

The matching process is of the type "one-job one-firm". The number of new matches in period t is:

$$m((1 - n_{t-1})S_t, V_t) \tag{1}$$

with $(1 - n_{t-1})$ the unemployment rate at the beginning of period t, S_t the mean search effort exerted by unemployed workers and V_t the number of vacancies posted in period t. $S_t(1 - n_{t-1})$ is the quantity of efficiency units of search. The matching function is strictly increasing and concave in each argument, features constant returns to scale and the number of new matches cannot exceed the number of potential matches $m((1 - n)S, V) \leq \min((1 - n)S, V)$.

As there is no on-the-job search, labor market tightness is defined as $\theta_t \equiv \frac{V_t}{(1-n_{t-1})S_t}$. The job finding rate per unit of search is defined as $f(\theta_t) \equiv m(1,\theta_t)$ so that the job finding rate is $S_t f(\theta_t) = m(.)/(1-n_{t-1})$, where $Sf(\theta) < 1$. The vacancy filling rate is $q(\theta_t) = m(.)/V_t$ and $q(\theta) < 1$. We have $f(\theta_t) = \theta_t q(\theta_t)$. The job finding rate perunit of search is increasing in the market tightness θ_t whereas the vacancy filling rate is decreasing in θ_t . We denote by s the exogenous exit rate.

The law of motion for employment is given by:

$$n_t = (1-s)n_{t-1} + f(\theta_t)S_t(1-n_{t-1})$$
(2)

2.3 Workers

At the end of the matching process, each one of the $1 - n_t$ unemployed workers determines his search effort, while the n_t employed workers choose a firm depending on the contract (wage and hours worked) it offers. There is no on-the-job search. Workers are characterized by the same preferences and have no access to financial markets.

Let w_t be the hourly wage of an employed worker in period t and let l_t be his intensive labor supply (hours worked) in period t. His budget constraint is given by $C_{e,t} = w_t l_t - T_t$, with $C_{e,t}$ his consumption (net labor income) and T_t taxes on gross labor income¹. In period t, an employed worker enjoys utility from consumption and suffers from disutility from work. His present discount value is given by:

$$V_{e,t} = (1-s) \left(u(C_{e,t}) - v(l_t) + \beta \mathbb{E}_t \left[V_{e,t+1} \right] \right) + s \left(u(C_{u,t}) + \beta \mathbb{E}_t \left[V_{u,t+1} \right] \right)$$
(3)

An unemployed worker only consumes unemployment benefits B_t paid by the Social Security system and bears the cost of search effort S_t . He chooses the intensity of his search activities in order to maximize his present discount value, which takes into account his probability $S_t f(\theta)$ to find a new job. The Social Security system is unable to monitor this decision, which gives rise to a moral hazard issue : unemployed workers have less incentives to search than required in the first-best economy. The Social Security system should therefore set its policy in order to provide incentives for unemployed workers to search.

$$V_{u,t} = \max_{S_t} \left\{ -\psi(S_t) + S_t f(\theta_t) \left(u(C_{e,t}) - v(l_t) + \beta \mathbb{E}_t \left[V_{e,t+1} \right] \right) + \left(1 - S_t f(\theta_t) \right) \left(u(C_{u,t}) + \beta \mathbb{E}_t \left[V_{u,t+1} \right] \right) \right\}$$
(4)

 $\beta \in (0,1)$ is the discount factor. The disutility of labor for employed workers $v : \mathbb{R}_+ \to \mathbb{R}$ and the cost of search for unemployed workers $\psi : \mathbb{R}_+ \to \mathbb{R}$ are both strictly continuous, increasing, convex, twice differentiable, $\psi'(0) = 0$ and v'(0) = 0. The utility of consumption $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly continuous, increasing, concave, twice differentiable and $\lim_{C \to 0} u(C) = +\infty$. The first-order condition of the Bellman equation provides the incentive constraint for the unemployed workers:

$$\psi'(S_t) = f(\theta_t) \Delta_{w,t} \tag{5}$$

¹Here, taxes take a general form, which can be summarized as the sum of a lump-sum and a linear tax in a representative agent model.

where $\Delta_{w,t}$ is defined as the surplus from being employed :

$$\Delta_{w,t} \equiv u(C_{e,t}) - v(l_t) - u(C_{u,t}) + \beta \mathbb{E}_t \left[V_{e,t+1} - V_{u,t+1} \right]$$
(6)

Due to the timing of the model, the surplus is the difference between the gain from employment and from unemployment after the matching happened, whereas the excess present value $V_{e,t} - V_{u,t}$ is related to the expected gain before the matching process.

According to this incentive constraint, the marginal cost of search is equal to the expected marginal revenue from search. From Equations (3) to (5), we rewrite the first order condition of the worker's optimization problem :

$$\frac{\psi'(S_t)}{f(\theta_t)} = \left[u(C_{e,t}) - v(l_t) - u(C_{u,t})\right] + \beta \mathbb{E}_t \left[\psi(S_{t+1}) + (1 - s - S_{t+1}f(\theta_{t+1})) \frac{\psi'(S_{t+1})}{f(\theta_{t+1})}\right]$$
(7)

The current surplus from employment is the sum of the current gain from employment in terms of utility and effort plus the weighted surplus from employment next period.

2.4 Firms

Firms are considered in a competitive search equilibrium (CSE), a framework developped by Moen [1997] allowing for search and matching frictions while ensuring an efficient surplus-splitting between the firm and the employed worker². In this approach, the economy is made of many islands, indexed by j, among which workers are perfectly mobile and have perfect information. On each island, firms post job offers in the form of a contract, made of a wage and a volume of hours worked, which means that the number of vacancies can vary ex-ante across islands, as well as labor tightness, number of workers, hours worked and wages. In equilibrium, symmetry is imposed. Search activities are directed by these posted contracts. A firm makes its choices in two stages. Firstly, on each island, a firm chooses whether to post a vacancy and enter production or not. Secondly, the firm selects the best contract in order to maximize the return on its vacancy under the incentive constraint given by (7), which materializes how the posted contract affects the queue on the local labor market.

Firms are made of one job and are either matched to a worker or vacant. A matched firm produces the consumption good under constant return to scale: $y_{j,t} = Z_t l_{j,t}$, where

 $^{^{2}}$ In a Nash bargaining equilibrium, the surplus sharing is determined by exogenous bargaining parameters and has thus no reason to be efficient. Hosios condition insures an efficiency at the steady-state but cannot be easily defined in a dynamic environment. We solve the model with Nash bargaining instead of competitive search equilibrium and show the implications of this choice in Appendix.

hourly productivity per capita Z_t is the same for every worker on each island and in each firm. Firms and workers have the same discount rate. A filled job generates immediate profits $y_{j,t} - w_{j,t}l_{j,t}$. To post a vacancy, a firm pays a fix cost c and finds a worker with probability $q(\theta_{j,t})$. $J_{ej,t}$ denotes the present discounted value of a firm on island jentering period t matched to a worker and $J_{uj,t}$ denotes the value of an unmatched firm on island j posting a vacancy. The present value of a filled job is:

$$J_{ej,t} = (1-s) \left((Z_t - w_{j,t}) l_{j,t} + \beta \mathbb{E}_t \left[J_{ej,t+1} \right] \right) + s \beta \mathbb{E}_t \left[J_{uj,t+1} \right]$$
(8)

while a vacant job produces the present value:

$$J_{uj,t} = -c + q(\theta_{j,t}) \left((Z_t - w_{j,t}) l_{j,t} + \beta \mathbb{E}_t \left[J_{ej,t+1} \right] \right) + (1 - q(\theta_{j,t})) \beta \mathbb{E}_t \left[J_{uj,t+1} \right]$$
(9)

In a symmetric competitive equilibrium, the value of an unemployed firm at any period t is equal to zero due to the free entry condition. Firms enter production on each island until the marginal cost of a vacancy is equal to its marginal benefit. They face the risk not to get a return on their investment, but they can perfectly insure on a perfect financial market. The present value of a matched firm is :

$$J_{ej,t} = (1-s)\frac{c}{q(\theta_{j,t})} \tag{10}$$

The surplus from a matched vacancy is equal to the average cost of a vacant job:

$$\Delta_{fj,t} \equiv (Z_t - w_{j,t})l_{j,t} + \beta \mathbb{E}_t \left[J_{ej,t+1} \right] = \frac{c}{q\left(\theta_{j,t}\right)}$$
(11)

2.5 Competitive search

Once entered in the market, a firm chooses a contract³ specifying both a wage $w_{j,t}$ and hours worked $l_{j,t}$ in order to maximize its return on investment. It takes into account the impact of its choice on local labor market tightness through reactions of job seekers.

 $^{^{3}}$ Pissarides [2000] explains that hours of work might be determined either by the worker maximizing his utility or simultaneously with the wage during the Nash bargaining process. In the first case, workers will choose a suboptimal quantity of work because they compare their marginal cost of effort to their marginal benefit in terms of hourly wage, whereas in the second case, they compare the same marginal cost to the marginal productivity of the firm, which is higher than the hourly wage. Parmentier [2006] argues that "considering that workers choose their working hours would introduce a bias toward a lower decrease in wage rates following an increase in the marginal tax rate. Firms would have to take into account the negative effect of higher marginal tax rates on hours and thus compensate by a smaller decrease in wages."

We build here on the work of Arseneau and Chugh [2008], who show how to implement a competitive search equilibrium in a DSGE model. The program of the firm is written:

$$\max_{\substack{w_{j,t}, l_{j,t}, \theta_{j,t} \\ s.t.}} q(\theta_{j,t}) \left((Z_t - w_{j,t}) \, l_{j,t} + \beta \mathbb{E}_t \left[J_{ej,t+1} \right] \right)$$

s.t.
$$\frac{\psi'(S_{j,t})}{f(\theta_{j,t})} = \left[u(C_{ej,t}) - v(l_{j,t}) - u(C_{u,t}) \right] + \beta \mathbb{E}_t \left[\psi(S_{j,t+1}) + (1 - s - S_{j,t+1}f(\theta_{j,t+1})) \frac{\psi'(S_{j,t+1})}{f(\theta_{j,t+1})} \right]$$

Optimal wage and hours worked are characterized by the first-order conditions of this program, where τ_t denotes the marginal tax rate and $\varepsilon_{\theta j,t}$ the elasticity of matches with respect to vacancies, which is also the elasticity of the job-finding rate per unit of search with respect to tightness.

$$\psi'(S_{j,t}) = \frac{1 - \varepsilon_{\theta_{j,t}}}{\varepsilon_{\theta_{j,t}}} c\theta_{j,t} (1 - \tau_t) u'(C_{ej,t})$$
(12)

$$(1 - \tau_t)u'(C_{ej,t}) = \frac{v'(l_{j,t})}{Z_t}$$
(13)

Equation (12) sheds light on the impact of the net-of-tax rate $1 - \tau_t$ on the division of the surplus from a filled job between the firm and the worker. The net of tax rate $1 - \tau_t$ can be interpreted as the ratio between the marginal rate of substitution (between consumption and leisure) and the marginal rate of transformation (between time spend searching for a job and production of goods). When the marginal taxe rate is high, any increase in the net wage will be expensive for firms whereas a reduction in the gross wage will be only partly passed on to the net wage. Thus labor supply is less sensitive to variations in the gross wage than labor demand. As a consequence, thanks to the tax system, firms get an upper hand in the bargaining process and can put a downward pressure on wage. This "wage moderating" effect has already been stressed in a static environment by a large microeconomic literature (Sorensen [1999], Parmentier [2006], Lehmann et al. [2013]). Rewriting this equation facilitates interpretation.

$$\psi'(S_{j,t}) = c\theta_{j,t} \frac{1 - \varepsilon_{\theta_{j,t}}}{\varepsilon_{\theta_{j,t}}} \frac{1 - \tau_t}{1 - \alpha_{j,t}} u'(C_{u,t})$$

where $\alpha_t = [u'(C_{u,t}) - u'(C_{ej,t})] / u'(C_{ej,t})$ is increasing in the degree of incentives. Search effort will be higher when incentives are strong, unemployment benefits low, when the marginal tax rate is tiny and when there are a lot of job opportunities. Equation (13) represents the equality between the marginal cost of hours worked and the net-of-tax marginal gain for the whole economy of an additional working hour (productivity expressed in terms of marginal utility). It may also be seen as the equality between the net-of-tax rate and the ratio of the marginal rate of substitution to the marginal rate of transformation.

Finally, the equilibrium is symmetric across submarkets, which allows to get rid of subscript j.

2.6 Product market equilibrium

The ressource constraint is given by:

$$n_t Z_t l_t = n_t C_{e,t} + (1 - n_t) C_{u,t} + n_t c \left(\frac{1}{q(\theta_t)} - \beta \mathbb{E}_t \left[\frac{1 - s}{q(\theta_{t+1})} \right] \right)$$
(14)

where the last term on the right hand side is the sum of vacancy cost and profits. (Cf. Appendix 8.1 for more details.)

2.7 Social Security system and tax policy

2.7.1 Budget constraint

In a first step, we consider as Burda and Weder [2010], Jung and Kuester [2011], Landais et al. [2010] that the government has to balance its budget constraint each period:

$$n_t T_t = (1 - n_t) B_t \tag{15}$$

In section 21, we look at the case where the government has access to perfect financial markets and is therefore able to smooth its budget over the cycle, as in Mitman and Rabinovich [2011].

2.7.2 Taxation

Most of the articles dealing with unemployment benefits over the business cycle generally highlight one specific fiscal policy to support employment. The present article shows how, with one very general tax, the government is able to both finance unemployment insurance benefits and stimulate the labor market.

Taxation follows a very general form so that the government is able to use a large panel of tools in order to implement its objectives. Lump-sum taxes are very useful to raise tax revenue or finance tax rebates without generating distortions. Equations (12) and (13) show that the marginal tax rate might impact work incentives. Tax revenues are expressed as the sum of a lump-sum and a linear component : $T_t = T_{0,t} + \tau_t w_t l_t$, where τ_t stands for the marginal tax rate in period t and $T_{0,t}$ for the lump-sum component.

As wages are fully flexible, taxes are paid by both employers and employees. They affect the global surplus from the matching process and induce direct reactions from both labor supply and demand.

Finally, even if the focus here seems to be purely on labor taxation, it is not the case. A lump-sum tax on production could be added in this economy, as in Jung and Kuester [2011] or in Mitman and Rabinovich [2011], but this would be of little interest as this instrument would be totally redundant with the lump-sum component of taxation. It is the same for social contributions for employers, vacancy subsidies and any other kind of taxes involving transactions that are taken into account in this model.

3 A Ramsey Primal Approach to Unemployment Insurance Benefits

We follow here the Ramsey primal approach, as defined by Chari and Kehoe [1998] and Ljungqvist and Sargent [2004], in order to find the optimal fiscal policy. At each period, the economy is characterized by a competitive equilibrium and the government is searching for the optimal policy in order to decentralize this equilibrium. According to Chari and Kehoe [1998], solving this problem is the same as considering that the government maximizes the social welfare in order to find the best allocation, subject to resource and implementability constraints. From the best allocation, it is straightforward to infer the optimal tax and transfer policy.

3.1 Competitive Equilibrium

A competitive equilibrium is defined as a feasible allocation $\{C_{e,t}, C_{u,t}, l_t, \theta_t, n_t, S_t\}_{t=0}^{\infty}$, a price system $\{w_t\}$ and a policy $\{B_t, T_t, T'_t\}$ such that, given the tax policy and the price system, the allocation solves (i) the employed worker budget constraint, (ii) the ressource constraint (14), (iii) condition (13) on hours worked, (iv) equation characterizing the present value of a filled job, (v) the law of motion of employment (2), (vi) the incentive constraint for search activities (5) and (vii) condition (12) on wage. For convenience, we rewrite the system of equations characterizing the competitive equilibrium:

$$\begin{split} C_{e,t} &= w_t l_t - T_t \\ n_t Z_t l_t &= n_t C_{e,t} + (1 - n_t) C_{u,t} + n_t c \left(\frac{1}{q(\theta_t)} - \beta \mathbb{E}_t \left[\frac{1 - s}{q(\theta_{t+1})}\right]\right) \\ v'(l_t) &= Z_t u'(C_{e,t})(1 - \tau_t) \\ \frac{c}{q(\theta_t)} &= (Z_t - w_t) l_t + \beta c \mathbb{E}_t \left[\frac{(1 - s)}{q(\theta_{t+1})}\right] \\ n_t &= (1 - s) n_{t-1} + S_t f(\theta_t)(1 - n_{t-1}) \\ \frac{\psi'(S_t)}{f(\theta_t)} &= \left[u(C_{e,t}) - v(l_t) - u(C_{u,t})\right] + \beta \mathbb{E}_t \left[\psi(S_{t+1}) + (1 - s - S_{t+1}f(\theta_{t+1})) \frac{\psi'(S_{t+1})}{f(\theta_{t+1})}\right] \\ \psi'(S_t) &= \frac{1 - \varepsilon_{\theta,t}}{\varepsilon_{\theta,t}} c \theta_t (1 - \tau_t) u'(C_{e,t}) \end{split}$$

Some additional remarks about the steady-state and dynamics of the competitive equilibrium are provided in Appendix 8.2.

3.2 Decentralization of the equilibrium

Taxes are wedges between marginal rates of transformation and marginal rates of substitution. According to the Ramsey primal approach, it is possible to express the price system and the fiscal policy of the competitive equilibrium as a function of the allocation $\{C_{e,t}, C_{u,t}, l_t, \theta_t, n_t, S_t\}_{t=0}^{\infty}$. Then decentralizing the competitive equilibrium is the same as choosing the best allocation under specific constraints.

Four constraints are necessary in order for the government to replicate the competitive equilibrium. Two of them characterize the ressources of the economy, the other two being implementability constraints. In Appendix 8.2, we demonstrate that the allocation and price system in a competitive equilibrium satisfies the resource and implementability constraints, and that, given allocation and period 0 policies satisfying these four constraints, we can construct an allocation, a price system and a fiscal policy constituting a competitive equilibrium.

The first resource constraint is given by Equation (14). The second resource constraint is a technological constraint : it is the law of motion of employment (2). The first implementability condition, related to the incentive problem, is given by Equation (7). The second implementability condition comes from wage (12) and working hours (13) setting processes in the competitive search equilibrium :

$$\psi'(S_t) = c \frac{\chi}{1-\chi} \theta_t \frac{v'(l_t)}{Z_t}$$
(16)

3.3 Optimal Policy

The optimal policy is defined as the best allocation a time-consistent government would set maximizing the weighted sum of net utilities of workers:

$$W_{t} = \max_{C_{e,t}, C_{u,t}, l_{t}, \theta_{t}, n_{t}, S_{t}} \left\{ n_{t} \left[u(C_{e,t}) - v(l_{t}) \right] + (1 - n_{t})u(C_{u,t}) - (1 - n_{t-1})\psi(S_{t}) + \beta \mathbb{E}_{t} \left[W_{t+1} \right] \right\}$$
(17)

subject to the ressource constraint (14), the two implementability constraints (7) and (16), and the technological constraint (2). Four dynamic Lagrange multipliers, $\lambda_{1,t}$, $\lambda_{2,t}$, $\lambda_{3,t}$, $\lambda_{4,t}$ are associated to these four constraints. The first-order conditions with respect to $C_{e,t}$, $C_{u,t}$, l_t , θ_t , n_t , S_t are respectively:

$$\frac{1}{\lambda_{1,t}} = \frac{n_t}{u'(C_{e,t})} + \frac{1-n_t}{u'(C_{u,t})}$$
(18a)

$$\lambda_{2,t} = n_t \left[\frac{\lambda_{1,t}}{u'(C_{e,t})} - 1 \right]$$
(18b)

$$\lambda_{3,t}\psi'(S_t) = -\frac{\lambda_{1,t}}{\eta} \left[1 - \frac{v'(l_t)}{Z_t u'(C_{e,t})} \right] l_t Z_t n_t$$
(18c)

$$-\lambda_{3,t}\psi'(S_t)\left(1+\frac{1}{\kappa}\right) = \frac{c}{q(\theta_t)}\left(\varepsilon_{\theta_t}-1\right)\left[n_t\lambda_{1,t}-(1-s)n_{t-1}\lambda_{t-1}\right]$$
(18d)

$$+ \frac{\psi'(S_{t})}{f(\theta_{t})} \varepsilon_{\theta_{t}} [\lambda_{2,t} - (1-s)\lambda_{2,t-1}] + \lambda_{4,t}(1-n_{t-1})S_{t}f(\theta_{t})\varepsilon_{\theta_{t}}$$

$$\lambda_{4,t} = [u(C_{e,t}) - v(l_{t}) - u(C_{u,t}) + \beta\psi(S_{t+1})]$$
(18e)

+
$$\beta (1 - s - S_{t+1}f(\theta_{t+1})) \lambda_{4,t+1} + \lambda_{1,t} \frac{C_{u,t}}{n_t}$$

$$S_{t}(1 - n_{t-1}) = \frac{\mu}{f(\theta_{t})} \left[(1 - s - S_{t}f(\theta_{t}))\lambda_{2,t-1} - \lambda_{2,t} \right]$$
(18f)
$$- \lambda_{3,t}\mu + \lambda_{4,t} \frac{(1 - n_{t-1})f(\theta_{t})S_{t}}{\psi'(S_{t})}$$

Together with the two technological constraints (14), (2) and the two implementability constraints (7), (16), these ten equations define the planner allocation.

The inverse marginal utility can be interpreted as the marginal resource cost of providing utility u(C) to a worker. Thus $1/\lambda_{1,t}$ is the total marginal resource cost of providing utility $u(C_{e,t})$ to employed workers and $u(C_{u,t})$ to unemployed workers, which is consistent with the common interpretation of $\lambda_{1,t}$ as the shadow cost of relaxing the resource constraint 14. The higher $\lambda_{1,t}$, the stronger the insurance motive for unemployed workers. The shadow cost of relaxing the incentive constraint is given by $\lambda_{2,t}$. In case of perfect insurance, we would have $C_{u,t} = C_{e,t}$, $\lambda_{1,t} = u'(C_{e,t})$ and $\lambda_{2,t} = 0$, thus the incentive constraint would not bind anymore. So $\lambda_{2,t}$ is positively related to the incentive motive. $\lambda_{3,t}$ is the shadow cost for the government of reducing the wedge between the MRS and the MRT, which is the loss in revenues from linear taxes (as $\tau_t = 1 - (v'(l_t)/Z_t u'(C_{e,t})) = 0$) and thus in resources in order to provide utility for workers. The last Lagrange multiplier $\lambda_{4,t}$ is the shadow cost of hiring more workers.

4 Steady-state analysis

4.1 Functional Form

Utility function is CRRA where γ stands for relative risk aversion: $u(C_t) = (C_t^{1-\gamma} - 1)/(1-\gamma)$. Disutility from work is modeled as $v(l_t) = b_1 l_t^{1+\eta}/1 + \eta$ where η is the inverse of the Frish elasticity of labor supply. Disutility from search is $\psi(S_t) = b_2 S_t^{1+\mu}/1 + \mu$ where μ is the inverse of the elasticity of the extensive labor supply. b_1 and b_2 are scale parameters.

The matching function is of the CES type:

$$m\left((1-n_t)S_t, V_t\right) \equiv \frac{(1-n_t)S_tV_t}{\left[\left((1-n_t)S_t\right)^{1/\kappa} + V_t^{1/\kappa}\right]^{\kappa}}$$

which restricts the job-finding rate per unit of search⁴ and the vacancy-filling rate to belong to the [0; 1] interval.

The log of productivity follows an AR(1), so that $Z_t = Z_{t-1}^{\rho} \epsilon_t$, where ϵ_t are independent and identically distributed exogenous shocks drawn from a Gaussian distribution and $0 \le \rho < 1$ is the persistence parameter.

4.2 Calibration

Baseline calibration is given by Table 1. The discount factor is equal to 0.99. κ is set to 1.7 so that the elasticity of matches with respect to vacancies is equal to 0.5, which is usual in the literature about tax incidence on the labor market. η is set to 2 which is consistent with a Frish elasticity of labor supply if 0.5. There is less consensus about the elasticity of the extensive labor supply. The meta-analysis of Chetty et al. [2011] advocates for an elasticity between 0.15 and 0.33, lower than the elasticity of the

⁴But we have to make sure that $S_t f(\theta_t) < 1$.

intensive labor supply. I set μ equal to 5.5 which is consistent with this fact and with the calibration of Mitman and Rabinovich [2011]. γ takes the standard value of 2 in order to account for risk-aversion and to shed light on the insurance motive. The exogenous exit rate is set to 1.5%. c, b_1, b_2 are calibrated internally in the model.

Productivity AR1 term	ρ	0.8
Risk aversion	γ	2
Inverse Frish elasticity	η	2
Inverse elasticity of the extensive margin	μ	5.5
Discount factor	β	0.99
Matching function parameter	κ	1.7
Vacancy cost	c	0.5
Exit rate	s	0.015
Scale parameter (intensive margin)	b_1	1.35
Scale parameter (extensive margin)	b_2	1.66

Table 1: Parameters

4.3 Steady state

The steady state value of the allocation and of the Lagrange multipliers (cf. Table 2) is given by the following system of equations :

$$nZl = nC_e + (1-n)C_u + n\frac{c}{q(\theta)} (1-\beta(1-s))$$
 (19a)

$$\frac{\psi'(S)}{f(\theta)} = \frac{1}{1 - \beta \left(1 - s - Sf(\theta)\right)} \left[u(C_e) - v(l) - u(C_u) + \beta \psi(S)\right] (19b)$$

$$\psi'(S) = c \frac{\chi}{1-\chi} \theta \frac{v'(l)}{Z}$$
(19c)

$$n = \frac{Sf(\theta)}{s + Sf(\theta)}$$
(19d)

$$\lambda_1 = \frac{u'(C_u)u'(C_e)}{nu'(C_u) + (1-n)u'(C_e)}$$
(19e)

$$\lambda_2 = n(1-n)\frac{u'(C_u) - u'(C_e)}{nu'(C_u) + (1-n)u'(C_e)}$$
(19f)

$$\lambda_3 = -\frac{nlZ}{\eta\psi'(S)} \left[1 - \frac{v'(l)}{Zu'(C_e)} \right] \lambda_1$$
(19g)

$$-\lambda_3 \psi'(S) \left(1 + \frac{1}{\kappa}\right) = \frac{c}{q(\theta)} (\varepsilon_\theta - 1) sn\lambda_1 + \frac{\psi'(S)}{f(\theta)} \varepsilon_\theta s\lambda_2 + \lambda_4 ns\varepsilon_\theta$$
(19h)

/

$$\lambda_4 = \frac{\psi'(S)}{f(\theta)} + \frac{1}{1 - \beta \left(1 - s - Sf(\theta)\right)} \frac{\lambda_1 C_u}{n}$$
(19i)

$$S(1-n) = -\frac{\mu}{f(\theta)} \left(s + Sf(\theta)\right) \lambda_2 - \lambda_3 \mu + \lambda_4 \frac{ns}{\psi'(S)}$$
(19j)

Table 2: Steady-state

C_e	C_u	n	l	θ	S	$f(\theta)$	$q(\theta)$
0.889	0.510	0.943	0.959	0.944	0.822	0.299	0.317

The difference between consumption of employed and unemployed workers is due to the incentive motive and the unability of the government to monitor search activities. When γ is higher, agents are more risk-averse and want to self-insure more, which reduces the discrepancy between these two consumption levels. The wage is equal to 0.96, inducing a gross replacement ratio of 53% and a net one of 57%. Unemployment rate is equal to 5.7%, which is quite low but close to many articles (Shimer and Werning) and consistent with the optimality of the framework. More informations about the steady-state are provided in Appendix 8.2.

5 Dynamic analysis of a transitory shock

The dynamics of taxes, transfers and allocations following a negative shock sheds light on the optimal fiscal policy during a crisis. This dynamics is given by the four constraints (Equations (2), (14), (7) and (16)) and the six first-order conditions (Equation (18)) of the planner's problem. Productivity per capita Z_t is exogenous. Among the 10 endogenous variables, three are forward-looking : search S_t , labor-market tightness θ_t and the fourth Lagrange multiplier $\lambda_{4,t}$. Indeed, search activities depend on the future state of the economy as expected by unemployed workers. Current tightness is a function of search and of vacancies, which are determined according to the expected future state of the labor market. The seven remaining endogenous variables are predetermined and employment is the main state variable. The focus here is on a 1% negative transitory and unexpected productivity shock at the steady state⁵.

Figure 1: 1% negative productivity shock at the steady state



Firms are the first concerned by this shock. As it occurs, they cut wages, hours worked and vacancies in order to maintain their margins, which lowers the disposable income of employed workers. The drop in labor market tightness θ reduces the jobfinding rate per-unit-of-search and thus the marginal gain from search. Unemployed

⁵The case of a permanent shock provides very similar conclusions and is developed in Appendix 8.3.

Figure 2: 1% negative productivity shock at the steady state : first and second Lagrange multipliers



workers react by reducing their search effort and employment falls as a consequence. In a time of higher risk to get stuck in an unemployment trap (as the job-finding rate is low), and due to their high level of risk aversion, employed workers are willing to finance more unemployment insurance benefits.

The objectives of the government are reflected by the reactions of the first two Lagrange multipliers (Figure 2). The increase in the first multiplier depicted by the graph on the left is due to a reduction in the cost to provide insurance for unemployed workers. The drop in the second multiplier expresses a temporary contraction of the moral hazard issue. Indeed, during a recession, severe matching conditions on the labor market make it harder for unemployed workers to get a job, so that there is less need for incentives to search (cf. Landais et al. [2010]). As the crisis hits the economy, the insurance motive is higher than in the steady-state and the incentive motive is lower, which favours higher unemployment benefits.

In order to finance this policy, the government has to raise taxes. The best way is to levy non-distortive lump-sum taxes on labor. However, such kind of taxes reduces the value from work. According to Equation (7), this policy might have a detrimental effect on job search. To compensate for this non-desirable side effect, the government reduces the marginal tax rate in order to stimulate the intensive and extensive labor supplies (Equation 12 and 13)⁶.

⁶Due to the calibration of γ , variation of taxes induce strong income effects, but those are annihilated by the government thanks to lump-sum taxes.

As the initial shock vanishes, the incentive issue becomes more and more important. The government react to this new objective by reducing UI benefits below their steadystate level, therefore lowering the value from unemployment. In addition, he sets up a tax credit (reduction of lump-sum taxes) in order to increase the value from work and to support the extensive margin. This policy is financed through a rise in the marginal tax rate, which has a limited impact on labor demand thanks to the previously mentionned "wage moderating" effect and a positive impact on intensive labor supply due to the income effect.

As a consequence of this policy, the surplus from employment is larger and unemployed workers raise their search effort. Hours worked increase due to an income effect. Firms anticipate these positive impacts of the tax policy and post more vacancies. The matching process is more efficient, tightness rises as well as employment.

	-	~	~			^	~	-		
	Z	C_e	C_u	l	n	θ	S	T'	au	T_0
Z	1	0.87	-0.70	0.74	0.81	0.77	0.64	-0.80	0.52	-0.67
C_e	-	1	-0.26	0.34	0.62	0.35	0.18	-0.44	0.09	-0.23
C_u	-	-	1	-0.98	-0.74	-0.99	-0.99	0.95	-0.83	0.93
l	-	-	-	1	0.82	0.96	0.95	-0.98	0.71	-0.85
n	-	-	-	-	1	0.73	0.66	-0.91	0.37	-0.58
θ	-	-	-	-	-	1	0.98	-0.94	0.86	-0.95
S	-	-	-	-	-	-	1	-0.92	0.87	-0.95
T	-	-	-	-	-	-	-	1	-0.68	0.84
au	-	-	-	-	-	-	-	-	1	-0.97
T_0	-	-	-	-	-	-	-	-	-	1

Table 3: Correlogram of the main variables

Table 3 summarizes correlations between the main variables. Unemployment insurance benefits are countercyclical because workers require more insurance during recessions. Unemployment is countercyclical as well, while consumption of employed workers, labor-market tightness, hours worked and search are procyclical: these findings are in line with economic theory and empirical facts. Tax revenue are countercyclical here due to the tight budget constraint the government faces. The negative correlation between Sand C_u reflects the moral hazard issue when insuring unemployed workers, whereas the one between θ and C_u is related to the insurance motive: when the labor market is less tight, it is harder to find a job and unemployed workers need more insurance. Finally, lump-sum taxes T_0 are a major instrument to finance UI benefits C_u , as shown by the positive correlation between these two variables. Otherwise, increasing lump-sum taxes has a negative impact on the main outcomes of the labor market, as it reduces the global surplus from work.

6 Alternative scenarios

6.1 Risk-neutral agents

In the baseline model, impulse-response functions reveal an overshooting. For instance, after the shock, as employment converges toward its steady state, it firstly exceeds this level before catching up with it. This mechanism might be due to the succession of the insurance and incentive objective, implemented sequentially by the government through, among other things, an increase followed by a drop in UI benefits.

Figure 3: Risk-neutral agents



To assess this intuition, the insurance motive for UI benefits is shutted down, assuming that workers are risk-neutral ($\gamma = 0$). UI benefits fall to a very low steady-state level because workers do not feel the need to insure against the unemployment risk. Figure 3 displays the impulse-response functions when workers are risk-neutral. As there is no more insurance motive, UI benefits immediatly drop when the shock hits the economy and then slowly go back to the steady state as the shock blurs. Therefore, the overshooting mechanism disappears. The main aggregates follow the same dynamics as in the benchmark case. The government directly implements the incentive objective, setting up a tax credit in order to stimulate the extensive margin (both on the demand and on the supply side). He finances this policy through an increase in the marginal tax rate, which does not harm labor demand thanks to the wage moderating effect.

6.2 Public debt

The government has no access to financial markets in this economy, he has to balance his budget each period and can only rely on taxes. If it were able to freely borrow on perfect financial markets, it might be easier for him to implement his different objectives: raising funds, insuring workers and providing incentives to work. Assuming the government has access to public debt, his budget constraint becomes :

$$D_0 = \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^{t-1} \left[n_t T_t - (1-n_t) C_{u,t}\right]$$
(20)

where r is the real interest rate, taken as constant, and D_0 is the initial level of debt before the negative shock occurs. We take $D_0 = 0$ and r is equal to the discount rate of the agents, so that $1/(1+r) = \beta$. The program of the planner is:

$$\max_{C_{e,t},C_{u,t},l_{t},\theta_{t},n_{t},S_{t}} \sum_{t=0}^{\infty} \beta^{t} \left\{ V\left(C_{e,t},C_{u,t},l_{t},\theta_{t},n_{t},S_{t},\lambda_{1}\right) + \lambda_{2,t} \left[\frac{\psi'(S_{t})}{f(\theta_{t})} - \left[u(C_{e,t}) - v(l_{t}) - u(C_{u,t}) \right] - \beta \mathbb{E}_{t} \left[\psi(S_{t+1}) + (1 - s - S_{t+1}f(\theta_{t+1})) \frac{\psi'(S_{t+1})}{f(\theta_{t+1})} \right] \right] + \lambda_{3,t} \left[\psi'(S_{t}) - c \frac{\chi}{1 - \chi} \theta_{t} \frac{v'(l_{t})}{Z_{t}} \right] + \lambda_{4,t} \left[n_{t} - (1 - s)n_{t-1} + f(\theta_{t})S_{t}(1 - n_{t-1}) \right] \right\}$$

$$(21)$$

with:

$$V(C_{e,t}, C_{u,t}, l_t, \theta_t, n_t, S_t, \lambda_1) = n_t \left(u(C_{e,t}) - v(l_t) \right) + (1 - n_t) u(C_{u,t}) - (1 - n_{t-1}) \psi(S_t) + \lambda_1 \left\{ n_t Z_t l_t - n_t C_{e,t} - (1 - n_t) C_{u,t} - n_t c \left(\frac{1}{q(\theta_t)} - \mathbb{E}_t \left[\frac{\beta(1 - s)}{q(\theta_{t+1})} \right] \right) \right\}$$

As before, the economy is characterized by the four constraints and the six first-order conditions of this optimization problem. We follow Ljungqvist and Sargent [2004] in order to find numerically the optimal allocation of this economy. Firstly we solve the steady-state. Secondly, for a value of λ_1 , we solve the dynamic economy. Thirdly, we compute the infinite-time ressource constraint. Finally, as long as this constraint is not equal to zero, we adjust λ_1 and repeat steps 2 and 3. The optimal allocation is obtained when the resource constraint holds⁷.

In the baseline calibration, the government had to increase taxes above the steadystate and then to reduce them under this level in order to successively fulfill his insurance and incentive objectives, which generated an overshooting reaction in the allocation. When he has access to public debt, he is able to complete these objectives simultaneously, as shown by Figure 4. As soon as the shock hits the economy, he raises unemployment benefits to insure workers and reduces lump-sum taxes at the same time, in order to stimulate labor demand and supply through a rise in the average return of a vacancy. Firms, expecting greater profits, immediatly raise vacancies, therefore stimulating search effort and employment from the first period on⁸. In order to reimburse public debt, the government raises the marginal tax rate, which has a limited negative impact on labor demand due to the wage moderating effect and a positive impact on labor supply resulting from an income effect.

At last, fluctuations of the main aggregates are much smaller than in the baseline scenario, meaning that an additionnal instrument such as public debt helps the government maximizing social welfare. Volatility of the shock is absorbed by lump-sum taxes.

6.3 Model without intensive margin

In order to shed light on the role of the intensive labor supply, we consider here an alternative model where there is only an extensive margin on the labor market.

Unemployment insurance benefits are now procyclical⁹. Figure 5 shows a tiny increase during the first quarters leaves quickly its place for a massive drop the following years, as in Mitman and Rabinovich [2011]. Contrary to the baseline scenario, the government is unable to raise enough lump-sum taxes to finance UI benefits without generating bad incentives. He cannot use the leverage of the intensive margin to raise tax revenue. Thus he sacrifices the insurance objective.

⁷We consider an horizon of 250 periods for public debt to come back to its steady-state level.

⁸A positive reaction of employment to a negative productivity shock is generally unlikely, but might be due in this framework to the absence of wage rigidity and of debt or deficit ceiling.

⁹This is confirmed by an analysis of the correlations.





6.4 Nash bargaining

In the baseline model, we consider firms in a Competitive Search Equilibrium (CSE) framework, in order to ensure an efficient surplus-splitting between the firm and the employed worker. However, most of the literature on unemployment insurance benefits over the business cycle consider wages resulting from Nash bargaining between the firm and its employee. We show here that the framework we consider for hours worked and wage setting has little impact on our main conclusions.

The maximization of the Nash product of the firm's and the worker's surplus $\Delta_{w,t}^{\chi} \Delta_{f,t}^{1-\chi}$ with respect to w_t and l_t gives the two following first order conditions :

$$\frac{\Delta_{w,t}}{\Delta_{f,t}} = \frac{\chi}{1-\chi} u'(C_{e,t})(1-\tau_t)$$
(22)

$$v'(l_t) = Z_t u'(C_{e,t})(1 - \tau_t)$$
(23)

where $\chi \in (0, 1)$ refers to the bargaining power of employees. The wage setting process has no impact on the determination of hours worked (Equation 23). Equation (22) is exactly the same as Equation (12), when the bargaining power of firms $1 - \chi$ is replaced by the local elasticity of matches with respect to vacancies $\varepsilon_{\theta j,t}$. A high elasticity means that a tiny variation of vacancies will have a major impact on matches, which is consistent with firms having a high bargaining power.

Figure 5: No intensive margin



The Ramsey approach to optimal taxation is not substantially affected by this assumption¹⁰. According to Lehmann and Linden [2007], Parmentier [2006] the bargaining power of employed workers χ is set to 0.5. κ is chosen so that the Hosios (1990) condition is satisfied at the steady state, which means that the steady-state value of the elasticity of job matches with respect to vacancies ε_{θ} is equal to the bargaining power of firms $1 - \chi$. Impulse-response functions are very similar to the benchmark case, the only difference being the size of these reactions. This result is consistent with Arseneau and Chugh [2008] finding that the competitive search equilibrium is equivalent to the Nash bargaining when the Hosios condition applies each period and that business cycle fluctuations under Nash bargaining when the Hosios condition is not binding are not far from those under competitive search equilibrium.

$$-\lambda_{3,t}\psi'(S_t)\left(1+\frac{1}{\kappa}\right) = \frac{c}{q(\theta_t)}\left(\varepsilon_{\theta_t}-1\right)\left[n_t\lambda_{1,t}-(1-s)n_{t-1}\lambda_{t-1}\right] \\ + \frac{\psi'(S_t)}{f(\theta_t)}\varepsilon_{\theta_t}\left[\lambda_{2,t}-(1-s)\lambda_{2,t-1}\right] + \lambda_{4,t}(1-n_{t-1})S_tf(\theta_t)\varepsilon_{\theta_t}$$

 $^{^{10}}$ Only the fourth first-order condition of the planner's program (18) is altered and becomes:

Figure 6: Nash bargaining



7 Conclusion

In this article, I focus on optimal unemployment insurance benefits and labor income taxes a government should implement during a recession. In line with most articles on the topic, I find that optimal unemployment benefits should be countercyclical, but this result depends strongly on the ability of the government to finance this policy.

As a negative shock occurs, priority is to insure workers through a bigger amount of UI benefits. However, the return to economic stability requires stimulating search activities as well as the labor demand, which is achieved through a fall in benefits together with a lump-sum tax credit. Considering a general form for taxation, I show that taxes are a very unseful tool to raise tax revenue and to stimulate the labor market, allowing unemployment insurance benefits to take care of the insurance objective. This general result might then be interpreted in terms of specific taxes, subsidies or tax credits, based on either firms or employees.

When the government has no access to public debt, he must increase lump-sum taxes sharply in order to finance the initial raise in UI benefits. This financing policy has a negative impact on employment, which can be compensated by a drop in the marginal tax rate in order to stimulate the labor supply. When the initial shock vanishes, the government sets up a tax credit in order to raise the value from employment and reduce labor costs. This policy is financed through an increase in the marginal tax rate whose negative impact on labor demand is limited thanks to the wage moderating effect. This chronological sequence of objectives is constrained by the tight budget the government has to observe: he cannot finance a tax credit when the negative shock occurs because he has to finance UI benefits.

In an alternative case where the government has access to perfect financial markets, I show that he is able to implement simultaneously the insurance and the incentive objective because benefits and taxes are not dependent anymore from each other in his budget constraint. On the contrary, in a scenario with tight budget constraint and no intensive margin, I find procyclical UI benefits. Depriving the government of a potential action on hours worked reduces his ability to raise tax revenue and reinforces the financing constraint he is facing. The reason why some articles find procyclical UI benefits might therefore be that they force the government to implement the optimal fiscal policy with only a restricted set of availiable fiscal instruments.

8 Appendix

8.1 Ressource constraint derivation

Equation (14) is derived from the equation defining the present value of a filled job and from both budget constraints of the government (15) and of the employed worker.

$$n_t Z_t l_t = n_t C_{e,t} + (1 - n_t) C_{u,t} + c V_t + c(1 - s) \left(\frac{n_{t-1}}{q(\theta_t)} - \beta \mathbb{E}_t \left[\frac{n_t}{q(\theta_{t+1})} \right] \right)$$

Once production has been used for consumption of both agents and investment in vacancies, firms make profits, which correspond to the last term in the previous equation. At the steady state, profits are equal to zero when $\beta = 1$. In the short run, they can be either positive or negative whether the growth of the vacancy filling rate exceeds the discounted employment growth or not. This illustrates the ability of firms to borrow on perfect financial markets in order to finance vacancy posting.

8.2 Competitive Equilibrium and steady-states

For an exogenous unemployment insurance policy¹¹, competitive equilibrium can be simulated. The steady-state is very similar to the case of optimal policy, except for the employment rate which is lower and for the intensive labor margin which is higher.

¹¹In the present case, $C_u = 0.5$ and $\tau = 0$.

Variable	Optimal	Optimal	Competitive		
	policy (NB)	policy (CSE)	equilibrium (CSE)		
C_e	0.888	0.889	0.891		
C_u	0.490	0.510	-		
n	0.947	0.943	0.940		
l	0.957	0.959	0.966		
θ	1.070	0.944	0.972		
S	0.846	0.822	0.832		
f(heta)	0.318	0.299	0.303		
q(heta)	0.297	0.317	0.312		
$\varepsilon_{ heta}$	0.490	0.508	0.504		
T	0.027	0.031	0.032		
au	0.026	0.020	-		
T_0	0.004	0.013	-		
w	0.956	0.959	0.956		

Table 4: Steady-states: competitive equilibrium and optimal policy (Nash Bargaining and Competitive Search Equilibrium)

In the optimal policy with competitive search, the firm is able to maximize the return of a vacancy while limiting the impact of its actions on labor market tightness. Its bargaining power is bigger than is the Nash bargaining equilibrium and labor market tightness is lower. Both of these reactions might have a strong negative impact on search if the government would not have chosen to reduce the marginal tax rate.

Figure 7: Competitive equilibrium



Compared to the optimal fiscal policy, the strict government budget constraint im-

poses taxes to be strictly above their steady-state. This prevent any incentive policy from happening, explaining therefore the slower return of employment toward its steady-state, and induces a strong income effect underlying the massive positive reaction of hours worked.

8.3 Demonstration of the Ramsey primal approach

We prove that: (i) the allocation and price system in a competitive equilibrium satisfies the resource and implementability constraints, (ii) given allocation and period 0 policies satisfying these two constraints, we can construct an allocation, a price system and a fiscal policy constituting a competitive equilibrium.

By construction, any allocation constituting a competitive equilibrium will satisfy both the resource constraint (14), the technology constraint (2) and the two implementability constraints (7) and (7). This proves the necessity.

The sufficiency requires to demonstrate that any allocation $\left\{C_{e,t}^*, C_{u,t}^*, \theta_t^*, n_t^*, l_t^*, S_t^*\right\}_{t=0}^{\infty}$ satisfying the resource constraint (14), the technology constraint (2) and the two implementability constraints (7) and (16), there is a sequence of policies $\{B_t, T_t, T_t'\}$ such that the allocation can be decentralized as a competitive equilibrium.

- 1. We define $B_t = C_{u,t}^*$, $T_t = (1 n_t)C_{u,t}^*/n_t$ and $T'_t = 1 \frac{v'(l_t^*)}{Z_t u'(C_{e,t}^*)}$. By construction, the allocation satisfies the budget constraint of the government (15).
- 2. The product market equilibrium is satisfied because it is the same as the resource constraint (14) of the Ramsey problem.
- 3. We define the wage so that it respects the employed worker budget constraint:

$$w_t = \frac{n_t^* C_{e,t}^* + (1 - n_t^*) C_{u,t}^*}{n_t^* l_t^*}$$

Replacing for this expression in the resource constraint (14), we find the value of the firm (10).

- 4. We have the law-of-motion of employment (2).
- 5. The first implementability condition (7) is the same as the incentive constraint.
- 6. The bargaining on hours (13) is given by the previous definition of the marginal tax rate T'_t .

7. Thanks to the expression of the marginal tax rate, replacing for $v'(l_t)/Z_t$ in the second implementability condition (16) provides the bargaining on wage (12).

Thus, for any allocation satisfying the resource constraint (14), the technology constraint (2) and the two implementability constraints (7) and (16), there is a sequence of fiscal policies $\{T_t, T'_t, B_t\}$ such that, when these policies are implemented, the allocation might be decentralized as a competitive equilibrium.

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