Mission-Oriented Firms, Motivated Workers, and Screening

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Abstract

We study non-linear optimal contracts offered by two firms competing for the exclusive services of workers, who are privately informed about their ability and motivation. Firms differ in their technology and in their mission, and motivated workers are keen to be hired by the mission-oriented firm. In equilibrium, the mission-oriented firm attracts fewer high-ability workers with respect to the competitor. We also find that workers exert more effort at the mission-oriented firm than at the standard one despite an upward distortion in effort levels being possible for the standard firm. Finally, a compensating wage differential emerges in that the mission-oriented firm, given the effort, offers lower wages than the standard firm.

JEL classification: D82, D86, J24, J31, M55.

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1 Introduction

There exists a well-established empirical evidence on compensating wage differentials that are uniquely generated by differences in job characteristics or attributes for which heterogeneous workers have different willingness to pay. For instance, an earnings penalty has been documented for public firms as opposed to private ones and for not-for-profit firms relative to for-profit organizations.[1]

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[1] For compensating wage differentials see Rosen (1986). The case of public versus private firms has been studied by Disney and Gosling (1998) and Melly (2005), among others. Lower average wages in not-for-profit firms relative to for-profit ones have been found by Preston (1989) and Gregg *et al.* (2011).

Among the prominent differences in jobs and/or firms determining wage gaps, Besley and Ghatak (2005) consider that some firms are often identified as mission-oriented because of the sector they operate in and/or because of the collective nature of the goods they produce (education, health and defence), whereas Bénabou and Tirole (2010) highlight the role of firms' explicit strategies in terms of corporate social responsibility: some firms take employee-friendly or environment-friendly actions, some employers are mindful of ethics, or they even have an investor-friendly behavior (as ethical banks). Those organizations have in common the pursuit of a mission or goal that is valuable for some workers, precisely those who share such objectives and who are characterized by non-pecuniary motivations, together with the standard extrinsic incentives.

The idea that intrinsic motivation for being employed by mission-oriented firms might be the source of wage gaps has been first proposed by Heyes (2005), for caring vs non-caring jobs in the health sector, and by Delfgaauw and Dur (2007) who analyze applicants' tastes for being employed at a specific firm. These authors predict that relatively low pay and weak monetary incentives endogenously emerge in jobs where intrinsic motivation matters.

However, another strand of empirical work points out that the wage differential might arise because of a selection bias, given that a wage gap can also reflect *unobservable* differences in workers' ability across sectors or firms.[2]

Therefore, an open question still remains. Suppose that a wage penalty for workers employed in mission-oriented sectors or firms is observed, although neither workers' intrinsic motivation nor ability can be directly measured: then, wages can be lower either because of the lower reservation wages of motivated workers or because of the lower productivity of workers self-selecting into such sectors or firms (or because of a combination of these two effects). In other words, when workers' productivity and motivation are the workers' private information, is it possible to disentangle the pure compensating wage differential from the selection effect of ability?

To this respect, Delfgaauw and Dur (2008) characterize the optimal incentive schemes offered by a public, cost-minimizing agency that faces a perfectly competitive private sector and that wants to hire workers with unknown laziness (the opposite of ability) and public service motivation. The public agency attracts all dedicated workers (i.e. agents characterized by high ability and high public service motivation) as well as the laziest workers (i.e. low-ability and not motivated agents). Lazy workers are paid less than in the private sector whereas dedicated workers are offered higher wages by the public agency. However, the model cannot account for the distribution of workers' laziness (ability) between the two sectors and is therefore not informative about the selection effect of ability.

In our paper, we consider a labor market characterized by two firms, a mission-oriented or non-profit firm and a standard for-profit firm. The two firms compete to attract workers who are heterogeneous

[2] See Goddeeris (1988), Hwang *et al.* (1992), Gibbons and Katz (1992), Goux and Maurin (1999).

with respect to both their skills and their intrinsic motivation. These two characteristics are the workers' private information and are not correlated. In particular, workers can have either high or low ability, whereas motivation is continuously distributed in the unit interval. The two firms simultaneously offer screening contracts defined by a task level (the observable effort) and a non-linear wage rate which depends on effort. Because of the strategic interaction between the two firms, the workers' outside options are type-dependent and endogenous and thus the analysis of a multi-principal framework with bidimensional screening is called for.

Motivated workers care about the mission pursued by the firm which employs them. More precisely, the payoff of motivated agents depends on their own type but also on the type of firm hiring them. When motivated workers are employed by the non-profit employer they enjoy a non-monetary benefit which is unrelated to effort exertion or output produced. Conversely, all workers experience a cost from effort provision, which can differ across workers types but which does not depend on the employer's organizational form.

Therefore, firms' heterogeneity stems from workers' motivation; moreover, the two firms are heterogeneous in their technologies because their marginal productivity of labor is different. Importantly, we take a general perspective in dealing with the differences in firms' technologies and study all possible environments: the one where the for-profit firm has a superior technology with respect to the non-profit one or vice-versa.[3]

Taking into account the combined effect of the two sources of firms heterogeneity (workers' motivation and firms' technology), we say that one firm is fully dominant when it succeeds in hiring all types of workers even when the rival firm offers the highest possible utility to all potential applicants.

To be continued

1.1 Related literature

Our work contributes to two different strands of literature: from an economic point of view, it adds to the recent and rapidly growing literature on the self-selection of workers with intrinsic motivation into different firms/sectors of the labor market; from a theoretical point of view, it explicitly solves a multi-principal game in a labor market where two firms compete to attract workers who are characterized by two different dimensions of private information.

The problem of the design of optimal incentive schemes for intrinsically motivated workers has been tackled by Murdock (2002), Besley and Gathak (2005) and Prendergast (2007), whose attention has primarily been devoted to moral hazard, while we consider the screening problem. Heyes (2005) and Delfgaauw and Dur (2007) are the first papers that address the issue of the selection of workers who

[3]Equivalently, one could consider the same technology for both firm but a different unit price for the final output. This is what Delfgauuw and Dur (2010) do.

are heterogeneous with respect to their motivation. They show that, as the wage increases, the average motivation of the workers who are willing to accept the job deteriorates. But workers' heterogeneity in ability is not considered.

Previous results from theoretical literature admitting for workers' private information are ambiguous on whether mission-oriented firms or sectors are characterized by lower or higher workers' productivity on average. In particular, Handy and Katz (1998) consider the selection of managers who differ in terms of ability and devotion to the non-profit firm. They impose an exogenously given ranking of both effort provisions and reservation wages for different types of managers and they find that lower wages attract managers that are more committed to the cause of the non-profit firm. But this comes at the cost of selecting less able managers who are unable to command higher wages in standard sectors.

The most closely related paper to ours is Delfgaauw and Dur (2010) that is framed in a full information setup. The article considers a perfectly competitive economy consisting of the public and the private sector. Workers are heterogeneous with respect to both productivity and motivation which are fully observable. Thus workers are paid their marginal product in both sectors. Moreover, output prices differ and the return to managerial ability is lower in the public than in the private sector. Hence, a public-private earnings differential exists, which is caused partly by a compensating wage differential (motivated workers evaluate more being employed in the public sector) and partly by selection arising endogenously from the adjustment in prices to differences in job attributes (on average more productive workers enter the private sector where remuneration is higher). Our model extends the setup in Delfgaauw and Dur (2010) in two ways: first, bidimensional adverse selection is considered rather than full information about the workers' characteristics and, second, firms interact strategically. We confirm the result of negative selection of workers' ability between firm, coupled with the existence of a wage gap penalizing workers employed at the non-profit firm.[4] Nonetheless, we also document either ability-neutrality or even positive selection; the latter allows for a wage differential favoring employees of non-profit firms.

Our paper is also related to Delfgaauw and Dur (2008), where, again, the problem of workers' self-selection into public vs private sectors is considered and the screening problem of the governmental agency is tackled. As for the setup, we depart from Delfgaauw and Dur (2008) in two main respects: first, their private sector is perfectly competitive and therefore firms do not interact strategically. Second, their screening mechanism is simplified because the public agency is constrained to hire at most two types of agents. As for the results, we find a different selection pattern of workers to firms and we are able to compare average ability of workers between the two firms, while Delfgaauw and Dur (2008) can not.

More recently, DeVaro et al. (2015) consider a non-profit firm that faces a non-distribution constraint and that is bound to offer flat wages to its employees. The non-profit firm competes with perfectly

[4] In our setup, the main determinant of the wage gap is not the difference in output prices but the non-inferior technology of the for-profit firm.

competitive for-profit rivals in hiring a worker who is heterogeneous in skills and who derives intrinsic motivation from the non-profit social mission. It is shown that the worker is hired by the non-profit firm if intrinsic motivation is sufficiently high and that a wage differential favoring for-profit firms emerges when the latter are more effective than the nonprofit firm in training workers.[5]

Finally, Bénabou and Tirole (2013) analyze a model where firms compete to attract workers that are heterogeneous with respect to their productivity and their work ethics, i.e. the extent to which agents "do the right thing" beyond what their material self-interest commands. In a framework with multitasking and moral hazard, they show how competition for the most productive workers interacts with the incentive structure inside firms to undermine work ethics. Besides the different focus of the two papers, Bénabou and Tirole (2013) assumes an affine compensation scheme, we instead consider non-linear contracts. Moreover, their screening is not bidimensional but it is performed by firms with respect to one dimension at a time (either productivity or work ethics).

From a technical point of view, our paper draws both from the literature on multidimensional screening and from the literature on multi-principals. Models where both problems are simultaneously considered are very few.

Screening when agents have several unobservable characteristics has been analyzed by some important papers that deal with continuous distributions of types: Armstrong and Rochet (1999), Armstrong (1996), Rochet and Chonè (1998), Basov (2001, 2005) and Deneckere and Severinov (2011). They all show that it is almost impossible to extend to the multidimensional environment the qualitative results and the regularity conditions of the unidimensional case.

Barigozzi and Burani (2013) considers the screening problem of a mission-oriented monopsonist willing to hire a worker of unknown ability and motivation. The present paper adds the important dimension of competition between two differentiated firms, a non-profit and a for-profit firm.

The multi-principal literature with asymmetric information was initiated by the seminal contributions of Martimort (1992) and Stole (1992). Within this literature, the paper that is most closely related to ours is Rochet and Stole (2002) which extends the analysis carried out in Stole (1995) and studies duopolists competing in nonlinear prices in the presence of both vertical and horizontal preference uncertainty. Consumers are heterogeneous and privately informed about their preference for quality and about their outside opportunity cost. Quality is the only screening instrument and contracts consist of quality-price pairs that only depend on consumers' (unidimensional) preference for quality. The other unknown characteristic, the outside opportunity cost, affects the consumers' decision about which firm to buy from.[6] We depart from Rochet and Stole (2002) because they only consider symmetric firms and find

[5]The paper also tests the theoretical results with data on California establishments showing that for-profits firms offer higher wages and higher incentive pay with respect to non-profits.

[6]A similar setup is analyzed in Lehmann et al. (2014) that considers optimal nonlinear income taxes levied by two

that incentive compatibility constraints are never binding for any firm, so that efficient quality allocation with cost-plus-fee pricing emerge as the equilibrium outcome.[7]

Another related model is Biglaiser and Mezzetti (2000), which studies an incentive auction in which multiple principals bid for the exclusive services of a single agent, who has private information about ability (or the cost of providing effort). They examine symmetric equilibria and show that only downward incentive constraints (whereby the high-ability type is attracted by the contract offered to the low-ability worker), if any, might be binding. They find that the "separation property" does not hold, meaning that the presence of multiple principals not only affects the agent's compensation, but also reduces the distortion in the agent's effort level. Unlikely, our setup is inherently asymmetric and we show that the upward incentive constraint (whereby low-ability workers have incentive to mimic high-ability types) might also be binding for the non-profit firm and this leads to an upward distortion in the optimal allocation for high-ability workers.

2 The model

We consider two firms that differ in the mission they pursue: one firm is a standard profit-oriented organization while the other firm is mission-oriented and can be interpreted either as a non-profit institution or as a Corporate Social Responsible organization (see below).[8] Technically we study a multi-principal setting with bidimensional adverse selection where the two principals (firms) compete to hire agents (workers). Each agent (she) can work exclusively for one principal (it). Principals and agents are risk neutral.

Effort supplied by the agent is the only input the two firms need in order to produce output. We call x the *observable and measurable* effort (task) level that the agent is asked to provide.[9] Both principals' production functions display constant returns to effort in such a way that

$$q_i(x) = k_i x,$$

where q_i is the amount of output produced by firm $i = S, MO$, S meaning standard and MO indicating mission-oriented, and k_i is the marginal productivity of effort for firm i. We do not impose any exogenous ranking of marginal productivities for the two firms, so that $k_S \gtreqless k_{MO}$. The principals' profit margins

competing governments in the presence of migration costs.

[7] Precisely the same result can be found in Armstrong and Vickers (2001) that model firms as supplying utility directly to consumers.

[8] One could rephrase the whole setup considering two sectors populated by a monopsonistic firm each.

[9] In particular, x can be interpreted as a job-specific requirement like the amount of hours of labor or the amount of services the agent is asked to provide.

(per-worker, conditional on the worker being hired) are given by the difference between revenues and costs and are equal to

$$\pi_i(x) = R_i(x) - w_i(x) = k_i q(x) - w_i(x), \tag{1}$$

where the price of output is assumed to be equal to 1 for both firms, and where $w_i(x)$ is the total salary paid to the worker hired by principal i exerting effort x.[10]

We also assume that the mission-oriented firm payoff per-worker is

$$\Pi_{MO}(x) = \alpha \pi_{MO}(x)$$

where the parameter $\alpha \in (0,1]$ has two possible interpretations: (i) if the firms's mission-orientation is stemming from Corporate Social Responsibility, then the firm is ready to sacrifice a share of its profit in the social interest.[11] (ii) If the mission-oriented firm is a nonprofit organization, then the manager can appropriate, in the form of perks, only the share α of profits because of the non-distribution constraint.[12]

Suppose that a unit-mass population of agents differ in two characteristics, ability and intrinsic motivation, that are independently distributed. Ability can only take two values, high and low ability. A worker characterized by high ability incurs in a low cost of providing a given effort level. Ability is denoted by $\theta_j \in \{\theta_1, \theta_2\}$ where $\theta_2 > \theta_1 \geq 1$. A fraction ν of employees has high ability (i.e. a low cost of effort) θ_1, the fraction $1-\nu$ is instead characterized by low ability (i.e. a high cost of effort) θ_2. We will denote by $\Delta\theta$ the difference in ability, whereby $\Delta\theta = \theta_2 - \theta_1$. Ability is the only relevant workers' characteristic for the standard firm, while the premium from intrinsic motivation can only be enjoyed when workers are employed by the mission-oriented organization, because workers share its goals. Indeed, we assume that workers derive a non-monetary benefit from being employed by the mission-oriented organization and that such benefit is unrelated to output produced or effort exerted.[13] As for intrinsic motivation, we assume that it is continuous and uniformly distributed in the interval $[0,1]$, so that $\Delta\gamma = 1$.

When a worker is not hired by any firm, we assume that her utility is zero.

If a worker is hired by one firm, her *reservation utility* is endogenous and it depends on the contract offered by the rival firm. When a worker is hired by the standard firm, her utility is given by the total salary gained less the cost of effort provision, which depends on the agent's ability type θ_j. Thus,

$$u_S = w_S - C(x_S, \theta_j) = w_S - \frac{1}{2}\theta_j x_S^2$$

In fact, motivated workers do not enjoy any benefit from motivation when hired by the standard firm. As a consequence, from the point of view of the standard principal, all workers with the same ability

[10] An equivalent specification of the model would be the one adopted by Delfgaauw and Dur (2010): firms have the same technology but output prices differ.

[11] See the discussion in Bénabou and Tirole (2010) and in Kitzmueller and Shimshack (2012).

[12] The definition of this objective function for nonprofit firms was first proposed in Glaeser and Shleifer (2001), following the ideas expressed in Hansmann (1996). See also Ghatak and Mueller (2011).

[13] The same representation of intrinsic motivation is used in Hayes (2005) and Delfgaauw and Dur (2010).

are the same, irrespective of their level of motivation. However, agents with the same ability potentially benefit from different outside options. In fact, given ability, the mission-oriented firm values highly motivated workers more than workers with lower motivation because the former provide the same effort in exchange for a lower salary. The latter is called the "labor donation assumption" in the literature on Labor Economics. Thus intrinsic motivation positively affects workers' outside options even though it does not alter effort exerted when hired by the standard firm.

Likewise, when a worker is hired by the mission-oriented firm, her utility takes the form

$$u_{MO} = w_{MO} - C\left(x_{MO}, \theta_j\right) = w_{MO} - \frac{1}{2}\theta_j x_{MO}^2 + \gamma,$$

where, as mentioned before, only ability θ_j is related to effort exertion while motivation γ is not action-oriented.

Observe that the marginal rate of substitution between effort and wage is given by

$$MRS_{x,w}^i = -\frac{\partial u_i/\partial x_i}{\partial u_i/\partial w_i} = \theta_j x_i,$$

which is always positive. So all workers' indifference curves have positive slope in the (x, w) plane and the single-crossing property holds for both firms.

Because a worker of type (θ_j, γ) has preferences over the pair effort-salary which are independent of γ (conditional on being hired by one firm), our problem with bidimensional screening reduces to a program with unidimensional screening on the ability parameter θ_j and endogenous outside options. Moreover, considering that the effort level x_i is observable and contractible, we use an indirect approach and restrict attention to the situation where firms offer two incentive-compatible transfer schedules, one for each type θ_j, that are conditional on the effort target, $\{(x_i(\theta_1), w_i(\theta_1)), (x_i(\theta_2), w_i(\theta_2))\}$, $i = S, MO$, and each agent selects the preferred pair $(x_i(\theta_j), w_i(\theta_j))$.[14]

Let $U_i(\theta_j)$ denote the indirect utility or information rent of an agent of type θ_j who is hired by firm i, absent the benefit accruing from intrinsic motivation. Then

$$U_i(\theta_j) = \max_{x_i} w_i(x_i) - C(x_i, \theta_j) = w_i(x_i) - \frac{1}{2}\theta_j x_i^2. \tag{2}$$

Denoting by $x_i(\theta_j)$ the solution to this program, one can write

$$U_i(\theta_j) = w_i(x_i(\theta_j)) - \frac{1}{2}\theta_j x_i^2(\theta_j)$$

and solve for the wage rate as

$$w_i(x_i(\theta_j)) = U_i(\theta_j) + \frac{1}{2}\theta_j x_i^2(\theta_j). \tag{3}$$

[14] We are in a multi-principals setting where, as it is well known, a general Revelation Principle is almost impossible to apply. For this reason we use the Taxation Principle and study the game where each firm offers a menu with two contracts at most and each agent chooses the contract assuring her the highest payoff. See the discussion in Biglaiser and Mezzetti (2000, page 149).

Using expression (3), one can eliminate the wage rate from the expression for the firm's profit and write profit margins related to each type θ_j worker as

$$\pi_i(\theta_j) = S_i(\theta_j) - U_i(\theta_j) = k_i x_i(\theta_j) - \frac{1}{2}\theta_j x_i^2(\theta_j) - U_i(\theta_j) \tag{4}$$

where

$$S_i(\theta_j) \equiv R_i(x_i(\theta_j)) - C(x_i(\theta_j), \theta_j) = k_i x_i(\theta_j) - \frac{1}{2}\theta_j x_i^2(\theta_j) \tag{5}$$

is the total surplus realized by a worker of type θ_j providing effort $x_i(\theta_j)$ for firm i (again, absent the benefit accruing from intrinsic motivation, when $i = MO$). Then $U_i(\theta_j) = S_i(\theta_j) - \pi_i(\theta_j)$.

Given $U_i(\theta_j)$ formally defined by (2), if a worker is hired by the mission-oriented firm her *total* indirect utility becomes

$$\mathcal{U}_{MO}(\theta_j) = U_{MO}(\theta_j) + \gamma.$$

Hence a worker of type (θ_j, γ) gets utility $U_{MO}(\theta_j) + \gamma$ if she works for the mission-oriented firm and utility $U_S(\theta_j)$ if she is hired by the standard firm.

Definition 1 *The* marginal worker of type θ_j *is the worker who is indifferent between working for either firm and is characterized by motivation* $\widehat{\gamma}(\theta_j)$ *such that:*

$$\widehat{\gamma}(\theta_j) = U_S(\theta_j) - U_{MO}(\theta_j), \ j = 1, 2 \tag{6}$$

Workers with $\gamma \geq \widehat{\gamma}(\theta_j)$ *prefer to work for the mission-oriented firm whereas workers with* $\gamma < \widehat{\gamma}(\theta_j)$ *prefer to work for the standard firm.*

Recall that intrinsic motivation is uniformly distributed in the interval $[0, 1]$, then the set of workers with ability θ_j being hired by the mission-oriented firm writes $\Pr(\gamma \geq \widehat{\gamma}(\theta_j)) = 1 - (U_S(\theta_j) - U_{MO}(\theta_j))$, $j = 1, 2$. Conversely, $\Pr(\gamma < \widehat{\gamma}(\theta_j)) = U_S(\theta_j) - U_{MO}(\theta_j)$, $j = 1, 2$, describes the mass of workers with ability θ_j attracted by the standard firm. Obviously, in order for both firms to have a positive labor supply of type-θ_j workers, it must be that:

$$0 < U_S(\theta_j) - U_{MO}(\theta_j) < 1, \ j = 1, 2 \tag{7}$$

This represents the most interesting situation to analyze where firms are sufficiently similar and no-firm is able to attract all the workers. In what follows we focus our attention precisely to the case in which an internal solution is obtained for both ability types or Condition (7) is satisfied.

Definition 2 *Different types of sorting inside the two firms can be observed:*

- Ability neutrality *occurs when:*

$$\widehat{\gamma}(\theta_1) = \widehat{\gamma}(\theta_2) \Longleftrightarrow U_S(\theta_1) - U_{MO}(\theta_1) = U_S(\theta_2) - U_{MO}(\theta_2) \tag{8}$$

- Negative selection of ability *into the mission-oriented firm occurs when:*

$$\widehat{\gamma}(\theta_1) > \widehat{\gamma}(\theta_2) \Longleftrightarrow U_S(\theta_1) - U_{MO}(\theta_1) > U_S(\theta_2) - U_{MO}(\theta_2) \tag{9}$$

- Positive selection of ability *into the mission-oriented firm occurs when:*

$$\widehat{\gamma}(\theta_1) < \widehat{\gamma}(\theta_2) \Longleftrightarrow U_S(\theta_1) - U_{MO}(\theta_1) < U_S(\theta_2) - U_{MO}(\theta_2) \tag{10}$$

Ability neutrality captures the situation in which the share of workers who are active in one firm is constant and does not depend on the workers' ability.[15] When Condition (9) holds, the mission-oriented firm attracts a workforce that is on average more motivated but less skilled with respect to the one hired by the standard firm.[16] Finally, when Condition (10) is instead satisfied, workers in the mission-oriented firm are both more motivated and more skilled on average.

Interestingly, Condition (9) can be rewritten as

$$U_S(\theta_1) - U_S(\theta_2) > U_{MO}(\theta_1) - U_{MO}(\theta_2) > 0$$

meaning that the *returns to ability* (that is the increase in indirect utility experienced by workers as their ability increases) are higher for the standard firm than for the mission-oriented firm, again absent the benefits accruing from intrinsic motivation.

The expected profit of the mission-oriented firm is the following:

$$\begin{aligned} E(\Pi_{MO}) = {} & \alpha\nu \left(1 - \left(U_S(\theta_1) - U_{MO}(\theta_1)\right)\right) \left(k_{MO} x_{MO}(\theta_1) - \tfrac{1}{2}\theta_1 x_{MO}^2(\theta_1) - U_{MO}(\theta_1)\right) + \\ & \alpha(1-\nu)\left(1 - \left(U_S(\theta_2) - U_{MO}(\theta_2)\right)\right) \left(k_{MO} x_{MO}(\theta_2) - \tfrac{1}{2}\theta_2 x_{MO}^2(\theta_2) - U_{MO}(\theta_2)\right) \end{aligned} \tag{$E(\Pi_{MO})$}$$

and, similarly, the expected profit of the standard firm is:

$$\begin{aligned} E(\Pi_S) = {} & \nu\left(U_S(\theta_1) - U_{MO}(\theta_1)\right)\left(k_S x_S(\theta_1) - \tfrac{1}{2}\theta_1 x_S^2(\theta_1) - U_S(\theta_1)\right) + \\ & (1-\nu)\left(U_S(\theta_2) - U_{MO}(\theta_2)\right)\left(k_S x_S(\theta_2) - \tfrac{1}{2}\theta_2 x_S^2(\theta_2) - U_S(\theta_2)\right). \end{aligned} \tag{$E(\Pi_S)$}$$

As noticed before, the mass of type-θ_j workers being hired by each firm depends on the difference between the indirect utilities $(U_S(\theta_1) - U_{MO}(\theta_1))$ or by the workers' difference in reservation utilities. Indeed, in each program the workers' reservation utility is treated as given but it is endogenous . Thus firms compete against each other in the utility space: an increase in the utility offered to a given type of worker reduces the principal's payoff when hiring this worker but increases the probability of hiring her. Moreover, motivation γ does not appear in these objective functions nor does affect the worker's reservation utility.

Because ability is not observable by the principals, one has to consider the workers' incentive compatibility constraints. Provided that both firms are able to hire workers with both ability levels, there

[15] In a model where intrinsic motivation is effort-related and descrete, Barigozzi and Burani (2014) show that, in equilibrium, sorting is ability neutral.

[16] This corresponds to the case analyzed in Delfgaauw and Dur (2010).

are two incentive compatibility constraints for each firm: the *downward incentive constraint* (henceforth DIC) whereby high-ability types should not be attracted by the contract offered to low-ability types and the *upward incentive constraint* (henceforth UIC) whereby the low-ability types are not willing to mimic the high-ability workers. For each principal $i = S, MO$ such constraints are given by

$$w_i\left(x_i\left(\theta_1\right)\right) - \frac{1}{2}\theta_1 x_i^2\left(\theta_1\right) \geq w_i\left(x_i\left(\theta_2\right)\right) - \frac{1}{2}\theta_1 x_i^2\left(\theta_2\right)$$

and

$$w_i\left(x_i\left(\theta_2\right)\right) - \frac{1}{2}\theta_2 x_i^2\left(\theta_2\right) \geq w_i\left(x_i\left(\theta_1\right)\right) - \frac{1}{2}\theta_2 x_i^2\left(\theta_1\right)$$

respectively. Notice that these constraints do not depend on γ because motivation enters both sides of the inequality and therefore it cancels out. One can use (3) in order to eliminate wages and rewrite both constraints as a function of effort and utility:

$$U_i\left(\theta_1\right) \geq U_i\left(\theta_2\right) + \frac{1}{2}\left(\theta_2 - \theta_1\right) x_i^2\left(\theta_2\right) \tag{DIC}$$

and

$$U_i\left(\theta_2\right) \geq U_i\left(\theta_1\right) - \frac{1}{2}\left(\theta_2 - \theta_1\right) x_i^2\left(\theta_1\right). \tag{UIC}$$

In the case of a monopsonist firm dealing with workers of unknown ability (the single-principal problem), the relevant constraint is typically (DIC), showing that high-ability workers receive an information rent for being able to mimic low-ability workers. Given that we are analyzing here a setting with competing principals, also the (UIC) constraint can be relevant as a consequence of the type-dependent and endogenous reservation utilities. As a consequence, also the low-ability workers can receive a rent in our setting.

Finally, putting DIC and UIC together yields

$$\frac{1}{2}\left(\theta_2 - \theta_1\right) x_i^2\left(\theta_2\right) \leq U_i\left(\theta_1\right) - U_i\left(\theta_2\right) \leq \frac{1}{2}\left(\theta_2 - \theta_1\right) x_i^2\left(\theta_1\right)$$

which makes it clear that $U_i\left(\theta_1\right) - U_i\left(\theta_2\right) > 0$, whenever a non-null contract with $x_i > 0$ is proposed by each firm $i = S, MO$. Intuitively, being more efficient, a high-ability type exerts an effort level which is higher than the one exerted by the low-ability type. In line with that, in order to prevent mimicking, the information rent of the high-ability type is higher than the one of the low-type.

To sum up, the two firms maximize their expected profits $E\left(\Pi_{MO}\right)$ and $E\left(\Pi_S\right)$ (with respect to the effort level x_{MO} and the indirect utility U_{MO} and with respect to the effort level x_S and the indirect utility U_S, respectively) under the two incentive compatibility constraints DIC and UIC illustrated above. Once the workers' effort levels x_i and utilities U_i are obtained, the related wages are immediately derived using equation (3).

Hence, the timing of the game is as follows. Each firm simultaneously offers a menu of contracts of the form $\left\{\left(x_i\left(\theta_1\right), w_i\left(\theta_1\right)\right), \left(x_i\left(\theta_2\right), w_i\left(\theta_2\right)\right)\right\}$, with $i = S, MO$. Workers observe the contracts in the menus, choose which firm (if any) to work for and select a contract. Then workers exert the effort level specified by the chosen contract, output is produced, and the contracted wages are paid.

An equilibrium is such that each firm chooses a menu of contracts that maximizes its expected profit, given the contracts offered by the rival firm and given the equilibrium choice of workers. Workers choose the contracts that maximize their utility.

3 The benchmark case

Let us first consider the benchmark case in which workers' ability is fully observable, so that contracts can be contingent on θ_j, while motivation is the workers' private information.[17] Each firm maximizes the expected profit given the worker's type $\theta_j \in \{\theta_1, \theta_2\}$ and ignoring the incentive compatibility constraints. In particular, firm MO solves

$$\max_{x_{MO}, U_{MO}} \alpha \left(1 - \left(U_S\left(\theta_j\right) - U_{MO}\left(\theta_j\right)\right)\right) \left(k_{MO} x_{MO}\left(\theta_j\right) - \frac{1}{2}\theta_j x_{MO}^2\left(\theta_j\right) - U_{MO}\left(\theta_j\right)\right)$$

taking $U_S\left(\theta_j\right)$ as given. In the same way firm S solves the problem:

$$\max_{x_S, U_S} \left(U_S\left(\theta_j\right) - U_{MO}\left(\theta_j\right)\right) \left(k_S x_S\left(\theta_j\right) - \frac{1}{2}\theta_j x_S^2\left(\theta_j\right) - U_S\left(\theta_j\right)\right)$$

given $U_{MO}\left(\theta_j\right)$. From the first-order conditions with respect to the effort levels, $x_{MO}\left(\theta_j\right)$ and $x_S\left(\theta_j\right)$, one obtain the effort levels:

$$x_{MO}^*\left(\theta\right) = \frac{k_{MO}}{\theta_j} = x_{MO}^{FB}\left(\theta_j\right) \tag{11}$$

$$x_S^*\left(\theta_j\right) = \frac{k_S}{\theta_j} = x_S^{FB}\left(\theta_j\right) \tag{12}$$

where the superindex FB stands for *first-best*. Intuitively, it is a dominant strategy for the firms to ask for the first-best effort level so that the highest amount of resources is available to attract the workers.

Furthermore, the first-order conditions with respect to $U_{MO}\left(\theta_j\right)$ and $U_S\left(\theta_j\right)$ allow us to obtain the reaction functions of the two firms. Each reaction function characterizes the optimal utility left by the firm to an agent of type θ_j given the utility that this agent receives from the competing firm:

$$U_{MO}\left(\theta_j\right) = \frac{1}{2}\left(\frac{k_{MO}^2}{2\theta_j} - \left(1 - U_S\left(\theta_j\right)\right)\right) \tag{13}$$

$$U_S\left(\theta_j\right) = \frac{1}{2}\left(\frac{k_S^2}{2\theta_j} + U_{MO}\left(\theta_j\right)\right) \tag{14}$$

Reaction functions have positive slopes so that utilities can be interpreted as "strategic complements" in this game. In a Nash equilibrium, the levels of utilities assured by each firm to type θ_j solve (13) and

[17] Notice that first-best contracts must also be contingent on intrinsic motivation. First-best contracts would be defined by a continuum of effort-wage pairs defined over (θ_j, γ) such that the firm having a comparative advantage in hiring one worker attracts that worker by meeting the best offer of the competitor.

(14) simultaneously so that

$$U^*_{MO}(\theta_j) = \frac{1}{3}\left(\frac{k^2_{MO}}{\theta_j} + \frac{k^2_S}{2\theta_j} - 2\right) \quad \text{and} \quad U^*_S(\theta) = \frac{1}{3}\left(\frac{k^2_S}{\theta_j} + \frac{k^2_{MO}}{2\theta_j} - 1\right) . \tag{15}$$

Furthermore we can use expression (6) to obtain the equilibrium value for the marginal worker of type θ_j:

$$\widehat{\gamma}^*(\theta_j) = U^*_S(\theta_j) - U^*_{MO}(\theta_j) = \frac{1}{3}\left(1 + \frac{\left(k^2_S - k^2_{MO}\right)}{2\theta_j}\right), \tag{16}$$

Finally, from (3) the equilibrium salaries are such that

$$w^*_{MO}(\theta_j) = \frac{1}{3}\left(\frac{5k^2_{MO}+k^2_S}{2\theta_j} - 2\right) \quad \text{and} \quad w^*_S(\theta_j) = \frac{1}{3}\left(\frac{k^2_{MO}+5k^2_S}{2\theta_j} - 1\right) . \tag{17}$$

How is sorting between firms affected by the level of ability?

Proposition 1 *When ability is observable (and motivation is the workers' private information), a Nash equilibrium is such that:*
(a) the sorting of workers between firms only depends on firms' difference in technology: if $k_S = k_{MO} = k$ there is ability-neutrality and $\widehat{\gamma}(\theta_1) = \widehat{\gamma}(\theta_2) = \frac{1}{3}$, if $k_S > k_{MO}$ there is a negative selection of ability into the mission-oriented firm and $\widehat{\gamma}(\theta_1) > \widehat{\gamma}(\theta_2)$ holds; whereas if $k_S < k_{MO}$ there is a positive selection of ability into the mission-oriented firm and $\widehat{\gamma}(\theta_1) < \widehat{\gamma}(\theta_2)$.
(b) a wage differential favoring workers in the standard firm is in place, i.e. $w_S(\theta_j) > w_{MO}(\theta_j)$ holds for all $\theta_j \in \{\theta_1, \theta_2\}$, unless there is positive selection and the technological advantage of the mission-oriented firm is sufficiently high that $k^2_{MO} - k^2_S \geq \frac{\theta_j}{2}$.

Proof. Point (a) is straightforward given the equilibrium expression for the marginal worker in (16). Point (b) is derived from the comparison of the equilibrium wage rate levels (17). ■

When the two firms have the same technology, then at the benchmark with skills observability, the marginal worker has motivation $\gamma = \frac{1}{3}$ independently of her ability. All workers with motivation higher than $\frac{1}{3}$ work for the mission-oriented while all workers with motivation lower than $\frac{1}{3}$ prefer to apply at the standard firm. Intuitively, the premium γ earned by motivated workers assures the mission-oriented firm a labor supply that is twice the one of the standard firm. Using inequality (7) one can check that, in this case, and interior solution exists if and only if technology is efficient enough or $k > \sqrt{\frac{4\theta_2}{3}}$. Moreover, for each level of workers' ability, a wage differential always occurs in favor of the mission-oriented firm.[18] Indeed, the standard firm asks its employees the same first-best effort that is required by the mission-oriented firm but in exchange for a higher salary.

Consider now the case where $k_S > k_{MO}$ so that the standard firm has a technological advantage over the mission-oriented rival. The marginal consumer with high ability has a higher motivation than the

[18] Wages are always positive provided that equilibrium utilities are and that condition $k > \sqrt{\frac{4\theta_2}{3}}$ is satisfied.

one with low ability, meaning that a negative selection of ability into the mission-oriented firm realizes. In different words, the mission-oriented firm hires a larger mass of low- than high-ability workers. Again from inequality (7), an interior solution is assured if and only if $k_S^2 - k_{MO}^2 \leq 4\theta_1$ or if the difference in technology is not too high. If the previous condition is not satisfied, then the mission-oriented firm only hires low-ability workers. A wage differential in favour of the mission-oriented firm always exists also in this case but workers now exert a higher effort when hired by the standard firm.

Finally, the case in which the mission-oriented firm has a technological advantage with respect to the rival and $k_{MO} > k_S$. Now we observe a positive selection of ability into the mission-oriented firm. Here the mission-oriented firm always hires a positive mass of both high- and low-ability workers, whereas it might be the case that the standard firm only hires high-ability workers. An interior solution, or a positive mass of high-ability workers for the standard firm, now requires that $k_{MO}^2 - k_S^2 \leq 2\theta_1$: again, the difference in technology must not be too high. In this scenario, different wage differential may arise according to the magnitude of the difference in technology. In particular, a wage differential favoring workers in the standard firm still exists provided that $k_{MO}^2 - k_S^2 < \frac{\theta_1}{2}$. Alternatively, if $\frac{\theta_1}{2} \leq k_{MO}^2 - k_S^2 < \frac{\theta_2}{2}$ then high-ability workers earn more when they are employed by the mission-oriented firm than by the standard firm whereas low-ability workers earn more when they are employed by the standard firm than by the mission-oriented firm. Finally, if $k_{MO}^2 - k_S^2 \geq \frac{\theta_2}{2}$ then all workers get a higher salary when applying for the mission-oriented firm. Thus, when the mission-oriented firm has a technological advantage, both types of wage differentials are possible. In particular, if the difference in technologies is not too high, then workers employed by the mission-oriented firm exert more effort and are paid less than workers hired by the standard firm. When instead the technological advantage of the mission-oriented firm is sufficiently important, then the much higher effort required by the mission-oriented firm is rewarded with a higher salary.

Before moving to the case in which ability is private information, we would like to emphasize the following. Independently of which firm has a technological advantage, an interior solution exists, meaning that both firms are able to hire workers of both ability types, provided that the difference in technology is not too high. If the previous condition is not satisfied, then the most efficient firm is fully dominant with respect to at least one type of workers' ability. In different words, full market segmentation according to skills never occurs: it is never the case that all workers of a given skill level prefer to work for one firm, whereas all workers with the other skill level prefer all to be hired by the rival firm.

4 Screening for ability

Suppose now that workers' ability is not observable. The two incentive compatibility constraints UIC and DIC must be considered by both firms and the two programs obviously becomes more complex.

However, we show below that the added complexity is limited because we can indeed focus on simplified programs: either all incentive constraints can be neglected so that we are back to the benchmark case analyzed before, or the upward incentive constraint at most is binding for the mission-oriented firm and the downward incentive constraint at most is binding for the standard firm. Which scenario realizes depends on the difference in firms' technologies or in the type of selection into the mission-oriented firm.

Lemma 1 below indicates sufficient conditions on the relative magnitude of ability levels and technologies such that three possible scenarios occur where a limited amount of incentive constraints (or not constraints at all) is binding.

Lemma 1 *According to the relative difference in technologies and in ability levels, three situations can realize:*

1. ***Envy-freeness.*** *Incentive constraints are slack for both firms if*

$$\frac{k_S^2 - k_{MO}^2}{3k_{MO}^2} < \frac{\Delta\theta}{\theta_1}. \tag{18}$$

2. ***Only** UIC **binding.*** *The upward incentive constraint might be binding for the mission-oriented firm whereas incentive constraints are still slack for the standard firm if*

$$\frac{k_S^2 - k_{MO}^2}{2k_S^2 + k_{MO}^2} < \frac{\Delta\theta}{\theta_1} \leq \frac{k_S^2 - k_{MO}^2}{3k_{MO}^2}. \tag{19}$$

3. ***Both** UIC **and** DIC **binding.*** *The upward incentive constraint might be binding for the mission-oriented firm whereas the downward constraint might be binding for the standard firm if*

$$\frac{\Delta\theta}{\theta_1} \leq \frac{k_S^2 - k_{MO}^2}{2k_S^2 + k_{MO}^2}. \tag{20}$$

Proof. See Appendix A.1. ■

Condition (18) is always satisfied when the mission-oriented firm has a non-inferior technology with respect to the standard firm, i.e. when $k_{MO} \geq k_S$. In other words, condition (18) is always satisfied for positive selection or for ability-neutrality. Moreover, envy-freeness still holds when $k_S \geq k_{MO}$, provided that the difference in technology is sufficiently low. Under such three scenarios, no incentive constraint is binding for neither firm and, as a consequence, we are back to the benchmark case. More precisely, if both agents' types exert the first-best effort level and are compensated as in the case of observable ability, then low-ability workers do not want to mimic high-ability agents and strictly prefers the contract $\left(x_i^{FB}(\theta_2), U_i(\theta_2)\right)$ to the contract $\left(x_i^{FB}(\theta_1), U_i(\theta_1)\right)$. But the reverse is also true: high-ability workers do not want to mimic low-ability agents. Hence, we can treat each firm's problem as two independent problems, one for each ability level, because the presence of types θ_1 does not influence the optimal contract that firm i offers to types θ_2 and vice-versa.[19] In different words, the agent's private information on θ_j does not affect the effort levels and the utilities that each firm offers her. From Point 1 in Lemma 1 and from the analysis developed in the previous Section 3, we can state the following Proposition:

Proposition 2 ***Equilibrium with envy-freeness.*** *Suppose that neither ability nor motivation is observable. When the technological advantage favours the mission-oriented firm and when it favours the standard firm but it is small enough, then equilibrium contracts are as in the Benchmark case: all effort levels are set at the first-best $x_i(\theta_j) = x_i^{FB}(\theta_j)$ and compensation schemes $w_i(x_i(\theta_j))$ are given by expression (17) for all $\theta_j \in \{\theta_1, \theta_2\}$ and $i = S, MO$.*

Importantly, when condition (18) holds, competition between two firms **endowed with different technologies** leads to an efficient allocation. Notice that this efficiency result is more likely to be attained when the difference in workers' types, i.e. in ability levels, is relatively high while the difference in firms' types, namely the technological advantage of the S firm, is relatively low.

When instead condition (18) fails to hold, but condition (19) is satisfied, it means that among workers hired by the mission-oriented firm, low-ability workers might find the contract designed for high-ability agents attractive. Then the upward incentive constraint must be considered for the mission-oriented firm, but independence across types still holds for the standard firm, for whom no incentive constraint is binding. This happens when $k_{MO} < k_S$ and $k_S - k_{MO}$ is higher than the threshold $\frac{\Delta\theta}{\theta_1}$. Here the technological advantage of the standard firm more than compensate the benefit from labor donation and the standard firm is able to attack a larger mass of both workers' types, with the share of high-ability workers higher than the share of low-ability types. Finally, when neither condition (18) nor condition (19) is satisfied whereas (20) holds, then among workers hired by the standard firm, high-ability workers might find the contract designed for low-ability agents attractive. Therefore, the upward incentive constraint must be considered for the mission-oriented firm and the upward incentive constraint must be considered for the standard firm.

The latter two cases are outlined in more detail in the Subsections that follow. Before beginning the analysis, let us introduce a restriction on skill levels which is needed in order to prevent firm MO from making negative profit margins on high-ability types.

Assumption 1 *The difference in ability is sufficiently low so that $2\theta_1 > \theta_2 > \theta_1 \geq 1$ holds.*

Moreover, let us anticipate a general feature of optimal allocations and compensation schemes when incentive compatibility constraints matter for at least one firm.

[19] The proof of Point 1 in Lemma 1 essentially follows the argument developed by Biglaiser and Mezzetti (2000, Lemma 5, page 152).

Proposition 3 ***Equilibrium without envy-freeness.*** *Suppose that neither ability nor motivation is observable. When the technological advantage favours the standard firm and it is sufficiently high, then equilibrium contracts are such that:* (i) *an upward distortion in optimal allocations arise, i.e.* $x^{*}_{MO}(\theta_1) > x^{FB}_{MO}(\theta_1)$; (ii) *a wage differential exists in that* $w_S(\theta_j) > w_{MO}(\theta_j)$ *for all* $\theta_j \in \{\theta_1, \theta_2\}$. *Such wage differential is partly due to adverse selection of ability for the N firm and partly due to a pure compensating effect given the positive selection of motivation for the N firm.*

Proof. See Appendix A.4. ■

4.1 UIC binds for the MO firm

Consider the case in which condition (18) fails to hold but condition (19) is satisfied, so that one can take independence between agents with different ability for S and interdependence between agents with different ability for MO. This is the case in which UIC might bind for the MO firm while all incentives constraints are slack for the S firm. Now, the program for the S firm is the unconstrained (SS) whereas the problem for the MO firm is (SMO) subject to UIC binding that is

$$U_{MO}(\theta_2) = U_{MO}(\theta_1) - \frac{1}{2}(\theta_2 - \theta_1)\, x^2_{MO}(\theta_1).$$

The simultaneous solution to both principal's programs is quite complex in this case, and closed-form solutions cannot be provided especially as far as compensation schemes are concerned. Therefore, we only provide here the most important qualitative results and refer the reader to Appendix A.2 for a detailed analysis.

With respect to the benchmark solution, what changes is that the difference in ability between types decreases. Then the contract offered by firm MO to type θ_1 becomes attractive for type θ_2. Thus, firm MO is forced to distort effort of type θ_1 upwards in order to make mimicking less attractive and, at the same time, to give more information rents to type θ_2 whose utility increases. Since utilities are strategic complements, and increase in $U_{MO}(\theta_2)$ also leads to an increase in $U_S(\theta_2)$, although the rate of change of $U_S(\theta_2)$ is half the rate of change of $U_{MO}(\theta_2)$. Then the probability of type θ_2 workers self-selecting into the MO firm increases as well with respect to the benchmark solution and the adverse selection of ability effect is reinforced.

Proposition 4 *When condition (18) is not satisfied whereas condition (19) holds, then optimal contracts are such that:* (i) *the profit-oriented firm sets effort levels at the first-best and* $x^{*}_{S}(\theta_j) = x^{FB}_{S}(\theta_j)$ *for all* $\theta_j \in \{\theta_1, \theta_2\}$, *and* (ii) *the non-profit firm sets an efficient allocation for low-ability workers, i.e.* $x_{MO}(\theta_2) = x^{FB}_{MO}(\theta_2)$ *whereas it distorts high-ability workers' effort upwards, i.e.* $x^{*}_{MO}(\theta_1) > x^{FB}_{MO}(\theta_1)$, *but* $x^{*}_{MO}(\theta_1) < x^{*}_{S}(\theta_1)$.

Proof. Appendix A.2 provides the set of conditions characterizing the solution to both principal's programs. ■

4.2 UIC binds for the MO firm and DIC binds for the S firm

Consider the case in which both conditions (18) and (19) fail to hold so that neither firm can treat its optimal contract offered to low-ability agents as independent of the contract offered to high-ability agent and vice-versa. In particular, UIC binds for the MO firm while DIC binds for the S firm. Now, the program of the MO firm is (SMO) subject to UIC binding that is

$$U_{MO}\left(\theta_2\right) = U_{MO}\left(\theta_1\right) + \frac{1}{2}\left(\theta_2 - \theta_1\right) x_{MO}^2\left(\theta_1\right),$$

as in the preceding case, whereas the program of the S firm is (SS) subject to DIC binding, that is

$$U_S\left(\theta_1\right) = U_S\left(\theta_2\right) + \frac{1}{2}\left(\theta_2 - \theta_1\right) x_S^2\left(\theta_2\right).$$

Again, the Proposition that follows highlights the most relevant qualitative features of this equilibrium. We refer the reader to Appendix A.3 for the detailed analysis of the system of first-order conditions that characterize the solution in this case.

Proposition 5 *When neither condition (18) nor (19) are satisfied whereas condition (20) holds, then: (i) the optimal contract of the profit-oriented firm is such that an efficient allocation is reached for high-ability workers, i.e. $x_S\left(\theta_1\right) = x_S^{FB}\left(\theta_1\right)$ whereas low-ability workers' effort is distorted downwards, i.e. $x_S\left(\theta_2\right) < x_S^{FB}\left(\theta_2\right)$, (ii) the optimal contract of the non-profit firm is such that an efficient allocation is reached for low-ability workers, i.e. $x_{MO}\left(\theta_2\right) = x_{MO}^{FB}\left(\theta_2\right)$ whereas high-ability workers' effort is distorted upwards, i.e. $x_{MO}\left(\theta_1\right) > x_{MO}^{FB}\left(\theta_1\right)$, and (iii) effort levels are such that $x_{MO}\left(\theta_2\right) < x_{MO}\left(\theta_1\right) < x_S\left(\theta_2\right) < x_S\left(\theta_1\right)$.*

Proof. See Appendix **??**. ■

The so-called "separation property" asserts that competition would only lead to a change in the optimal compensation schemes while optimal allocations would be the same as in the absence of competition. We show that the separation property fails in our context. In particular, competition between principals allows to reach a more efficient allocation with respect to the absence of competition. In particular, when the difference in ability levels is high relative to the technological gap for the mission-oriented firm then competition between firms completely restores efficiency.

5 Concluding remarks

To be written

A Appendix

A.1 Proof of Lemma 1

Let us go back to the equilibrium utilities (15) of the benchmark case and let us rewrite them as follows

$$\begin{array}{rclcl} U_{MO}^{*}(\theta_j) = & \frac{k_{MO}^2}{2\theta_j} - \frac{1}{6}\left(4 - \frac{\left(k_S^2 - k_{MO}^2\right)}{\theta_j}\right) & = & S_{MO}\left(x_{MO}^{FB}(\theta_j), \theta_j\right) - \frac{1}{6}\left(4 - \left(R_S\left(x_S^{FB}(\theta_j)\right) - R_{MO}\left(x_{MO}^{FB}(\theta_j)\right)\right)\right) \\ U_S^{*}(\theta_j) = & \frac{k_S^2}{2\theta_j} - \frac{1}{6}\left(2 + \frac{\left(k_S^2 - k_{MO}^2\right)}{\theta_j}\right) & = & S_S\left(x_S^{FB}(\theta_j), \theta_j\right) - \frac{1}{6}\left(2 + \left(R_S\left(x_S^{FB}(\theta_j)\right) - R_{MO}\left(x_{MO}^{FB}(\theta_j)\right)\right)\right) \end{array}, \tag{21}$$

with $\theta_j \in \{\theta_1, \theta_2\}$. Thus, we can interpret $U_i^{*}(\theta_j)$ as the equilibrium indirect utility that each principal leaves to each type θ_j agent as the total surplus attained when equilibrium effort levels correspond to the first-best ones (this is the term $S_i\left(x_i^{FB}(\theta_j), \theta_j\right) = \frac{k_i^2}{2\theta_j}$) less an amount by which each firm shades his offer below total surplus, which depends on the difference between firm's revenues, again evaluated at first-best effort levels, $R_S\left(x_S^{FB}(\theta_j)\right) - R_{MO}\left(x_{MO}^{FB}(\theta_j)\right)$.

Furthermore, Let us define the function $\phi_i^{FB}(\theta_1, \theta_2)$ as the difference between the utility that type θ_1 receives from firm i when revealing her true type and the utility that type θ_1 would receive from the same firm i when claiming that her type is θ_2, if exerting the first-best levels of effort and receiving a compensation as in the benchmark case. Thus, if $\phi_i^{FB}(\theta_1, \theta_2) > 0$, then the DIC is slack for firm i at the benchmark allocation because type θ_1 is not attracted by the contract that firm i offers to type θ_2, conditional on firm i requiring all agents to exert first-best effort levels and giving compensation schemes as in the benchmark case. For the MO firm

$$\begin{aligned} \phi_{MO}^{FB}(\theta_1, \theta_2) &= S_{MO}\left(x_{MO}^{FB}(\theta_1), \theta_1\right) - \frac{1}{6}\left(4 - \left(R_S\left(x_S^{FB}(\theta_1)\right) - R_{MO}\left(x_{MO}^{FB}(\theta_1)\right)\right)\right) + \\ &\quad - S_{MO}\left(x_{MO}^{FB}(\theta_2), \theta_1\right) - \frac{1}{6}\left(4 - \left(R_S\left(x_S^{FB}(\theta_2)\right) - R_{MO}\left(x_{MO}^{FB}(\theta_2)\right)\right)\right) \end{aligned}$$

whereas for the S firm

$$\begin{aligned} \phi_S^{FB}(\theta_1, \theta_2) &= S_S\left(x_S^{FB}(\theta_1), \theta_1\right) - \frac{1}{6}\left(2 + \left(R_S\left(x_S^{FB}(\theta_1)\right) - R_{MO}\left(x_{MO}^{FB}(\theta_1)\right)\right)\right) \\ &\quad - S_S\left(x_S^{FB}(\theta_2), \theta_1\right) - \frac{1}{6}\left(2 + \left(R_S\left(x_S^{FB}(\theta_2)\right) - R_{MO}\left(x_{MO}^{FB}(\theta_2)\right)\right)\right). \end{aligned}$$

Notice that, in the above expressions, the consequence of type θ_1 mimicking type θ_2 is visible in the term $S_i\left(x_i^{FB}(\theta_2), \theta_1\right)$, where θ_1 directly affects the surplus through the cost of effort. All other effects are mediated by type θ_1 choosing effort $x_i^{FB}(\theta_2)$ instead of effort $x_i^{FB}(\theta_1)$.

Likewise, one can obtain functions $\phi_i^{FB}(\theta_2, \theta_1)$ for both principals, reverting the roles of the ability types. When $\phi_i^{FB}(\theta_2, \theta_1) > 0$, the UIC is slack for principal i, again conditional on firm i requiring all agents to exert first-best effort levels and giving compensation schemes as in the benchmark case.

Let us rewrite functions ϕ_i^{FB} extensively. For the MO firm, one has

$$\phi_{MO}^{FB}(\theta_1, \theta_2) = \frac{k_{MO}^2}{2\theta_1} - \frac{1}{3}\left(2 - \frac{\left(k_S^2 - k_{MO}^2\right)}{2\theta_1}\right) - \left(\frac{k_{MO}^2}{\theta_2} - \frac{1}{2}\theta_1\frac{k_{MO}^2}{\theta_2^2} - \frac{1}{3}\left(2 - \frac{\left(k_S^2 - k_{MO}^2\right)}{2\theta_2}\right)\right)$$

and

$$\phi_{MO}^{FB}(\theta_2,\theta_1) = \quad \tfrac{k_{MO}^2}{2\theta_2} - \tfrac{1}{3}\left(2 - \tfrac{\left(k_S^2-k_{MO}^2\right)}{2\theta_2}\right) - \left(\tfrac{k_{MO}^2}{\theta_1} - \tfrac{1}{2}\theta_2\tfrac{k_{MO}^2}{\theta_1^2} - \tfrac{1}{3}\left(2 - \tfrac{\left(k_S^2-k_{MO}^2\right)}{2\theta_1}\right)\right) \ .$$

Rearranging terms, one has

$$\phi_{MO}^{FB}(\theta_1,\theta_2) = \quad \tfrac{\left(2k_{MO}^2\theta_2 - 3k_{MO}^2\theta_1 + k_S^2\theta_2\right)(\theta_2-\theta_1)}{6\theta_2^2\theta_1} \ ,$$

where $\phi_{MO}^{FB}(\theta_1,\theta_2) > 0$ always holds, showing that DIC is not relevant for firm MO, whereas

$$\phi_{MO}^{FB}(\theta_2,\theta_1) = \quad \tfrac{\left(3k_{MO}^2\theta_2 - 2k_{MO}^2\theta_1 - k_S^2\theta_1\right)(\theta_2-\theta_1)}{6\theta_2\theta_1^2}$$

with $\phi_{MO}^{FB}(\theta_2,\theta_1) > 0$ if and only if

$$\frac{k_S^2 - k_{MO}^2}{3k_{MO}^2} < \frac{\theta_2-\theta_1}{\theta_1},$$

which corresponds to condition (18) in the main text. This shows that UIC is not a problem for firm MO if $k_S < k_{MO}$ or if $k_S > k_{MO}$ and $k_S - k_{MO}$ is sufficiently low. Instead, UIC becomes relevant for firm MO when $k_S > k_{MO}$ and $k_S - k_{MO}$ is sufficiently high.

Considering the S firm, one has

$$\begin{aligned}\phi_S^{FB}(\theta_2,\theta_1) &= \left(\tfrac{k_S^2}{2\theta_2} - \tfrac{1}{3}\left(1 + \tfrac{\left(k_S^2-k_{MO}^2\right)}{2\theta_2}\right)\right) - \left(\tfrac{k_S^2}{\theta_1} - \tfrac{1}{2}\theta_2\tfrac{k_S^2}{\theta_1^2} - \tfrac{1}{3}\left(1 + \tfrac{\left(k_S^2-k_{MO}^2\right)}{2\theta_1}\right)\right)\\ &= \tfrac{\left(3k_{MO}^2\theta_2 - 2k_{MO}^2\theta_1 - k_S^2\theta_1\right)(\theta_2-\theta_1)}{6\theta_2\theta_1^2}\end{aligned}$$

with $\phi_S^{FB}(\theta_2,\theta_1) > 0$ that always holds, showing that UIC is not relevant for firm S, and

$$\begin{aligned}\phi_S^{FB}(\theta_1,\theta_2) &= \left(\tfrac{k_S^2}{2\theta_1} - \tfrac{1}{3}\left(1 + \tfrac{\left(k_S^2-k_{MO}^2\right)}{2\theta_1}\right)\right) - \left(\tfrac{k_S^2}{\theta_2} - \tfrac{1}{2}\theta_1\tfrac{k_S^2}{\theta_2^2} - \tfrac{1}{3}\left(1 + \tfrac{\left(k_S^2-k_{MO}^2\right)}{2\theta_2}\right)\right)\\ &= \tfrac{\left(2k_S^2\theta_2 - 3k_S^2\theta_1 + k_{MO}^2\theta_2\right)(\theta_2-\theta_1)}{6\theta_2^2\theta_1}\end{aligned}$$

with $\phi_S^{FB}(\theta_1,\theta_2) > 0$ that holds if and only if

$$\frac{k_S^2 - k_{MO}^2}{2k_S^2 + k_{MO}^2} < \frac{\theta_2-\theta_1}{\theta_1}. \tag{22}$$

Thus, the latter inequality implies that DIC is binding for S when $k_S > k_{MO}$ and $k_S - k_{MO}$ sufficiently high.

Since, **for** $k_S \gtrless k_{MO}$,

$$\frac{k_S^2 - k_{MO}^2}{2k_S^2 + k_{MO}^2} < \frac{k_S^2 - k_{MO}^2}{3k_{MO}^2}$$

is true, then the firms' problems are independent across skill levels when condition (18) holds, whereas only firm S's problem has independence across skills levels, whereas MO's UIC is binding, when

$$\frac{k_S^2 - k_{MO}^2}{2k_S^2 + k_{MO}^2} < \frac{\theta_2-\theta_1}{\theta_1} \leq \frac{k_S^2 - k_{MO}^2}{3k_{MO}^2},$$

which corresponds to condition (19) in the main text.

Finally, when $\frac{k_S^2-k_{MO}^2}{2k_S^2+k_{MO}^2} > \frac{\theta_2-\theta_1}{\theta_1}$, then MO's UIC is binding together with S's DIC.

We are then able to provide the sufficient conditions guaranteeing that both incentive constraints are slack for both firms. Indeed, both $\phi_{MO}^{FB}(\theta_1,\theta_2) > 0$ and $\phi_S^{FB}(\theta_2,\theta_1) > 0$ are always satisfied, whereby DIC cannot be binding for the MO firm and UIC cannot be binding for the S firm. If also $\phi_{MO}^{FB}(\theta_2,\theta_1) > 0$ and $\phi_S^{FB}(\theta_1,\theta_2) > 0$ are satisfied, then for both firms $i = S, MO$ all incentive constraints are slack and effort levels are set at the first-best $x_i(\theta) = x_i^{FB}(\theta)$, with compensation schemes $w_i(x_i(\theta))$ being as in (17).

A.2 Optimal contracts when UIC binds for the MO firm

Consider the MO firm and assume that UIC is binding while DIC is slack. Its program is (SMO) and the Lagrangian associated with it is

$$\mathcal{L}_{MO} = E(\Pi_{MO}) + \lambda_{MO}^U\left(U_{MO}(\theta_2) - U_{MO}(\theta_1) + \frac{1}{2}(\theta_2-\theta_1)x_{MO}^2(\theta_1)\right)$$

with $\lambda_{MO}^U > 0$ being the Lagrange multiplier associated with UIC and

$$\begin{aligned} E(\Pi_{MO}) &= \alpha\nu\left(k_{MO}x_{MO}(\theta_1) - \frac{1}{2}\theta_1 x_{MO}^2(\theta_1) - U_{MO}(\theta_1)\right)(1-(U_S(\theta_1)-U_{MO}(\theta_1))) \\ &\quad +\alpha(1-\nu)\left(k_{MO}x_{MO}(\theta_2) - \frac{1}{2}\theta_2 x_{MO}^2(\theta_2) - U_{MO}(\theta_2)\right)(1-(U_S(\theta_2)-U_{MO}(\theta_2))) \end{aligned}$$

The first-order conditions with respect to effort levels are

$$\begin{aligned} \tfrac{\partial\mathcal{L}_{MO}}{\partial x_{MO}(\theta_1)} &= \alpha\nu(k_{MO}-\theta_1 x_{MO}(\theta_1))(1-(U_S(\theta_1)-U_{MO}(\theta_1))) + \lambda_{MO}^U(\theta_2-\theta_1)x_{MO}(\theta_1) = 0 \\ \tfrac{\partial\mathcal{L}_{MO}}{\partial x_{MO}(\theta_2)} &= \alpha(1-\nu)(k_{MO}-\theta_2 x_{MO}(\theta_2))(1-(U_S(\theta_2)-U_{MO}(\theta_2))) = 0 \end{aligned}$$

where, from the second line, one gets that the first-best effort level is required for low-ability types and $x_{MO}(\theta_2) = x_{MO}^{FB}(\theta_2)$, whereas from the first line one has that $k_{MO} - \theta_1 x_{MO}(\theta_1) < 0$ whereby

$$x_{MO}(\theta_1) > \frac{k_{MO}}{\theta_1} = x_{MO}^{FB}(\theta_1).$$

In particular,

$$x_{MO}(\theta_1) = \frac{\alpha\nu k_{MO}(1-(U_S(\theta_1)-U_{MO}(\theta_1)))}{\alpha\nu\theta_1(1-(U_S(\theta_1)-U_{MO}(\theta_1))) - \lambda_{MO}^U(\theta_2-\theta_1)}. \tag{24}$$

The first-order conditions with respect to utilities are

$$\tfrac{\partial\mathcal{L}_{MO}}{\partial U_{MO}(\theta_1)} = -\alpha\nu(1-(U_S(\theta_1)-U_{MO}(\theta_1))) + \alpha\nu\left(k_{MO}x_{MO}(\theta_1) - \tfrac{1}{2}\theta_1 x_{MO}^2(\theta_1) - U_{MO}(\theta_1)\right) - \lambda_{MO}^U = 0 \qquad (25$$

$$\tfrac{\partial\mathcal{L}_{MO}}{\partial U_{MO}(\theta_2)} = -\alpha(1-\nu)(1-(U_S(\theta_2)-U_{MO}(\theta_2))) + \alpha(1-\nu)\left(k_{MO}x_{MO}(\theta_2) - \tfrac{1}{2}\theta_2 x_{MO}^2(\theta_2) - U_{MO}(\theta_2)\right) + \lambda_{MO}^U = 0 \qquad (25$$

Substituting $x_{MO}^{FB}(\theta_2)$ into (2) yields

$$\lambda_{MO}^U = \alpha(1-\nu)\left((1-(U_S(\theta_2)-U_{MO}(\theta_2))) - \left(\frac{k_{MO}^2}{2\theta_2} - U_{MO}(\theta_2)\right)\right), \tag{26}$$

whereby, because $\lambda_{MO}^{U} > 0$,

$$(1 - (U_S(\theta_2) - U_{MO}(\theta_2))) > \left(\frac{k_{MO}^2}{2\theta_2} - U_{MO}(\theta_2)\right). \tag{27}$$

Let

$$x_{MO}^{CI}(\theta_1) = \frac{\nu k_{MO}}{\nu\theta_1 - (1-\nu)(\theta_2 - \theta_1)}$$

be the optimal allocation for high-ability workers that solves program (SMO) when UIC is binding and when outside options are exogenous and the participation constraint of the high-ability type is binding. Note that the superindex CI stands for *countervailing incentives* and that $x_{MO}^{CI}(\theta_1) > x_{MO}^{FB}(\theta_1)$ whenever $x_{MO}^{CI}(\theta_1) > 0$ that is for $\frac{\nu}{(1-\nu)} > \frac{(\theta_2 - \theta_1)}{\theta_1}$. Then

$$x_{MO}(\theta_1) < x_{MO}^{CI}(\theta_1)$$

holds if and only if

$$\alpha(1-\nu)(1 - (U_S(\theta_1) - U_{MO}(\theta_1))) > \lambda_{MO}^{U}$$

or else, taking (26) into account, if and only if the following inequality is satisfied

$$(1 - (U_S(\theta_1) - U_{MO}(\theta_1))) > (1 - (U_S(\theta_2) - U_{MO}(\theta_2))) - \left(\frac{k_{MO}^2}{2\theta_2} - U_{MO}(\theta_2)\right),$$

which, in the case of adverse selection of ability for the MO firm, can be rewritten as

$$\left(\frac{k_{MO}^2}{2\theta_2} - U_{MO}(\theta_2)\right) > (U_S(\theta_1) - U_{MO}(\theta_1)) - (U_S(\theta_2) - U_{MO}(\theta_2)) > 0. \tag{28}$$

Consider now the problem of the S firm. It is the same as in the benchmark case, therefore firm S solves

$$\begin{aligned} \max_{x_S, U_S} E(\Pi_S) = \;& \nu\left(k_S x_S(\theta_1) - \tfrac{1}{2}\theta_1 x_S^2(\theta_1) - U_S(\theta_1)\right)(U_S(\theta_1) - U_{MO}(\theta_1)) \\ & + (1-\nu)\left(k_S x_S(\theta_2) - U_S(\theta_2) - \tfrac{1}{2}\theta_2 x_S^2(\theta_2)\right)(U_S(\theta_2) - U_{MO}(\theta_2)) \end{aligned}$$

under no additional constraints, whereby the system of first-order conditions to this problem is

$$\begin{aligned} \tfrac{\partial E(\Pi_S)}{\partial x_S(\theta_1)} &= \nu(k_S - \theta_1 x_S(\theta_1))(U_S(\theta_1) - U_{MO}(\theta_1)) = 0 \\ \tfrac{\partial E(\Pi_S)}{\partial x_S(\theta_2)} &= (1-\nu)(k_S - \theta_2 x_S(\theta_2))(U_S(\theta_2) - U_{MO}(\theta_2)) = 0 \\ \tfrac{\partial E(\Pi_S)}{\partial U_S(\theta_1)} &= -\nu(U_S(\theta_1) - U_{MO}(\theta_1)) + \nu\left(k_S x_S(\theta_1) - \tfrac{1}{2}\theta_1 x_S^2(\theta_1) - U_S(\theta_1)\right) = 0 \\ \tfrac{\partial E(\Pi_S)}{\partial U_S(\theta_2)} &= -(1-\nu)(U_S(\theta_2) - U_{MO}(\theta_2)) + (1-\nu)\left(k_S x_S(\theta_2) - U_S(\theta_2) - \tfrac{1}{2}\theta_2 x_S^2(\theta_2)\right) = 0 \end{aligned}$$

The first two conditions yield first-best effort levels, whereby $x_S^*(\theta) = \frac{k_S}{\theta} = x_S^{FB}(\theta)$ for all $\theta \in \{\theta_1, \theta_2\}$. The last two conditions can be rewritten substituting for optimal effort levels in order to obtain

$$U_S(\theta_1) = \tfrac{1}{2}\left(\tfrac{k_S^2}{2\theta_1} + U_{MO}(\theta_1)\right) \quad \text{and} \quad U_S(\theta_2) = \tfrac{1}{2}\left(\tfrac{k_S^2}{2\theta_2} + U_{MO}(\theta_2)\right). \tag{29}$$

Notice that, combining the binding UIC for the MO firm with the negative selection of ability for the MO firm, one gets

$$\frac{1}{2}\left(\theta_2-\theta_1\right)x_{MO}^2\left(\theta_1\right)=U_{MO}\left(\theta_1\right)-U_{MO}\left(\theta_2\right)<U_S\left(\theta_1\right)-U_S\left(\theta_2\right).$$

Using (29), one gets

$$x_{MO}\left(\theta_1\right)<\frac{k_S}{\sqrt{\theta_1\theta_2}}$$

whereby,the following chain of inequality holds

$$x_{MO}\left(\theta_2\right)=x_{MO}^{FB}\left(\theta_2\right)<x_{MO}^{FB}\left(\theta_1\right)<x_{MO}\left(\theta_1\right)<\frac{k_S}{\sqrt{\theta_1\theta_2}}<\frac{k_S}{\theta_1}=x_S^{FB}\left(\theta_1\right) \tag{30}$$

Substituting for conditions (29) and (26) into the remaining equations yields a system of two equations in two unknowns $x_{MO}\left(\theta_1\right)$ and $U_{MO}\left(\theta_2\right)$ which is the following

$$\begin{aligned}
&\nu\left(k_{MO}-\theta_1 x_{MO}\left(\theta_1\right)\right)\left(1-\tfrac{k_S^2}{4\theta_1}+\tfrac{1}{2}U_{MO}\left(\theta_2\right)+\tfrac{1}{4}\left(\theta_2-\theta_1\right)x_{MO}^2\left(\theta_1\right)\right)\\
&+\left(1-\nu\right)\left(\theta_2-\theta_1\right)x_{MO}\left(\theta_1\right)\left(1-\tfrac{k_S^2}{4\theta_2}+\tfrac{3}{2}U_{MO}\left(\theta_2\right)-\tfrac{k_{MO}^2}{2\theta_2}\right)=0\\
&-\nu\left(1-\tfrac{k_S^2}{4\theta_1}+\tfrac{1}{2}U_{MO}\left(\theta_2\right)+\tfrac{1}{4}\left(\theta_2-\theta_1\right)x_{MO}^2\left(\theta_1\right)\right)+\nu\left(k_{MO}x_{MO}\left(\theta_1\right)-U_{MO}\left(\theta_2\right)-\tfrac{1}{2}\theta_2 x_{MO}^2\left(\theta_1\right)\right)\\
&-\left(1-\nu\right)\left(1-\tfrac{k_S^2}{4\theta_2}+\tfrac{3}{2}U_{MO}\left(\theta_2\right)-\tfrac{k_{MO}^2}{2\theta_2}\right)=0
\end{aligned}$$

As an example, consider the uniform distribution of abilities, whereby $\nu=\frac{1}{2}$, let $k_S=2$ and $k_{MO}=1$ and assume that $\theta_2=\frac{3}{2}$. Then condition (19) is satisfied and the solution is such that, for firm MO

$$x_{MO}\left(\theta_1\right)=1.089>x_{MO}^{FB}\left(\theta_1\right)=1 \text{ and } x_{MO}\left(\theta_2\right)=x_{MO}^{FB}\left(\theta_2\right)=\frac{2}{3}$$

moreover

$$U_{MO}\left(\theta_2\right)=0.017094 \text{ and } U_{MO}\left(\theta_1\right)=0.313\,57.$$

For firm S instead

$$x_S\left(\theta_1\right)=x_S^{FB}\left(\theta_1\right)=2 \text{ and } x_S\left(\theta_2\right)=x_S^{FB}\left(\theta_2\right)=\frac{4}{3}$$

with

$$U_S\left(\theta_2\right)=0.67521 \text{ and } U_S\left(\theta_1\right)=1.1568.$$

Then, the probability of high-ability workers accepting employment at the MO firm is $1-\left(U_S\left(\theta_1\right)-U_{MO}\left(\theta_1\right)\right)=1-\left(1.1568-0.31357\right)=0.15677$ which is smaller than the probability of low-ability workers accepting employment at the MO firm $1-\left(U_S\left(\theta_2\right)-U_{MO}\left(\theta_2\right)\right)=1-\left(0.67521-1.7094\times10^{-2}\right)=0.341\,88$, in line with adverse selection of ability for the MO firm. Finally wages paid by the MO firm are $w_{MO}\left(\theta_1\right)=0.906\,53$ and $w_{MO}\left(\theta_2\right)=0.350\,43$ whereas wage paid by the S firm are given by $w_S\left(\theta_1\right)=3.1568$ and $w_S\left(\theta_2\right)=2.0085$ with $w_i\left(\theta_1\right)>w_i\left(\theta_2\right)$ for $i=MO,S$ but also $w_S\left(\theta_1\right)-w_S\left(\theta_2\right)>w_{MO}\left(\theta_1\right)-w_{MO}\left(\theta_2\right).$

Finally, notice that Assumption 1 in the main text is needed because the difference in ability must be sufficiently low that

$$2\theta_1>\theta_2,$$

otherwise profits for firm MO from type θ_1 are negative. Indeed, consider

$$\pi_{MO}(\theta_1) = \left(k_{MO} x_{MO}(\theta_1) - \frac{1}{2}\theta_1 x_{MO}^2(\theta_1) - U_{MO}(\theta_1)\right)$$

and substitute for

$$U_{MO}(\theta_1) = U_{MO}(\theta_2) + \frac{1}{2}(\theta_2 - \theta_1) x_{MO}^2(\theta_1)$$

from the binding UIC. This yields to

$$\pi_{MO}(\theta_1) = \left(k_{MO} x_{MO}(\theta_1) - \frac{1}{2}\theta_2 x_{MO}^2(\theta_1) - U_{MO}(\theta_2)\right).$$

Since profits are decreasing in $x_{MO}(\theta_1)$ and since $x_{MO}(\theta_1) > x_{MO}^{FB}(\theta_1)$ it is true that

$$\pi_{MO}(\theta_1) < \left(k_{MO} x_{MO}^{FB}(\theta_1) - \frac{1}{2}\theta_2 x_{MO}^{FB}(\theta_1)^2 - U_{MO}(\theta_2)\right) = -\frac{(\theta_2 - 2\theta_1) k_{MO}^2}{2\theta_1^2} - U_{MO}(\theta_2)$$

The right-most term is strictly negative when $\theta_2 \geq 2\theta_1$ and hence a necessary condition for principal MO to make non-negative profits on the θ_1 type is that

$$\theta_2 < 2\theta_1.$$

The same observation holds for the Case that follows.

A.3 Optimal contracts when UIC binds for the MO firm and DIC binds for the S firm

For the MO firm, UIC is binding while DIC is slack. Its program (SMO), the Lagrangian associated with it and the first-order conditions are the same as in the preceding case.

Consider now the problem (SS) of the S firm under the constraint that DIC binds. Then firm S solves

$$\begin{aligned} \max_{x_S, U_S} x(\Pi_S) = \quad & \nu\left(k_S x_S(\theta_1) - \tfrac{1}{2}\theta_1 x_S^2(\theta_1) - U_S(\theta_1)\right)(U_S(\theta_1) - U_{MO}(\theta_1)) \\ & + (1-\nu)\left(k_S x_S(\theta_2) - U_S(\theta_2) - \tfrac{1}{2}\theta_2 x_S^2(\theta_2)\right)(U_S(\theta_2) - U_{MO}(\theta_2)) \end{aligned}$$

subject to

$$U_S(\theta_1) = U_S(\theta_2) + \frac{1}{2}(\theta_2 - \theta_1) x_S^2(\theta_2).$$

The Lagrangian associated with this problem is

$$\mathcal{L}_S = x(\Pi_S) + \lambda_S^D \left(U_S(\theta_1) - U_S(\theta_2) - \frac{1}{2}(\theta_2 - \theta_1) x_S^2(\theta_2)\right)$$

with associated first-order conditions

$$\begin{aligned} \tfrac{\partial \mathcal{L}}{\partial x_S(\theta_1)} &= \nu(k_S - \theta_1 x_S(\theta_1))(U_S(\theta_1) - U_{MO}(\theta_1)) = 0 \\ \tfrac{\partial \mathcal{L}}{\partial x_S(\theta_2)} &= (1-\nu)(k_S - \theta_2 x_S(\theta_2))(U_S(\theta_2) - U_{MO}(\theta_2)) - \lambda_S^D(\theta_2 - \theta_1) x_S(\theta_2) = 0 \\ \tfrac{\partial \mathcal{L}}{\partial U_S(\theta_1)} &= -\nu(U_S(\theta_1) - U_{MO}(\theta_1)) + \nu\left(k_S x_S(\theta_1) - \tfrac{1}{2}\theta_1 x_{MO}^2(\theta_1) - U_S(\theta_1)\right) + \lambda_S^D = 0 \\ \tfrac{\partial \mathcal{L}}{\partial U_S(\theta_2)} &= -(1-\nu)(U_S(\theta_2) - U_{MO}(\theta_2)) + (1-\nu)\left(k_S x_S(\theta_2) - \tfrac{1}{2}\theta_2 x_S^2(\theta_2) - U_S(\theta_2)\right) - \lambda_S^D = 0 \end{aligned}$$

Form the first two conditions one gets $x_S(\theta_1) = \frac{k_S}{\theta_1} = x_S^{FB}(\theta_1)$ and $x_S(\theta_2) < x_S^{FB}(\theta_2)$. In particular, one could write

$$x_S(\theta_2) = \frac{(1-\nu)(U_S(\theta_2) - U_{MO}(\theta_2))\, k_S}{(1-\nu)\,\theta_2\,(U_S(\theta_2) - U_{MO}(\theta_2)) + \lambda_S^D(\theta_2 - \theta_1)}.$$

From the third condition, substituting for $x_S^{FB}(\theta_1)$ and solving for the Lagrange multiplier yields

$$\lambda_S^D = \nu\left((U_S(\theta_1) - U_{MO}(\theta_1)) - \left(\frac{k_S^2}{2\theta_1} - U_S(\theta_1)\right)\right),$$

where it must be the case that

$$(U_S(\theta_1) - U_{MO}(\theta_1)) > \left(\frac{k_S^2}{2\theta_1} - U_S(\theta_1)\right)$$

because $\lambda_S^D > 0$. Finally, substituting λ_S^D into the expression for $x_S(\theta_2)$ one obtains

$$x_S(\theta_2) = \frac{(1-\nu)(U_S(\theta_2)-U_{MO}(\theta_2))k_S}{\left((1-\nu)\theta_2(U_S(\theta_2)-U_{MO}(\theta_2))+\nu(\theta_2-\theta_1)\left((U_S(\theta_1)-U_{MO}(\theta_1))-\left(\frac{k_S^2}{2\theta_1}-U_S(\theta_1)\right)\right)\right)}. \tag{31}$$

Let

$$x_S^{SB}(\theta_2) = \frac{(1-\nu)\,k_S}{(1-\nu)\,\theta_2 + \nu\,(\theta_2 - \theta_1)}$$

be the optimal allocation for low-ability workers that solves program (SS) when DIC is binding and when outside options are exogenous and the participation constraint of the low-ability type is binding. Note that the superindex SB stands for *second best* and that $x_S^{SB}(\theta_2) < x_S^{FB}(\theta_2)$. Then

$$x_S(\theta_2) > x_S^{SB}(\theta_2)$$

if and only if

$$\left(\frac{k_S^2}{2\theta_1} - U_S(\theta_1)\right) > (U_S(\theta_1) - U_{MO}(\theta_1)) - (U_S(\theta_2) - U_{MO}(\theta_2)) > 0$$

where the right-most inequality comes from the fact that $\widehat{\gamma}(\theta)$ is decreasing in θ.

Notice that, combining the two binding incentive compatibility constraints, i.e. DIC for the S firm and UIC for the MO firm, and adding negative selection of ability for the MO firm, one gets

$$\frac{1}{2}(\theta_2 - \theta_1)\, x_S^2(\theta_2) = U_S(\theta_1) - U_S(\theta_2) > U_{MO}(\theta_1) - U_{MO}(\theta_2) = \frac{1}{2}(\theta_2 - \theta_1)\, x_{MO}^2(\theta_1)$$

whereby the following chain of inequalities holds with respect to optimal effort levels

$$x_S(\theta_1) = x_S^{FB}(\theta_1) > x_S(\theta_2) > x_{MO}(\theta_1) > x_{MO}(\theta_2) = x_{MO}^{FB}(\theta_2). \tag{32}$$

For the MO firm, the solution solves the same equations as in the preceding Section A.3, whereby $x_{MO}(\theta_1) > x_{MO}^{FB}(\theta_1)$ and $x_{MO}(\theta_2) = x_{MO}^{FB}(\theta_2)$. More precisely, the complete system of equations to be solved in this case thus consists of

$$
\begin{aligned}
&-\nu\left(1-U_S(\theta_2)+U_{MO}(\theta_2)-\tfrac{1}{2}(\theta_2-\theta_1)\left(x_S^2(\theta_2)-x_{MO}^2(\theta_1)\right)\right)(\theta_1 x_{MO}(\theta_1)-k_{MO})\\
&+(1-\nu)\left(1-U_S(\theta_2)+2U_{MO}(\theta_2)-\tfrac{k_{MO}^2}{2\theta_2}\right)(\theta_2-\theta_1)\,x_{MO}(\theta_1)=0\\
&-\nu\left(1-U_S(\theta_2)+U_{MO}(\theta_2)-\tfrac{1}{2}(\theta_2-\theta_1)\left(x_S^2(\theta_2)-x_{MO}^2(\theta_1)\right)\right)\\
&+\nu\left(k_{MO}x_{MO}(\theta_1)-\tfrac{1}{2}\theta_2 x_{MO}^2(\theta_1)-U_{MO}(\theta_2)\right)-(1-\nu)\left(1-U_S(\theta_2)+2U_{MO}(\theta_2)-\tfrac{k_{MO}^2}{2\theta_2}\right)=0\\
&(1-\nu)\left(U_S(\theta_2)-U_{MO}(\theta_2)\right)(k_S-\theta_2 x_S(\theta_2))\\
&-\nu\left((\theta_2-\theta_1)\,x_S^2(\theta_2)+2U_S(\theta_2)-\tfrac{1}{2}(\theta_2-\theta_1)\,x_{MO}^2(\theta_1)-U_{MO}(\theta_2)-\tfrac{k_S^2}{2\theta_1}\right)(\theta_2-\theta_1)\,x_S(\theta_2)=0\\
&-(1-\nu)\left(U_S(\theta_2)-U_{MO}(\theta_2)\right)+(1-\nu)\left(k_S x_S(\theta_2)-\tfrac{1}{2}\theta_2 x_S^2(\theta_2)-U_S(\theta_2)\right)\\
&-\nu\left((\theta_2-\theta_1)\,x_S^2(\theta_2)+2U_S(\theta_2)-\tfrac{1}{2}(\theta_2-\theta_1)\,x_{MO}^2(\theta_1)-U_{MO}(\theta_2)-\tfrac{k_S^2}{2\theta_1}\right)=0
\end{aligned}
$$

where the relevant variables are $x_S^2(\theta_2)$ and $x_{MO}^2(\theta_1)$ on the one hand and $U_S(\theta_2)$ and $U_{MO}(\theta_2)$ on the other hand.

As an example, consider the uniform distribution of abilities, whereby $\nu=\frac{1}{2}$, let $k_S=2$ and $k_{MO}=1$ and assume that $\theta_2=\frac{6}{5}$. Then condition (20) is satisfied and the solution is such that, for firm MO

$$x_{MO}(\theta_1)=1.0932>x_{MO}^{FB}(\theta_1)=1 \text{ and } x_{MO}(\theta_2)=x_{MO}^{FB}(\theta_2)=\frac{5}{6}$$

moreover

$$U_{MO}(\theta_2)=0.18877 \text{ and } U_{MO}(\theta_1)=0.30828.$$

For firm S instead

$$x_S(\theta_1)=x_S^{FB}(\theta_1)=2 \text{ and } x_S(\theta_2)=1.6492<x_S^{FB}(\theta_2)=\frac{5}{3}$$

with

$$U_S(\theta_2)=0.90489 \text{ and } U_S(\theta_1)=1.1769.$$

Then, the probability of high-ability workers accepting employment at the MO firm is $1-(U_S(\theta_1)-U_{MO}(\theta_1))=1-0.86862=0.13138$ which is smaller than the probability of low-ability workers accepting employment at the MO firm $1-(U_S(\theta_2)-U_{MO}(\theta_2))=1-0.71612=0.28388$, in line with adverse selection of ability for the MO firm. Finally wages paid by the MO firm are $w_{MO}(\theta_1)=0.90582$ and $w_{MO}(\theta_2)=0.60544$ whereas wage paid by the S firm are given by $w_S(\theta_1)=3.1769$ and $w_S(\theta_2)=2.5368$ with $w_i(\theta_1)>w_i(\theta_2)$ for $i=MO,S$ but also $w_S(\theta_1)-w_S(\theta_2)>w_{MO}(\theta_1)-w_{MO}(\theta_2)$.

A.4 Proof of Proposition 1

Take expression (3) for the wage rate in the main text

$$w_i(x_i(\theta))=U_i(\theta)+\frac{1}{2}\theta x_i^2(\theta)$$

and assume that some incentive constraint matters for at least firm MO, i.e. assume that condition (18) is violated. It is immediate to check that, for any $\theta \in \{\theta_1, \theta_2\}$, a sufficient condition for $w_S(\theta) > w_{MO}(\theta)$ is that both $U_S(\theta) > U_{MO}(\theta)$ and $x_S(\theta) > x_{MO}(\theta)$ hold, which is precisely the case (see inequalities 30 and 32).

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RETURNS TO ABILITY

Moreover

$$w_S(\theta_1) - w_S(\theta_2) > w_{MO}(\theta_1) - w_{MO}(\theta_2)$$

holds if and only if

$$U_S(\theta_1) - U_S(\theta_2) + \frac{1}{2}\theta_1 x_S^2(\theta_1) - \frac{1}{2}\theta_2 x_S^2(\theta_2) > U_{MO}(\theta_1) - U_{MO}(\theta_2) + \frac{1}{2}\theta_1 x_{MO}^2(\theta_1) - \frac{1}{2}\theta_2 x_{MO}^2(\theta_2).$$

Sufficient conditions are that both $U_S(\theta_1) - U_S(\theta_2) > U_{MO}(\theta_1) - U_{MO}(\theta_2)$ and $\theta_1(x_S(\theta_1) - x_{MO}(\theta_1)) > \theta_2(x_S(\theta_2) - x_{MO}(\theta_2))$ hold. The first inequality is always satisfied and the latter

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